CGN 6933-002
Transport in Porous Media

Spring 2005 — University of South Florida
Homework #4 — Civil & Environmental Eng.
Due Mon., Feb. 21, 2005 — J. A. Cunningham

(1) Consider the transport of a chemical species i through a column of porous medium. The column has cross-sectional area A, porosity n, and volumetric flow rate Q in the $+x$-direction. The porous medium has a grain density ρ_g and a bulk density ρ_b. Chemical species i undergoes sorption and biodegradation as it is transported through the column. The biodegradation reaction follows *Monod kinetics*.

(a) What is the relationship between the volumetric flow rate, Q, and the average pore velocity, v?

(b) What is the relationship between the grain density, ρ_g, and the bulk density, ρ_b?

(c) Look up the expression for Monod kinetics. What is the rate of reaction for the biodegradation of species i? (You may assume that species i is the limiting concentration for the biodegradation reaction, i.e., your Monod kinetic expression does not need to include the concentrations of any species other than i.) Be sure to define all terms in your expression. Cite your reference(s).

(d) Derive the partial differential equation that describes transport of species i through the column. Be sure that your equation accounts for the processes of advection, dispersion, sorption, and biodegradation. Hint: start with a mass balance around a differential volume of the porous medium.

(e) Suppose that the sorption process is very fast compared to the other processes occurring in the column. Further suppose that, at equilibrium, the sorption isotherm for species i is linear. Simplify your expression from part (d).

(f) Now suppose that you know the concentration of species i, C_i, is much less than the half-saturation coefficient (also called a half-velocity concentration) from your Monod kinetic expression. Simplify your expression from part (e).

What you now have is a partial differential equation describing the transport of species i through the porous medium, subject to certain assumptions, limitations, or conditions that were stipulated above.

(2) Consider the one-dimensional transport of a conservative tracer through a porous medium. Suppose that the porous medium is long enough to be approximated as infinitely long, and that the tracer mass is added to the medium as a rapid pulse. Then we could describe the transport of the tracer with the following partial differential equation (PDE):

$$n\,\frac{\partial C(x,t)}{\partial t} = n\,D\,\frac{\partial^2 C(x,t)}{\partial x^2} - n\,v\,\frac{\partial C(x,t)}{\partial x}$$

with the following boundary and initial conditions:

$$C(x \to \pm\infty, t) = 0$$
$$C(x, t=0) = \frac{M}{n\,A}\,\delta(x).$$

The solution to the PDE with these initial and boundary conditions is:

$$C(x,t) = \frac{M}{n\,A}\,\frac{1}{\sqrt{4\,\pi\,D\,t}}\,\exp\left(-\frac{(x-vt)^2}{4\,D\,t}\right).$$

(a) Verify that this solution satisfies the PDE.
(b) Verify that this solution satisfies the boundary conditions.
(c) Argue that the solution satisfies the initial condition. This part is tricky because of the Dirac delta function. If you can't think of a good way to show it mathematically, then try to explain it in words.

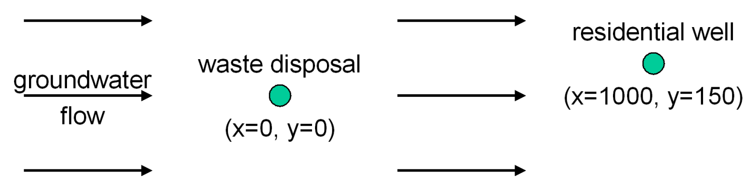
(3) A manufacturing company has a well on their property, which is drilled into a confined aquifer. The well is fully screened across the thickness b of the aquifer. In the past, the well was used for water supply, but now the company doesn't use it for water any more. Instead, when nobody is looking, the company dumps their waste into the well. The problem is, a nearby resident has his drinking-water well drilled into the same aquifer formation. The drinking well is 1000 m downgradient and 150 m off-center. See the figures on the next page.

(a) Suppose that the company drops a 10-kg slug of soluble waste down the well. Assume that the waste dissolves in the water instantaneously – probably not too likely, but we'll suppose it's OK for this problem. Also assume that the waste is distributed evenly over the thickness b of the aquifer – this also is questionable unless the aquifer is thin (i.e., unless b is small), but it helps us by changing the problem from 3-D to 2-D. Graph the resulting concentration history of the waste product in the resident's well. Use the following parameters: $n = 0.3$, $v = 0.2$ m/day, $\alpha_L = 10$ m, $\alpha_T = 2$ m, $b = 10$ m. You may ignore sorption and degradation for this particular waste compound.
(b) Instead of adding a 10-kg slug instantaneously, suppose that the company dumps waste into the well continuously. The rate of waste addition is 100 g/day. Find the steady-state concentration that will result in the resident's well. About how long will it take to reach this steady-state concentration?
(c) If you were the resident, would you be concerned about the company's activities? Why or why not?

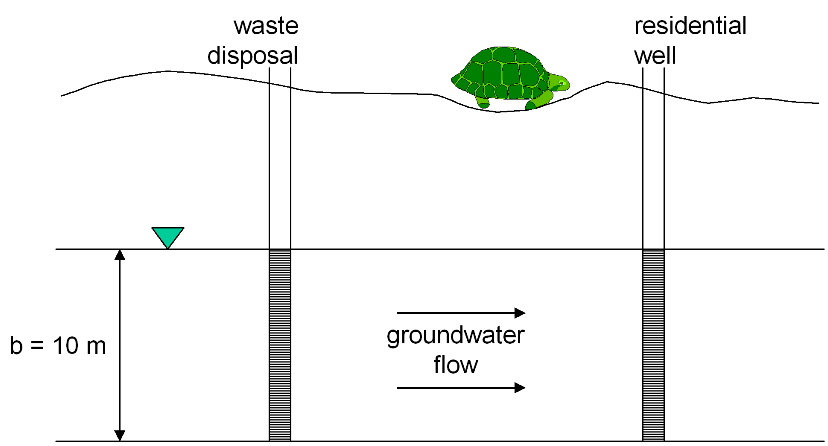
(3) continued

Just for your information, this is a good example of the type of problem you might have to solve as a practicing engineer. If your client were paying you enough, you would probably use some sophisticated modeling tools (like MODFLOW or MODPATH) to perform your analysis. However, if your client can't afford that level of analysis, then this exercise shows how to get a pretty reasonable estimate in just an hour (or two).

Plan View



Side View



(4) About how long (measured in hours) did it take you to complete this homework?