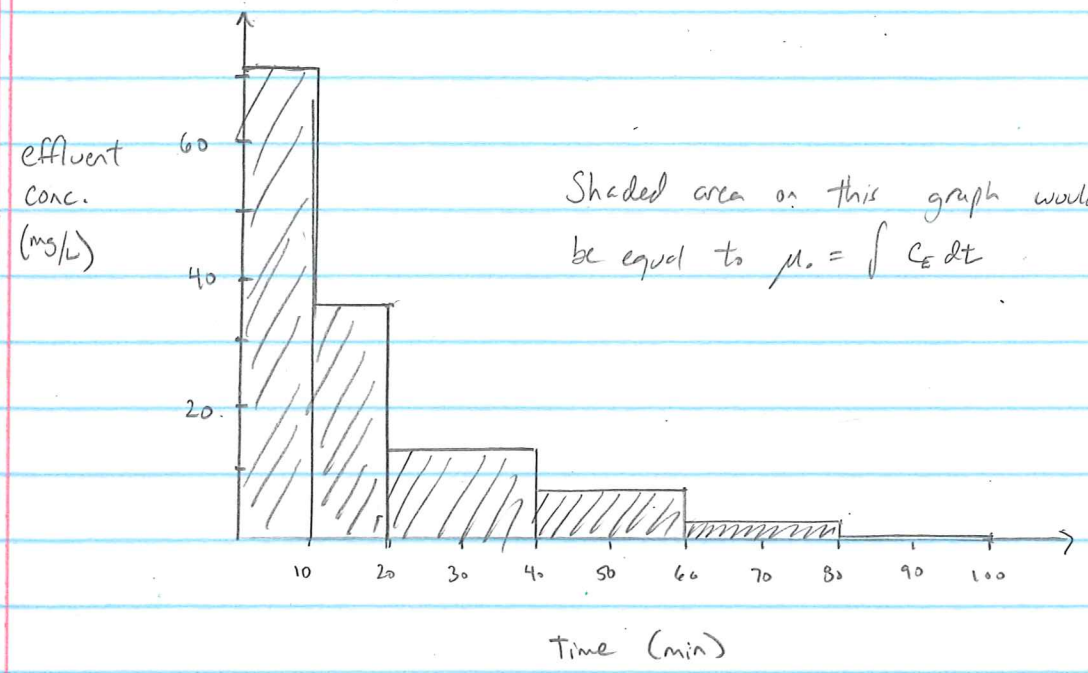
1. Estimate residence time from trace test



Shaded area on this graph would be equal to $\mu_0 = \int C_E \, dt$

$$\tau \approx \bar{t} = \frac{\int_0^\infty t \, C_E(t) \, dt}{\int_0^\infty C_E(t) \, dt} \approx \frac{\sum t \, C_E(t) \, \Delta t}{\sum C_E(t) \, \Delta t}$$

Denominator, $\sum C_E(t) \, \Delta t = \left(71.7 \frac{mg}{L}\right)(10 \text{ min} - 0 \text{ min}) + \left(36.8 \frac{mg}{L}\right)(20 \text{ min} - 10 \text{ min})$
$+ \left(13.5 \frac{mg}{L}\right)(40 \text{ min} - 20 \text{ min}) + \left(8.2 \frac{mg}{L}\right)(60 \text{ min} - 40 \text{ min})$
$+ \left(3.0 \frac{mg}{L}\right)(80 \text{ min} - 20 \text{ min}) + \left(0.2 \frac{mg}{L}\right)(100 \text{ min} - 80 \text{ min})$

$\sum C_E(t) \, \Delta t = 1583 \left(\frac{mg}{L}\right)(\text{min})$

Numerator, $\sum t \, C_E \, \Delta t = (5 \text{ min})\left(71.7 \frac{mg}{L}\right)(10 \text{ min}) + (15 \text{ min})\left(36.8 \frac{mg}{L}\right)(10 \text{ min})$
$+ (30 \text{ min})\left(13.5 \frac{mg}{L}\right)(20 \text{ min}) + (50 \text{ min})\left(8.2 \frac{mg}{L}\right)(20 \text{ min})$
$+ (70 \text{ min})\left(3.0 \frac{mg}{L}\right)(20 \text{ min}) + (90 \text{ min})\left(0.2 \frac{mg}{L}\right)(20 \text{ min})$

