ENV 6438: Physical & Chemical Processes for Drinking Water Treatment
Department of Civil & Environmental Engineering
University of South Florida

Cunningham **Spring 2020**

Homework #8 Due Mon., April 27

Topic: Membrane filtration and reverse osmosis

Assignment for 2020: Choose any *five* of the 15-point problems (questions 1 through 7), ***and*** any *one* of the 25-point problems (questions 15 through 17). That adds up to 100 points.

1. (15 pts) Answer problem 12-5 in the Crittenden text book.

2. (15 pts) Answer problem 12-6 in the Crittenden text book. Answer parts (a) and (b) for all 5 data sets. Then graph the calculated membrane resistance coefficients versus temperature and versus pressure to answer part (c) of the question.

3. (15 pts) Answer problem 12-14b in the Crittenden text book. (See also problem 17-11; you don't have to complete that one, it is just for comparison.)

4. (15 pts) Answer problem 17-6 in the Crittenden text book.

5. (15 pts) Answer problem 17-8 in the Crittenden text book.

6. (15 pts) Imagine that you are designing a reverse-osmosis system for a city's water treatment plant. The city's water source is brackish groundwater.
   - The design flow rate is 6.0 million gallons per day, which is equivalent to 15.8 m^3/min.
   - The brackish groundwater contains 2,300 mg/L of Na^+ and 3,545 mg/L of Cl^-. Concentrations of other ions can be ignored for the purposes of this problem.
   - The temperature of the brackish groundwater is 15 °C.
   - The city got a good deal from a vendor on RO4U spiral-wound membrane elements. Each element has a membrane area of 35 m^2. The mass-transfer coefficient for water through the membrane is 0.87 L/(m^2•hr•bar). The mass-transfer coefficient for salt through the membrane is 0.61 L/(m^2•hr).

   (a) Assume that your design criterion is that salt rejection must be at least 98.5%. Specify the *trans-membrane pressure* that you recommend to provide this rejection. Report your answer in units of bar.

   (b) The city has enough money in the budget to purchase 100 spiral-wound RO4U membrane elements from the vendor. Assuming the trans-membrane pressure you specified in part (a), will the city be able to meet the design flow rate of 6.0 MGD with 100 membranes? Show your calculations to support your answer.

7. (15 pts) The residents of Grant City have long complained about how hard the water is. When the mayor and the city manager both had to purchase new Keurig coffee brewers, they finally decided it was time to expand the water-treatment plant to treat the water further. Imagine that you are a consultant that has been hired to perform some initial analysis of what treatment processes might be viable for the city. One option under consideration is reverse osmosis (RO). This is under consideration because it is considered a robust process: if the source water quality deteriorates over time, the plant can just increase the pressure to maintain desired water quality in the product water. The city is considering a two-stage RO process, with a goal of 60% recovery in each stage. For each stage, a number of pressure vessels would be operated in parallel. Each pressure vessel holds 7 membrane elements in series. It is possible to show that if the recovery of each membrane element is 12%, then the recovery of each pressure vessel will achieve the desired 60%. (Trust me on that.)

   (a) Based on a two-stage configuration with 60% recovery in each stage, what would be the overall recovery of the plant?

   (b) Based on the overall recovery from part (a), at what rate must groundwater be supplied to the plant, to produce the necessary 950 m^3/hr? If the first stage contains 30 pressure vessels, at what rate must groundwater be fed to each pressure vessel?

   (c) Assuming that each membrane element has an area of 35 m^2, and assuming that the feed pressure on the first stage is 40 bar = 4×10^6 Pa, estimate/calculate the mass transfer coefficient k_w required to give 12% recovery in each membrane element. Report your answer in units of L/(m^2•bar•s). The osmotic pressure of the feed water is 0.23 bar.

   NOTE: I might actually prefer nanofiltration to reverse osmosis for Grant City, if the main problem is water hardness. Nanofiltration is a viable alternative for softening. RO might be a bit more than is needed. However, if the water quality of Grant City deteriorates over time, RO is a safer bet than nanofiltration, so maybe this plan is OK.

