Synthesizing Energy Minimizing Quantum-dot Cellular Automata Circuits for Vision Computing

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***Abstract*— We harness the energy minimization aspects of the quantum-dot cellular automata (QCA) computing model to synthesize QCA circuits to solve the vision problem of perceptual grouping. Unlike logic computing, vision computing problems are error-tolerant, but are hard to solve on existing computing platforms. The cost of failure of not finding *the* optimal solution is not high; even solutions that are close to optimal can suffice. The problem of perceptual grouping concerns with selecting, based on Gestaltic perceptual cues, salient subsets of low-level features, such as straight line boundary segments, that are most likely to belong to objects in the scene. We formulate a method to map this problem, which can be cast in terms of energy minimization, onto an arrangement of QCA cells. The QCA cells correspond to the straight lines, and the kink energies between them model the Gestaltic cue affinities. The magnitude of the polarizations of the QCA cells denote the saliency of the corresponding image features. We use classical multi-dimensional scaling (MDS) to synthesize the QCA cell layout. We demonstrate the ability of this arrangement to compute salient groups in real images by simulating the QCA layout using iterative, self consistent analysis, based on the Hartree-Fock approximation.**

***Index Terms*— QCA circuits, nanocomputing, energy minimization, computer vision**

I. Introduction

Quantum-dot cellular automata (QCA) [1] have potential for radically different form of computing. Recent advances in molecular implementations [2], which will enable room temperature operation, is particularly exciting. Although there are many open scientific and technical challenges at device level, strong case can be and has been made for also conducting circuits and systems level research in parallel. Understanding the types of circuits that need to be built and problems that can be solved with it will provide new directions for device-level research.

Unlike most work on QCA circuits that seeks to replicate traditional computing involving logic and arithmetic operations [3], [4], we consider computing scenarios where we can harness the energy minimizing aspects of QCA operations. One such context is the mid-level computer vision problem of perceptual grouping, which can be cast as an energy minimization problem. These minimization problems are computationally hard to solve on traditional computers. In this paper, we explore the possibility of mapping the vision minimization problem onto the QCA minimization problem, such that the ground state of the QCA arrangement gives us the solution to the vision problem. One attractive aspect of the perceptual grouping problem is that it is an *error-tolerant application*, where the cost of failure of not finding *the* optimal solution is not high; even solutions that are close to optimal ones suffices in practice. This aspect is significant in light of the expectation that nano-domain computing is expected to be error prone.

Although QCAs have not been used for image processing, arrays of quantum-dot (QD) arrays have been explored for solving image processing applications [5]. However, unlike image processing that deals with image pixels arranged in a regular gird, mid-level vision algorithms deals with symbolic representations, arranged in an irregular grid. The computational expense of image processing arise due to the sheer large number of pixels involved. The complexity of the operations themselves are rather simple. Therefore, there are many fast architectures with traditional CMOS based computing devices for low-level processing. In contrast, mid-level vision algorithms are inherently computationally complex.

We arrive at the mapping between the vision problem of grouping and the QCA layout by observing that the aggregate energy minimized by the ground state of the QCA circuit is a function of the spatial arrangement. This, we summarize in Section II. The problem of perceptual grouping and its energy minimization formulation is described in Section III. Given pairwise energies between image primitives or tokens, we have to find the spatial arrangement of QCA cells, each representing an image primitive, whose Coulombic interaction best approximates the grouping energies. In Section IV, we draw upon the statistical body of work on Multi-Dimensional Scaling (MDS) [6] to synthesize the energy minimizing QCA "circuit" layout. We present results of QCA circuit simulations, solving the grouping problem, in Section V. We conclude with Section VI.

II. QCA Arrays as Energy Minimizing Engines

The operation of a QCA cell can be abstracted in a fairly simple manner. Each QCA cell consists of one or more electrons that can exist in two or more dots, with two ground states configurations that can be taken to represent the logic states of zero or one. Two or more cells interact by Coulombic interaction, with an arrangement of cells settling to the lowest energy state. The QCA computing model is based on mapping the computed logic onto to the ground state, i.e. minimum energy state, of arrays of quantum-dot cells.