$\sum t \, C_E \, \Delta t = 29{,}965 \left(\frac{mg}{L}\right)(\text{min}^2)$

$$\tau \approx \frac{29{,}965 \left(mg/L\right) \text{min}^2}{1583 \left(mg/L\right) \text{min}} = 18.929 \text{ min} \approx \boxed{19 \text{ min}}$$

2. Flocculation

(a) Diameter of flocs

$$\Omega = N_F \frac{\pi}{6} (d_f)^3$$

$$3.0 \times 10^{-5} = \left(1.0 \times 10^{10} \frac{\#}{m^3}\right)\left(\frac{\pi}{6}\right) d^3$$

$$d = 1.7894 \times 10^{-5} \text{ m} \Rightarrow \boxed{d = 1.8 \times 10^{-5} \text{ m} = 18 \mu m}$$

(b) Material balance for concentration of pathogens

Accumulation = Flow in − Flow out + Sources − Sinks

Steady-state ⇒ Accumulation = 0

$$0 = Q N_p^{in} - Q N_p^{out} + 0 - V \cdot r$$

sink term is the flocculation of the pathogens with the flocs

$r = \beta N_F N_p \alpha$ ... second-order process

$$\boxed{0 = Q N_p^{in} - Q N_p^{out} - V \beta N_F N_p \alpha}$$

CMFR ⇒ $N_p = N_p^{out}$.

Divide entire equation through by Q. $\frac{V}{Q} = \tau$.

$$0 = N_p^{in} - N_p^{out} - \tau \alpha \beta N_F N_p^{out}$$

$$0 = N_p^{in} - N_p^{out}\left[1 + \alpha \beta \tau N_F\right]$$

$$\boxed{N_p^{out} = \frac{N_p^{in}}{1 + \alpha \beta \tau N_F}}$$

viruses: $N_v^{out} = N_v^{in} / (1 + \alpha \beta_{FV} \tau N_F)$

bacteria: $N_B^{out} = N_B^{in} / (1 + \alpha \beta_{FB} \tau N_F)$

cysts: $N_c^{out} = N_c^{in} / (1 + \alpha \beta_{Fc} \tau N_F)$

2. continued

(c) Estimate β for the collisions

Viruses: $\beta = \frac{\pi \Delta\rho\, g}{72\,\mu}\left[(d_i + d_j)^3 (d_i - d_j)\right]$

$$\beta = \frac{\pi\,(1700\ kg/m^3 - 998.5\ kg/m^3)(9.81\ m/s^2)}{72\ (1.06\times10^{-3}\ kg/m\cdot s)}\left[(1.79\times10^{-5}\ m + 1.0\times10^{-7}\ m)^3 (1.79\times10^{-5}\ m - 1.0\times10^{-7}\ m)\right]$$

$$\boxed{\beta_{FV} = 2.9\times10^{-14}\ m^3/s}$$

Bacteria: $\beta = \frac{\pi\,(1700\ kg/m^3 - 998.5\ kg/m^3)(9.81\ m/s^2)}{72\ (1.06\times10^{-3}\ kg/m\cdot s)}\left[(1.79\times10^{-5}\ m + 1.0\times10^{-6}\ m)^3 (1.79\times10^{-5}\ m - 1.0\times10^{-6}\ m)\right]$

$$\boxed{\beta_{FB} = 3.2\times10^{-14}\ m^3/s}$$

Cysts: $\beta = \left(\frac{1}{6}\right)\bar{G}\,(d_i + d_j)^3$

$$= \left(\frac{1}{6}\right)(20\ s^{-1})\left(1.79\times10^{-5}\ m + 1.0\times10^{-5}\ m\right)^3$$

$$\boxed{\beta_{FC} = 7.2\times10^{-14}\ m^3/s}$$

Microscale flocculation is not important because the flocs are too big... possibly virus-virus collisions would be controlled by microscale flocculation, but not virus-floc collisions.