8. (10 pts) (taken from *Water and Wastewater Engineering: Design, Principles, and Practice* by ML Davis) Each pressure vessel in a proposed design for the softening of brackish groundwater is rated at 45% recovery at flow rates between 750 and 1000 m^3/d. Design an array system that will yield at least 4,000 m^3/d (*about 1 mgd*) of treated water for a town of 7,000 people at a permeate recovery of at least 80%.

9. (5 pts) Consider a reverse-osmosis pressure vessel that contains seven spiral-wound RO elements. If the recovery of each element is 10%, what is the overall recovery of the pressure vessel?

10. (10 pts) (taken from *Water and Wastewater Engineering: Design, Principles, and Practice* by ML Davis) Each pressure vessel in a proposed design for the softening of brackish groundwater is rated at 45% recovery at flow rates between 750 and 1000 m^3/d. Design an array system that will yield at least 4,000 m^3/d (*about 1 mgd*) of treated water for a town of 7,000 people at a permeate recovery of at least 80%.

11. (10 pts) (taken from *Water and Wastewater Engineering: Design, Principles, and Practice* by ML Davis) The city of Yeehaw has a water demand of 2.75 mgd during the winter and 4.12 mgd during the summer. Yeehaw is installing a membrane filtration system to remove particles from the water during treatment. The winter and summer conditions are as follows:
    - maximum trans-membrane pressure = 152 kPa = 1.52 bar
    - operating trans-membrane pressure = 75% of maximum
    - membrane resistance coefficient = 2.94×10^{12} m^{-1}
    - winter water temperature = 4 °C
    - summer water temperature = 20 °C

    (a) Suppose Yeehaw is designing the membrane filtration system to meet the winter demand. What total area of membranes is required?
    (b) Will Yeehaw be able to meet their summer demand with this design?

12. (10 pts) Answer problem 17-14 in the Crittenden text book. Report your answer as the β value at which the solubility would be exceeded at the membrane surface.

13. (10 pts) (taken from *Water Supply & Pollution Control*, 8th ed., by Viessman et al.) A reverse-osmosis membrane has a k_w value of 2.5×10^{-4} L/(m^2•s•bar) and an average k_s value of 3.5×10^{-4} L/(m^2•s) for total dissolved solids (TDS). Find the expected permeate TDS concentration of chloride if the feed concentration is 4500 mg/L and the system operates at a gauge pressure of 35 bar on the feed-concentrate side of the membrane. Assume that the permeate has negligible osmotic pressure and is at atmospheric pressure.

14. (10 pts) (taken from *Water Supply & Pollution Control*, 8th ed., by Viessman et al.) A low-pressure RO membrane system treating a brackish groundwater is operating at 85% recovery. The rejection for both barium and sulfate is 90%. The feed water contains 0.05 mg/L of barium and 20 mg/L of sulfate, and K_{sp} for $BaSO_4$ is 1×10^{-10}. Find the concentration of barium and sulfate in the concentrate and determine if the solubility of $BaSO_4$ is exceeded.

15. (25 pts) Reverse osmosis is being used to treat a stream of salty water, flow rate 1500 L/hr, that contains 1% NaCl by mass. The water is treated with a spiral-wound RO module that has a membrane surface area of 5.0 m^2. The water temperature is 15 °C and the applied trans-membrane pressure is 35 bar. At these conditions, the recovery of the RO system is 10% and the rejection is 98%. The osmotic coefficient of the feed is $\phi = 0.95$, and the osmotic coefficient of the permeate is close enough to 1.0 that we can just assume it is 1.0.
    a. Estimate/calculate the concentration of NaCl in both the feed and the permeate in units of mol/L and in units of mg/L.
    b. Estimate/calculate the osmotic pressure of both the feed and the permeate, in units of bar.
    c. Estimate/calculate the mass transfer coefficient, k_w, for the water flux through the RO membrane, in units of $L/(m^2\cdot hr\cdot bar)$. Hint: find the water flux, J_w, through the membrane.
    d. Estimate/calculate the mass transfer coefficient, k_s, for the salt flux through the RO membrane, in units of $L/(m^2\cdot hr)$. Hint: find the salt flux, J_s, through the membrane.
    e. Suppose the concentration of NaCl in the feed stream doubled to 2%. If the trans-membrane pressure is not changed, estimate the *recovery* that you would expect for the new feed condition. You can assume that the osmotic coefficient ϕ is still 0.95 for the saltier feed. Hint #1: use the mass-transfer coefficient that you found in part (c), above. Hint #2: for this part of the problem, assume that the rejection is still 98%, as it was under the original condition.
    f. For the saltier feed (as in part e), estimate the *rejection* that you would expect. Hint: use the mass-transfer coefficient that you found in part (d) and the water flux that you found in part (e). Is the rejection still close to the original 98% (as you assumed in part e)? If so, then your answer to part (e) is probably a good estimate; but if the rejection is very different from 98%, then your assumption in part (e) was poor, and your answer from part (e) is probably off.
    g. Comment briefly on the effect that the quality of the feed water has on the performance of reverse osmosis.

