ENV 6002: Physical & Chemical Principles of Environmental Engineering

Fall 2021 — University of South Florida
Homework #10 -- **OPTIONAL** — Civil & Environmental Engineering
Due Tues., Nov. 23 — J.A. Cunningham

1. (20 pts) Consider a long stretch of river with the following characteristics.

   Channel depth, $H = 0.5$ m — Channel width, $W = 2.0$ m
   Average velocity, $u = 2.5$ m/min — Longitud. dispersion coefficient, $D = 5.0$ m^2/min

   At time $t = 0$, a pulse of chemical tracer is added very quickly to the river (i.e., as a Dirac pulse) at location $x = 0$. The mass of the pulse is $M = 50$ kg.

   a. On a single graph, plot the tracer's spatial concentration distribution (C vs x) at $t_1 = 500$ min and at $t_2 = 2000$ min. On your graph, make the range of the x-axis go from $x = 0$ to $x = 6000$ m. Print out your graph, label it problem 1a, and submit it as part of your assignment. Hint: use Excel, Matlab, or a similar tool. I think Matlab is easier/better for this assignment than Excel...but only if you know how to use Matlab.

   b. For both t_1 and t_2, determine the peak concentration, C_{max}, and the value of x where it occurs.

   c. Calculate the standard deviation, σ_x, of the spatial distribution (C vs. x) for both t_1 and t_2, and indicate on the graph (i.e., draw in by hand) the x-values corresponding to ± one and two standard deviations from the mean x for both t_1 and t_2. The peaks are relatively narrow so it is a little tricky to draw in the locations of $\pm\sigma_x$ -- you might want to make separate graphs for t_1 and t_2 so that you can clearly draw in these lines. Print out your graphs, label them as problem 1c, and submit them as part of your assignment.

   d. How does the elapsed time affect the peak concentration, C_{max}? How does elapsed time affect the x location of the peak? the spread, $(\Delta x)_{plume}$, i.e., the range of x over which $C > 0.01\ C_{max}$?

2. (20 pts) Consider a long stretch of river with the same characteristics as in problem 1. Again assume that at time $t = 0$, a pulse of chemical tracer is added very quickly to the river at location $x = 0$. Imagine that you are monitoring the concentration of the tracer at the location $x = 5000$ m.

   a. Graph the tracer concentration versus time (C vs t) for the location $x = 5000$ m. Make the time axis go 0 to 3000 min.

   Notice that in problem 1, you graphed concentration profiles. Now in problem 2, you are graphing concentration histories (also called "breakthrough curves").

problem 2 continues →

2. continued

   b. On the same graph, add in the curve that you would expect to see if the 5-km stretch of river were a completely mixed flow reactor. Hint: what is the effluent concentration as a function of time if you add a tracer pulse to a CMFR?

   c. On the same graph, add in the curve that you would expect to see if the 5-km stretch of river were a plug flow reactor. You might have to draw this one in by hand rather than using Excel or Matlab.

   d. Briefly compare the stretch of river to the two ideal reactor types.

3. (20 pts)

   a. Evaluate the validity of assuming plug flow in a stream for a tracer in a segment where: $u_x = 0.33$ m/s; $H = 2$ m; $W = 20$ m; $L = 200$ m; $D_x = 12$ m^2/s; $D_y = D_z = 0.15$ m^2/s. Is longitudinal dispersion negligible? Is transverse mixing complete?

   b. Repeat the assessment for the tracer with $L = 2$ km and with $L = 20$ km. How does our confidence in assuming plug flow change with increasing stream segment length? Explain in terms of the theory of advective/dispersive transport.

   c. To back up your findings from parts (a) and (b), graph the residence-time distribution of the 200-m stretch of river (from part a), and graph the residence-time distribution of the 20-km stretch of river (from part b). In both cases, graph the residence-time distribution in the fully dimensionless form, E_θ versus θ. Recall that $\theta = t/\bar{t}$, and in this problem, $\bar{t} = L/u_x$. Compare the residence-time distributions from parts (a) and (b). Which one looks more like the RTD for a plug-flow reactor?

4. (20 pts) Consider the same stretch of river as in problem 1. However, instead of a non-reacting tracer, suppose that 50 kg of a degradable chemical is added as a sharp infinitesimal (i.e., Dirac) pulse at $t = 0$. Other input parameters are the same as in problem 1. The chemical undergoes first-order degradation with $k = 0.001$ min^{-1}.

   a. Plot the waste's spatial concentration distribution at $t_1 = 500$ min and at $t_2 = 2000$ min. For comparison, include both the tracer and the degradable chemical on this graph (so you will have four pulses on the graph in total). Print out the graph, label it problem 2a, and submit it as part of your assignment. Indicate clearly which curves are for the conservative tracer and which are for the degradable waste.

   b. Compare the degradable waste to the non-reacting tracer: how does degradation affect the *peak concentration* and the *spread of the concentration distribution*?

problem 4 continues →

4. continued
   c. For the degradable chemical, what fraction of the original injected mass remains after a travel time t ? Hint: the mass remaining is given by $m(t) = \int_{-\infty}^{\infty} A\, C(x,t)\, dx$. You can evaluate this integral if you take out the part of $C(x, t)$ that doesn't depend on x.
   d. Compare your answer from part (c) with the result you would get for an ideal CMFR and for an ideal PFR.

5. (20 pts) Given: the same stream segment as in Question 1. A conservative tracer is added, but this time it is added as a finite-duration pulse, over the interval $-5 < t < 5$ min., i.e., $\Delta t_p = 10$ min. The mass added is 50 kg as before.
   a. Plot the tracer's spatial concentration distribution (C vs x) at $t_1 = 500$ min and at $t_2 = 2000$ min. Print out your graph, label it as problem 3c, and submit it as part of your assignment.
   b. Compare the peak height and the spread to those in the infinitesimal scenario, i.e., Question 1a, for $t_1 = 500$ min and $t_2 = 2000$ min. Does the form of the pulse (infinitesimal vs short finite) appear to make a difference once we have reached $t = 500$ min?
   c. Now examine what each pulse (infinitesimal and short finite) looks like at $t = 20$ min. What is the peak concentration of each? For $t = 20$ min, does the form of the pulse appear to make a difference?
   d. Based on your observations from parts b and c, discuss briefly the effect of the pulse form (infinitesimal vs. short finite) on the C vs x distribution, focusing on the effect of elapsed time.

6. *Skip this one in 2021*

   (10 pts) Now consider the same degradable chemical as in Question 2 (degradation rate constant, $k = 0.001$/min), but added as a finite pulse from $t = -5$ to $t = 5$ min. Plot the concentration distribution (C vs x) at times $t_1 = 500$ min and $t_2 = 2000$ min. Print out your graph, label it as problem 4, and submit it as part of your assignment. Compare to your results for the infinitesimal pulse (from problem 2a).