So differential settling and macroscale flocculation are dominant.

(d) Estimate fractional removal

Use formula from part (b) ... $N^{out} = N^{in} / (1 + \alpha \bar{\tau} N_F \beta)$

$\alpha = 0.8$ $\quad \bar{\tau} = 19\ min = 1140\ s$ $\quad N_F = 1.0\times10^{10}\ \frac{\#}{m^3}$

Use β values from part (c)

Viruses: 21% removal Bacteria: 23% removal Cysts: 40% removal

Not so great. Most pathogens exit flocculation <u>without</u> sticking to a floc.

3. Sedimentation

(a) What size particle is 100% removed?

100% removal if $V_s >$ OR

$V_s > 3.0\ m/hr \times 1\ hr/3600\ s = 8.33 \times 10^{-4}\ m/s$

Guess that $Re < 1$

Then $V_s = \frac{g\ \Delta\rho\ d^2}{18\ \mu} \geq 8.33 \times 10^{-4}\ \frac{m}{s}$

$$d^2 \geq \frac{(8.33 \times 10^{-4}\ m/s)(18)(1.06 \times 10^{-3}\ kg/m \cdot s)}{(9.81\ m/s^2)(1700\ kg/m^3 - 998.5\ kg/m^3)}$$

$$\boxed{d \geq 4.8 \times 10^{-5}\ m = 48\ \mu m}$$ for 100% removal

Check that $Re < 1$

$$Re = \frac{(998.5\ kg/m^3)(8.33 \times 10^{-4}\ m/s)(4.8 \times 10^{-5}\ m)}{1.06 \times 10^{-3}\ kg/m \cdot s} = 0.038 < 1\ \checkmark$$

So it is valid. Stokes law applies.

(b) How effectively will the particle types be removed?

Things do not look good. All four particle types are smaller than our hypothetical 100% - removed particle. That means we will get only partial removal for each particle type. Since removal goes as $\frac{V_s}{OR}$, and V_s goes as d^2, we can say:

Flocs: $\left(\frac{18\ \mu m}{48\ \mu m}\right)^2 = 0.14 \Rightarrow$ 14% removal by sedimentation

Cysts: $\left(\frac{10\ \mu m}{48\ \mu m}\right)^2 = 0.043 \Rightarrow$ 4% removal by sedimentation

Bacteria: $\left(\frac{1\ \mu m}{48\ \mu m}\right)^2 = 0.04\%$ removal ... very low!

Viruses: 0.0004% removal (essentially no removal)

4. Is the situation OK?

Things look pretty bad. The flocculation basin is only "sticking" 20–40% of the pathogens onto the flocs, and then the flocs are only 14% removed anyway. Overall removal of viruses is thus only $(0.21)(0.14) = 0.03 = 3\%$. Terrible! It is not safe to drink the water. Citizens should boil water or use bottled water until the facility can come back on line properly. The facility needs to start <u>filtering</u> and <u>disinfecting</u> its water as soon as possible.

(If they can't get filtration and disinfection back, there would be some benefit to improving the existing processes. If we can get another maturation basin on-line, growing the flocs up to, say, 50 μm, and if we can get $\bar{G}$ up to about 60 s^{-1}, then

$$\beta = \frac{1}{6} G (d_p + d_f)^3 = \left(\frac{1}{6}\right)(60\ s^{-1})\left[(d_p + 50\times10^{-6}\ m)^3\right] \approx 1.25\times10^{-12}\ m^3/s.$$

Thus $N_p^{out} = N_p^{in} / \left[1 + (0.8)(1140\ s)\left(1.0\times10^{10}\ \frac{\#}{m^3}\right)\left(1.25\times10^{-12}\ \frac{m^3}{s}\right)\right]$

$\Rightarrow$ 92% removal!

bigger

That'd be quite a bit better, and also the flocs would settle out in sedimentation, especially if we can get the overflow rate down to about 1.5 m/hr.

92% removal still isn't good enough, but it's much better than 3%!)