16. (25 pts) I was wondering how important it is to account for concentration polarization when estimating recovery and rejection during reverse osmosis (RO). Let's do some calculations and decide if this is actually an important phenomenon. Consider RO treatment of salty water according to the following. For the purposes of this problem, we will not sub-divide the membrane module into increments; we will assume that the following conditions apply to the module as a whole.
   - The membrane module is a spiral-wound module with a feed-concentrate channel height of 0.25 mm.
   - The membrane surface area is 35.0 m^2. The module is 1.0 m long.
   - The feed pressure is 30.0 bar and the permeate pressure is 1.0 bar.
   - The influent flow rate is 12.0 m^3/hr.
   - The water velocity in the feed-concentration channel is 0.381 m/s.
   - The feed water contains 1,840 mg/L of Na^+ and 2,840 mg/L Cl^-. Other ions can be neglected for the purposes of this problem.
   - The temperature of the feed water is 18 °C.
   - The water mass-transfer coefficient is $k_w = 2.6$ L/(m^2·bar·hr).
   - The salt mass-transfer coefficient is $k_s = 0.75$ L/(m^2·hr).
   - For the purposes of this problem, assume that the osmotic coefficient is 0.95 for the feed water and 1.0 for the permeate.

   a. Estimate/calculate the osmotic pressure of the feed water, in units of bar.

   b. Ignoring concentration polarization, and temporarily assuming that the permeate water contains no salt, estimate/calculate the water flux through the membrane, in units of L/hr. Estimate/calculate the recovery percentage.

   c. Estimate/calculate the salt flux through the membrane, in units of mg/hr.

   d. Estimate/calculate the salt concentration in the permeate, in units of mg/L. Estimate/calculate the rejection achieved by the membrane.

   e. Estimate/calculate the osmotic pressure of the permeate, in units of bar.

   f. Assume that the mass transfer coefficient for concentration polarization k_{CP}, is 2.0×10^{-4} m/s. Estimate/calculate the concentration polarization parameter, β.

   g. Re-calculate the water flux through the membrane and the recovery. This time, account for the osmotic pressure of the permeate and also the concentration polarization. How much did your estimates change?

   h. Re-calculate the salt flux through the membrane and the rejection, accounting for concentration polarization. How much did your estimates change?

   i. So, what do you think? Is it important to account for concentration polarization?

17. (25 pts) In example problem 17-5 in the Crittenden text, the authors determined that the recovery of the RO module would be 9.9% (3.1×10^{-4} m^3/s permeate from 3.125×10^{-3} m^3/s feed). Suppose we decided that we need at least 15% recovery from that module. What feed pressure would be required? Hint: set up the example problem in a spreadsheet; then it will be easy to run "what-if" scenarios to determine what feed pressure meets our goal. ***Note***: As part of this problem, you must calculate a parameter called k_{CP}. The value calculated in the example problem in the text book is not correct. It is not "wrong", but it was calculated using a formula from a different text-book; so when you do your calculations, your calculated value of k_{CP} might not agree with the one in the example. If that is the case, do not panic, you probably did it correctly; but you can check with me to be sure. It turns out that the final answer does not depend very strongly on the value of k_{CP}, so you get basically the same final answer with either the "right" or the "wrong" value of k_{CP}.