CGN 6933 Groundwater Engineering

Homework #4 — Fall 2019

"Due" Tues., Oct. 1, 2019 — Prof. J. A. Cunningham

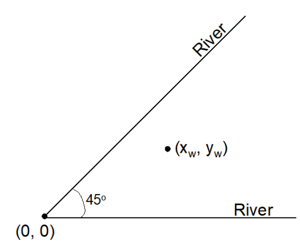
This assignment will not be collected or graded.
However, diligent completion of this assignment will help prepare you for the examinations.

(1) An aquifer is contaminated with some nasty chemicals, including TCE and hexavalent chromium. To ensure that people would not be exposed to the contaminated groundwater, a geotechnical company dug a deep trench along the line $x = 0$, and filled the trench with a very impermeable clay material. All the contaminated groundwater is located in the region $x < 0$. The groundwater in the region $x > 0$ is clean. Groundwater cannot cross the clay barrier at $x = 0$ because the clay is too impermeable.

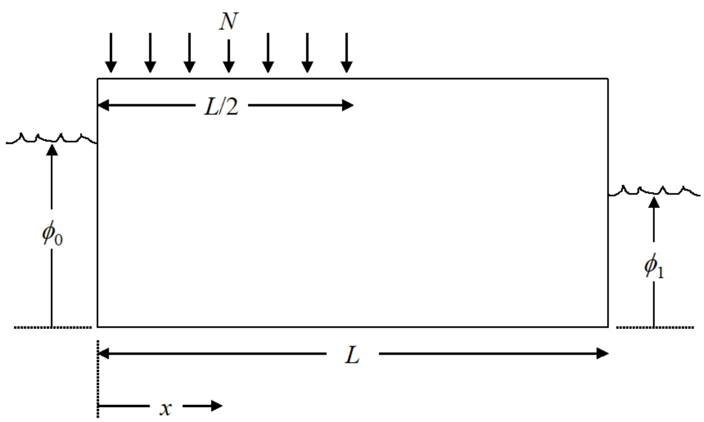
(a) Now suppose you drill a well in the clean portion of the aquifer, located at $(x = x_1, y = 0)$. The well pumps at a volumetric flow rate Q. Find the discharge potential $\Phi(x, y)$ in the "clean" side of the aquifer.

(b) For the scenario in part (a), suppose that $x_1 = 5$ m and $Q_1 = 50$ m^3/day. Sketch the equipotentials in the region $0 < x < 10$ and $-5 < y < 5$. You will probably want to use MatLab to make it easier.

(2) A pumping well is located near a 45^o bend of a river, as shown in the figure below. The well is located at (x_w, y_w). The river has a constant head ϕ_0 and a constant discharge potential Φ_0. Derive the expression for the potential at any point (x, y) within the bend. Proceed in the following manner: Find the location of the seven image wells that are needed to satisfy the constant head along the river. Show that $\Phi(x, y) = \frac{Q}{2\pi} \ln(\frac{r_1 r_3 r_5 r_7}{r_2 r_4 r_6 r_8}) + \Phi_0$, where Q is the pumping rate, r_1 is the distance from the well, and r_2 through r_8 are the distances from the image wells.



(3) An aquifer is situated between two parallel rivers, as shown in the figure below. Groundwater flow in the aquifer is one-dimensional in the x-direction. The hydraulic head h is known at the two rivers. The formation is unconfined, is underlain by a horizontal layer of bedrock, and has a hydraulic conductivity K. The distance between the two rivers is L. In the region $0 < x < \frac{L}{2}$, it rains a lot, and there is a recharge rate N. In the region $\frac{L}{2} < x < L$, it does not rain much, and there is no recharge.