Following Tougaw and Lent [7] and other subsequent works on QCA, we use the two-state approximate model of a single QCA cell. An array of cells can be modeled fairly well by considering cell-level quantum entanglement of the two states and just Coulombic interaction between nearby cells, modeled using the Hartee-Fock (HF) approximation [7]. We denote the eigenstates of a cell corresponding to the 2-states by $|0\rangle$ and $|1\rangle$. The state at time t, which is referred to as the wave-function and denoted by $|\Psi(t)\rangle$, is a linear combination of these two states, i.e. $|\Psi(t)\rangle = c_0(t)|0\rangle + c_1(t)|1\rangle$. Each cell polarization is defined to the difference of probabilities of the two states, $\delta = |c_0|^2 - |c_1|^2$. The Hamiltonian of the i-th cell, given by [7]

$$\mathbf{H}_i = \begin{bmatrix} -\frac{1}{2}\sum_{j\neq i} E_{ij}^k \delta_j & -\gamma \\ -\gamma & \frac{1}{2}\sum_{j\neq q} E_{ij}^k \delta_i \end{bmatrix} \quad (1)$$

where the sums along the diagonal are over the neighboring cells, computing the average effective polarizations of the neighbors. The tunneling energy between the two states of a cell, which is controlled by the clocking mechanism, is denoted by γ. E_{ij}^k is the "kink energy" or the energy cost of two cells having opposite polarizations, as determined by Coulombic interaction.

An equivalent representation of the cell wave function can be formulated in terms of δ and a quantum mechanical phase angle ψ using the transformations: $c_0 = \sqrt{(1+\delta)/2}$ and $c_1 = \sqrt{(1-\delta)/2}e^{i\psi}$. It can be shown that the energy (expectation of the Hamiltonian) of each cell is given by [8]

$$E_i = -2\gamma cos(\psi)\sqrt{(1-\delta_i^2)} - \delta_i \sum_{j\neq i} E_{ij}^k \delta_j \quad (2)$$

The total energy of the system is given by $E = \sum_i E_i$.

$$E = -2\gamma cos(\psi) \sum_i \sqrt{(1-\delta_i^2)} - \sum_i \sum_{j\neq i} E_{ij}^k \delta_i \delta_j \quad (3)$$

Dissipation to the environment adsorbs excess energy and minimizes the energy of the system towards a steady state, where $\psi = 0$. By increasing or decreasing γ, the tunneling energy, one can control which of the two energy terms dominate the minimization. This γ is controlled by the clocking signal. But increasing it, the steady state polarization of a cell can be driven to zero. We can use this effect to deselect cells in a regular QCA grid arrangement to implement the synthesized layout. For the selected cells, the γ will be close to zero and the second term will dominate the minimization. Or equivalently, the following energy functional is maximized.

$$\hat{E} = \sum_i \sum_{j\neq i} E_{ij}^k \delta_i \delta_j \quad (4)$$

It is a quadratic cost term involving pairwise kink energies that are determined by the relative locations between the cells. By controlling the placement of the cells these pairwise kink energy terms can be made to match the problem at hand. We outline an approach for this process in Section IV. But, before that we look at the problem of perceptual grouping.

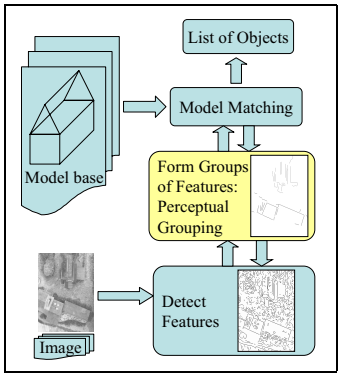

Fig. 1. Levels of processing involved in a solution to the problem of object recognition in computer vision. Samples of data involved at each level are shown. The goal for the image shown is to find buildings in the images.

III. The Task of Perceptual Organization

Object recognition from images is one of the primary goals of computer vision. As illustrated in Fig. 1, this involves three basic kinds of processes. The low-level process extracts low-level primitives such as edge pixels, extended edge structures such as straight line segments and arcs, region patches, or image pixel based local measures. High-level processes are concerned with the problem of inferring object match or pose, given a subset of the low-level features found in an image. The intermediate level is concerned with forming the low-level feature subsets to be input to the high-level process. A brute selection of all possible subsets of N features will result in exponential complexity. Fig. 1 shows an example of the best grouping that one can find in the edge image, with each group corresponding to a building. The mid-level perceptual grouping process are vital in controlling the combinatorics of the vision problems. Similar processes are also impart robustness to the human visual system.

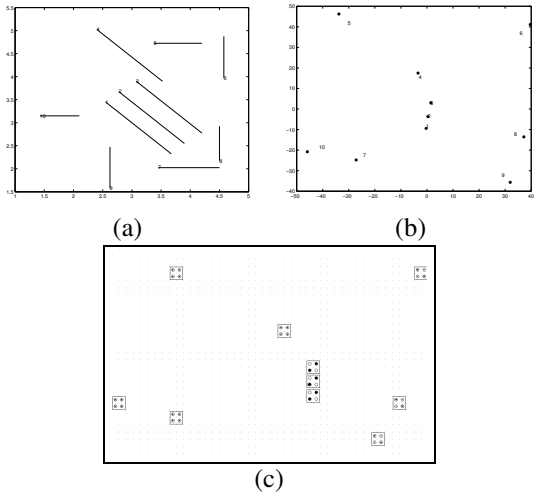

Fig. 2. (a) An arrangement of straight lines found in an image that needs to be separated into object and background features. (b) Spatial layout of the line tokens synthesized by classical MDS based on the Gestalt energies. (c) Arrangement QCA cells, with each cell representing a straight line token, to match the underlying Gestalt energies. The polarization of the cells at the ground state are depicted.

