CGN 6933-002 Transport in Porous Media

MIDTERM EXAM — University of South Florida

Mon., March 7, 2005 — Spring 2005

60 points total — J. A. Cunningham

Instructions:

(1) You may read these instructions, but do not turn the page or begin working until instructed to do so.

(2) Answer all questions in the exam booklet provided, and write your name conspicuously on the exam booklet.

(3) You are allowed one sheet of 8.5-by-11-inch paper with hand-written notes. You may write on both sides of that paper. However, mechanical reproductions (photocopying, laser printing, scanning, faxes, etc.) are not allowed; all notes must be hand-written. A calculator is recommended, but it may not be pre-programmed with formulae from the class.

(4) Time limit: until the end of class (at least 60 minutes, but no more than 75). Stop working when asked. If you continue working after time has been called, you will be penalized at a rate of 1 point per minute.

(5) Show all work and state all assumptions in order to receive maximum credit for your work.

(6) Make sure your answers include units if appropriate. Watch your units!!

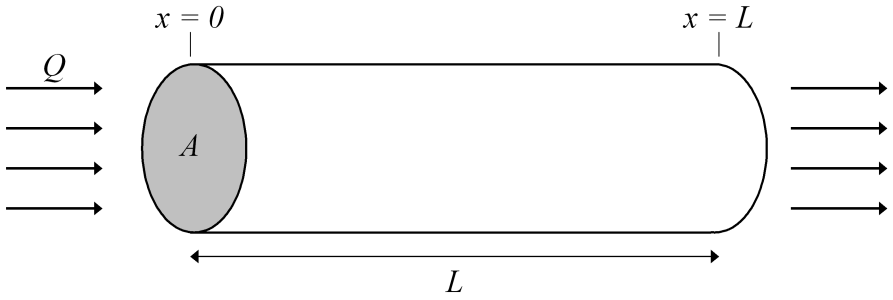
(7) This exam contains 7 questions, plus one bonus question, for a total of 8. The point value of each question is indicated. The total number of points is 60. You will have a little more than an hour to complete the exam. This works out to about one point per minute, so gauge your time accordingly.

(8) You may read all the information on this side of the page, but do not start working on the exam until instructed to do so.

(9) Use a reasonable number of significant digits when reporting your answers. *You are likely to be graded down* if you report an excessive number of significant digits.

(10) Don't cheat. Cheating will result in appropriate disciplinary action according to university policy.

Suppose we have a column filled with a saturated porous medium. The column has a length $L = 50$ cm and a cross-sectional area $A = 30$ cm^2. The porous medium has porosity $n = 0.33$, hydraulic conductivity $K = 10$ cm/day, longitudinal dispersivity $\alpha_L = 0.4$ cm, grain density $\rho_g = 2.54$ g/cm^3, and fraction of organic carbon $f_{OC} = 0.0023$. Water flows through the column at a volumetric flow rate $Q = 100$ cm^3/day. See the figure below.

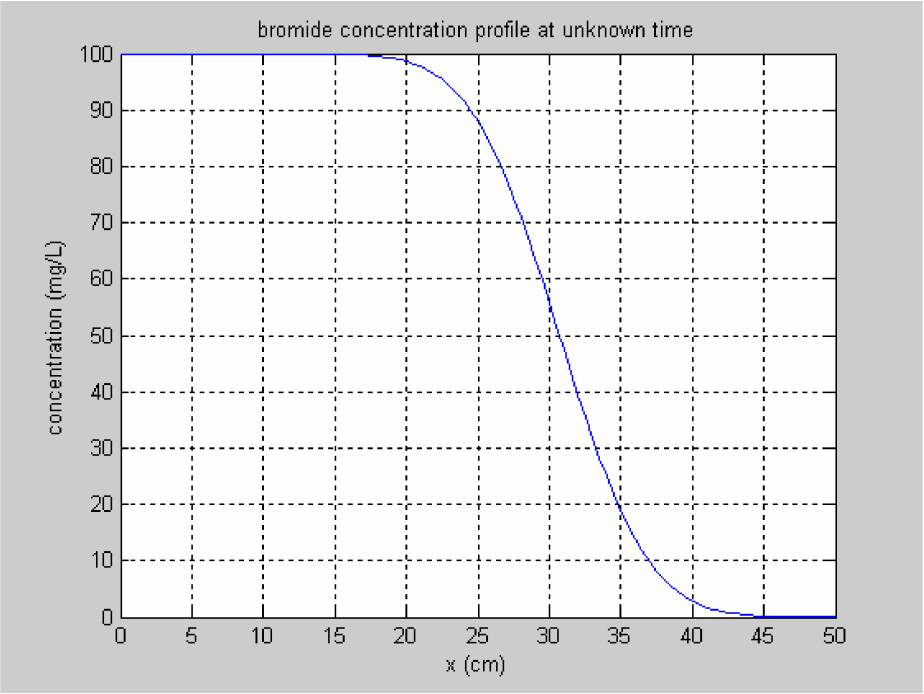


(1) (8 pts) Estimate/calculate the head loss Δh across the column. (If you are not familiar with the term, "head loss" means the difference in hydraulic head between water entering the column and water exiting the column.)

(2) (6 pts) At time $t = 0$, we start adding bromide (a conservative tracer) to the column inlet, with an influent concentration $C_0 = 100$ mg/L. You can consider it a continuous ("step") input that begins at $t = 0$. Write the partial differential equation that describes the transport of bromide through the column. Identify the terms in your equation.

(3) (BONUS QUESTION – NOT REQUIRED – UP TO 3 POINTS) Write an equation for the initial condition that there is no bromide in the column until time $t = 0$. Write the boundary condition for the bromide input at the column influent. Write the boundary condition that says there is no dispersive flux of bromide out of the column at $x = L$.

(4) (7 pts) After a while, the bromide concentration profile in the column looks like this:



At what time t would you expect the concentration profile to look like this? Explain your reasoning and/or show your calculations.

(5) (8 pts) For the concentration profile shown, estimate/calculate the *advective flux* of bromide at $x = 30$ cm. In which direction is this flux (i.e., from right to left, or from left to right)? How do you know which direction? Hint: 1 L = 1000 cm^3.

(6) (9 pts) For the concentration profile shown, estimate/calculate the *dispersive flux* of bromide at $x = 30$ cm. In which direction is this flux (i.e., from right to left, or from left to right)? How do you know which direction? Hint: 1 L = 1000 cm^3.

(7) (12 pts) Suppose that, at time $t = 0$, we started adding trichloroethene (TCE) along with the bromide. TCE undergoes sorption, but not reaction, as it travels through the column. You can assume that the sorption process is fast and that the sorption isotherm is linear. The inlet TCE concentration is 100 μg/L. (In case you don't know, 1 μg/L is 0.001 mg/L.) TCE has an octanol-water partition coefficient $K_{OW} = 10^{2.42} = 263$. Draw the concentration profile of TCE in the column for the same time as the bromide profile shown above. Make your graph as quantitative as you can. Hint: it is possible to do this problem *without* using the analytical solution for $C(x, t)$.

(8) (10 pts)

(a) (2 pts) What is the relationship between hydrodynamic dispersion, mechanical dispersion, and molecular diffusion?

(b) (6 pts) List and *briefly* explain the mechanism(s) that cause(s) *mechanical dispersion* during mass transport through porous media. You can use pictures if it will help your discussion. Hint: don't spend more than 5–10 minutes on this question.

(c) (2 pts) What relationship is commonly observed between mechanical dispersion and fluid velocity, and how do we usually describe this relationship mathematically?

END OF EXAMINATION