(a) Verify that the discharge potential in the formation is:

$$\Phi(x) = -\frac{N}{2}x\left(x - \frac{L}{2}\right) + \frac{\frac{L}{2} - x}{\frac{L}{2}}\Phi_0 + \frac{2x}{L}\left[\frac{\Phi_0 + \Phi_1}{2} + \frac{NL^2}{16}\right] \qquad \text{for} \quad 0 < x < \frac{L}{2}$$

$$\Phi(x) = \frac{L - x}{\frac{L}{2}}\left[\frac{\Phi_0 + \Phi_1}{2} + \frac{NL^2}{16}\right] + \frac{x - \frac{L}{2}}{\frac{L}{2}}\Phi_1 \qquad \text{for} \quad \frac{L}{2} < x < L$$

where Φ_0 and Φ_1 are the values of the discharge potential at the two rivers, i.e., at $x = 0$ and at $x = L$, respectively. Hint: You can derive these equations, but it isn't necessary. It is sufficient to show that the governing equations are satisfied, that the boundary conditions are met, that the discharge potential is continuous at $x = \frac{L}{2}$, and that the discharge is continuous at $x = \frac{L}{2}$.

(b) Suppose that $N = 10^{-9}$ m/sec, $K = 10^{-6}$ m/sec, $\phi_0 = 12$ m, $\phi_1 = 10$ m, and $L = 2$ km. Draw the water table, $h(x)$.

(c) For the same parameter values as given in part (b), graph the discharge, $Q_x(x)$, between $x = 0$ and $x = L$.

(d) Where is the phreatic surface (the water table) highest? Hint: for a monotonic relationship between h and Φ, the peak of h and the peak of Φ are at the same location.

(4) A river flows along the line $y = -\frac{1}{2}x$. The river is flowing along a valley floor, and represents a hydraulic sink within the valley. That means that the regional groundwater flow in the surrounding aquifer flows towards the river. The hydraulic head of the river is ϕ_0 and the discharge potential is Φ_0. The overall discharge rate in the aquifer is $\sqrt{{Q_x}^2 + {Q_y}^2} = 1$ m²/day. Write the expression for the discharge potential, $\Phi(x, y)$, within the aquifer. Hint: draw a picture that shows the river and the groundwater flow towards the river. That will help you identify lines of constant Φ and, hence, an expresssion for $\Phi(x, y)$.

(5) In this problem, you are given either the potential $\Phi(x, y)$ or the streamfunction $\Psi(x, y)$. Verify that the given quantity satisfies the Laplace equation, and, if it does, find the conjugate function as stated. For each function, plot a nice-looking flownet of streamlines and equipotentials in the domain $(0 < x < 1, 0 < y < 1)$.

(a) $\Phi(x, y) = xy$. Find $\Psi(x, y)$.

(b) $\Phi(x, y) = x^3 - 3xy^2$. Find $\Psi(x, y)$.

(c) $\Psi(x, y) = \ln(r)$, where $r = \sqrt{x^2 + y^2}$. Find $\Phi(x, y)$. What kind of flow is described through this pair of Φ and Ψ?

(6) Consider an aquifer with regional flow in the $+x$ direction. The regional discharge rate is Q_{x0}. A well located at $(x = 0, y = 0)$ is extracting water at a rate Q. This is the classic "well in regional flow" example.

(a) Write the discharge potential, $\Phi(x, y)$, and the streamfunction, $\Psi(x, y)$.

(b) Find the stagnation point(s).

(c) Identify any streamlines that pass through the stagnation point(s). Show that the boundary of the capture zone of the well can be given by

$$x = y \cot\left(\frac{2\pi Q_{x0} y}{Q}\right)$$

(d) What is the width of the capture zone at a large distance upgradient?

(e) The "natural" length scale for this problem is $\frac{Q}{2\pi Q_{x0}}$. Use this length scale to non-dimensionalize the capture zone equation, above. Then, plot the capture zone of the well in the non-dimensionalized coordinates.

(f) For the non-dimensionalized coordinate system, plot the flownet in the vicinity of the well. Choose your domain appropriately so that the most important features of the flownet are visible.