We consider the specific version of the perceptual grouping problem of grouping straight line segments found in the edge images. One illustration of the input is shown in Fig. 2(a). The task is to select the subset of the straight line features that are most likely to come from an object, *without* the knowledge of the specific object identity. Some of the principles that are used to group features are based on the findings of the Gestalt school of psychology, who found that groups that exhibit symmetry, similarity, proximity, closure, and continuity are salient. Perceptual grouping can be formulated as an energy minimization problem [9], [10]. We represent a group using the vector $\mathbf{x}$, whose i-th component, x_i, is 1 if f_i is part of the group and $x_i = 0$ if it is not. Between every pair of straight line we associate affinity energies, A_{ij} to capture the perceptual saliency of the relationship between them. For example, if two straight lines are parallel to each other then according to the Gestalt principles they are likely to belong to one object and hence the affinity should be high. Similarly, lines that are close together are more likely to associated together, i.e. the proximity principle. Lines that form one straight arrangement are also likely to be grouped, i.e. the continuity principle. The quantitative forms of the affinity function varies in different implementation, but qualitatively they capture similar aspects. The particular form we use is

$$A_{ij} = \sqrt{l_i l_j} e^{-\frac{o_{ij}}{\max(l_i,l_j)}} e^{-\frac{d_{min}}{\max(l_i,l_j)}} \sin^2(2\theta_{ij}) \quad (5)$$

where l_i and l_j are the lengths of the i-th and j-th features, o_{ij} is the overlap, θ_{ij} is the angle, and d_{min} is the minimum distance between the two straight lines. We can see that this form of the affinity function agrees with the strengths of the Gestalt affinities that we want to model.

The goal is to find a group, $\mathbf{x}$, such that total affinity energy is maximized.

$$A = \sum_i \sum_{j \neq i} A_{ij} x_i x_j \quad (6)$$

IV. Synthesis of Energy Minimizing QCA Circuit Layout

We can notice the correspondence between the objective function for the grouping problem (Eq. 6) and that the energy function for an QCA circuit (Eq. 4). Each low-level primitive or token, f_i, can be represented by a QCA cell, whose polarization magnitude, $|\delta_i|$ would determine the membership in the salient grouping, x_i. Given pairwise affinity energies between N low-level primitives, how do we embed the corresponding QCA cells in a two (or three) dimensional space so that the distance between them would result in Coulombic kink energies that is proportional to the Gestalt affinities? For this we use the statistical method of multidimensional scaling [6], which is commonly used to embed distance matrices in low-dimension spaces and have so far been used extensively for data visualization. Let N image tokens have pairwise affinity energies specified by $\{A_{ij}\}$ between them. The goal of multidimensional scaling (MDS) is to find a configuration of points, representing these tokens, in a $p = 2$ or 3 dimensional space such that the distance between two points r and s, denoted by d_{rs}, will be proportional to $\frac{1}{A_{ij}^{1/8}}$. If QCA cells are placed at these point coordinates then the pairwise interaction between them will be proportional to the given energies, i.e. $E_{ij}^{QCA} \propto \frac{1}{d_{ij}^8} \propto A_{ij}$. Let the matrix Λ be constructed out of given energies such that: $\Lambda_{ij} = d_{rs}^2 = \frac{1}{(A_{ij})^{1/4}}$. We desire to find the coordinate of each point in a p dimensional match, which we denote by the matrix of coordinate vector, $\mathbf{X}_{MDS} = [\mathbf{x}_1, \cdots, \mathbf{x}_N]$, such that

$$(\mathbf{x}_i - \mathbf{x}_j)^T (\mathbf{x}_i - \mathbf{x}_j) = c\Lambda_{rs} \quad (7)$$

or equivalently

$$\mathbf{X}_{MDS}^T \mathbf{X}_{MDS} = -c\frac{1}{2}\mathbf{H}\Lambda\mathbf{H}, \quad \text{where} \quad \mathbf{H} = (\mathbf{I} - \frac{1}{N}\vec{1}\vec{1}^T) \quad (8)$$

