CGN 6933-002
Transport in Porous Media

Spring 2005 | University of South Florida
Homework #9 | Civil & Environmental Eng.
Due Mon., April 25, 2005 | J. A. Cunningham

(1) For the past few weeks, we have been considering the "test problem" of one-dimensional transport of a conservative tracer through a column of porous medium. We have described the transport mathematically as

$$\frac{\partial C(x,t)}{\partial t} = D\,\frac{\partial^2 C(x,t)}{\partial x^2} - v\,\frac{\partial C(x,t)}{\partial x}$$

with the following initial and boundary conditions:

$$C(x, t = 0) = 0$$

$$C(x = 0, t \geq 0) = C_0$$

$$C(x = L, t) = 0$$

where L is the length of the column, and C_0 is an input concentration. We have found that the finite difference method can be used to obtain an approximate solution $C(x,t)$ if the Peclet number is low enough, but that the method doesn't work well at high Peclet number.

(a) Write a *finite element* code to solve this problem, and graph the concentration profiles at $t = 1$ d, $t = 3$ d, and $t = 5$ d. Use the following physical parameters: length $L = 50$ cm; specific discharge $q = 1.5$ cm/d; porosity $n = 0.3$; dispersion coefficient $D = 0.33$ cm²/d; input concentration $C_0 = 100$ mg/L. Use the following numerical parameters: element size $\Delta x = 1$ cm; time step $\Delta t = 0.05$ d. Submit your code and your graph of the concentration profiles.

(b) Note that this is the same problem and the same set of physical parameters that you considered in problem (1b) of homework 7. Compare your profiles from the finite-element code to the profiles you obtained on homework 6. What is different? Is one of the methods (finite differences or finite elements) clearly superior for this problem, or are the profiles about the same?

(2) In class, when we developed the finite element method for this transport problem, we encountered nine integrals that needed to be evaluated to yield the necessary coefficients. You were given the values of these nine integrals without proof or demonstration. Choose any one of the nine integrals, and verify that its value is equal to the value you were given by the instructor. Note: If you choose to work with your classmates on this problem, that is fine, but in that case, please do not choose the same integral as your co-worker(s). You have nine integrals to choose from, so I should not see every person in the class evaluating exactly the same one.

(3) Modify your finite element code from problem (1) to consider the transport of a compound that undergoes *first-order decay*. You may need to do some pencil-and-paper derivations before you modify your code. Check that your code gives you the proper answer when you set the decay coefficient k to zero. Then, use a decay constant $k = 0.16$/d, and graph the concentration profiles at $t = 1$ d, $t = 3$ d, and $t = 5$ d. Use the same physical and numerical parameters as in problem (1), above. Submit your graph and your code. At $t = 5$ d, about how much of the injected mass remains in the column, and how much has decayed? (Hint: compare the profiles obtained in this problem to those from problem (1)).

(4) About how long (measured in hours) did it take you to complete this homework?