with $\mathbf{I}$ as the identity matrix and $\vec{1}$ as the vector of ones. This operator $\mathbf{H}$ is referred to as the centering operator. These coordinates $\mathbf{X}$ can be arrived at by singular value decomposition of the centered distance matrix $\frac{1}{2}\mathbf{H}\Lambda\mathbf{H} = \mathbf{V}_{MDS}\Delta_{MDS}\mathbf{V}_{MDS}^T$ where $\mathbf{V}_{MDS}$, Δ_{MDS} are the eigenvectors and eigenvalues respectively. Assuming that centered distance matrix represents the inner product distances of an Euclidean distance matrix, the coordinates which are given by

$$\mathbf{X}_{MDS} = (\mathbf{V}_{MDS}\Delta_{MDS}^{\frac{1}{2}})^T \quad (9)$$

Note that we have dropped the constant of proportionality, c, since the energy minimizing solutions are invariant to scaling of the original function.

V. Results

For the simple arrangement of straight lines in Fig. 2(a) the embedding of the corresponding QCA cells in a 2D space is shown in Fig. 2(b); if the QCA cells are placed at these locations, the Coulombic interactions will be proportional to the Gestalt energies. Fig. 2(c) shows the polarization of the QCA cells in the ground state configurations. The extent of the polarization, δ_i, denotes participation of the corresponding straight line in the salient group. We see that the central 3 QCA cells, which correspond to the 3 parallel lines, are fully polarized.

In Fig. 3 we present result on one real image among the many we have tested. The low-level edges that are to be grouped are shown in Fig. 3(b). Each straight line segment (not shown) detected in the edge map is represented by a corresponding QCA cell. The synthesized layout to match the corresponding Gestaltic grouping energy are shown in Fig. 3(d). The location of each cell is marked by a cross and the corresponding ground state polarization values are shown as spikes associated with each cell. The ground state was computed by self consistent analysis based on the Hartree-Fock approximation. Fig. 3(c) highlight the edges corresponding to the highly polarized QCA cells. Notice how the salient edges corresponding to the dominant objects in the scene, such as the mailbox and building facade, are separated from the background clutter.

VI. Conclusion

We have presented a novel context for nanocomputing with QCAs that exploits their energy minimization aspects. Vision computing is an error-tolerant application that can tolerate near optimal solutions. In particular, we considered the problem of perceptual grouping that is computationally hard to solve on traditional computers. We presented a method for synthesizing energy minimization QCA circuits to solve the grouping problem and demonstrated their effectiveness using iterative, self-consistent analysis Hartree-Fock simulations. Results on real images are promising.

This is just the first step in designing a novel form of QCA "circuits." There are, of course, many issues that need to be considered in future. For instance, the layout of the QCA cells are dependent on the specific instance of the energy minimization problem. One would have to consider building the required QCA arrangement by deselecting cells from a regular arrangement of cells. We are looking into using the clocking mechanism for this. Read-out of the cell polarizations is also another issue, for which are looking into constructing a solution based on layered QCAs [11].

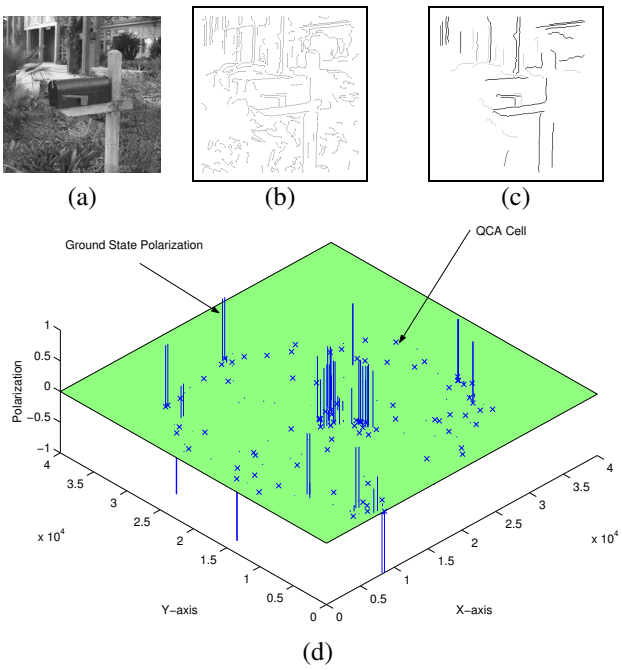

Fig. 3. Grouping of edges using QCA circuits. (a) Input image. with (b) the corresponding edge images. (c) The salient groups of edges selected based on the ground state polarizations of the synthesized circuit. (d) Synthesized QCA circuit layout with each QCA cell corresponding to a straight edge segment and the distances between the cells chosen to match the Gestaltic affinities. Location of each cell is marked with a cross and the corresponding ground state polarizations are shown as spikes associated with each cell.

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