CGN 6933-002
Transport in Porous Media

Spring 2005
Homework #2
Due Wed., Feb. 2, 2005

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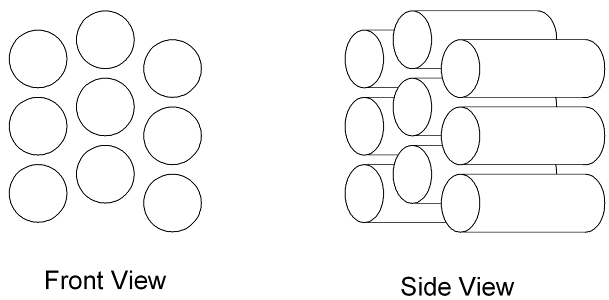
(1) Perhaps you wonder why hydraulic conductivity is lower for fine-grained porous media than for coarse-grained media. Keep in mind that, for a porous medium, the size of the pores between grains is about the same as the size of the grains. Now, let's pretend that the pores of the medium look like a bundle of tubes, like this:



This is a pretty good (not perfect) model of the pores in a porous medium. Suppose each tube has a radius R and a length L. There is a pressure gradient ΔP across the bundle of tubes. Water flows through the tubes in response to this pressure gradient. You may assume that the flow is laminar, and you may treat the water as incompressible. We can use this model to investigate the permeability of porous media.

(a) Find the average velocity of water flow through a single tube. You can derive it, or you can look it up in a fluid-mechanics text-book under Hagen-Poiseuille flow, whichever you prefer.

(b) Use this velocity in conjunction with Darcy's Law to find the hydraulic conductivity, K, and the intrinsic permeability, k, for the bundle of tubes. Don't forget to think about porosity.

(c) Use your result from part (b) to argue that fine-grained media have lower conductivities than coarse-grained media.

(2) Look up the *Kozeny-Carman equation*, which relates the permeability or hydraulic conductivity of an unconsolidated porous medium to the medium's grain size and porosity. Use the equation to estimate the range of hydraulic conductivities that we might expect for clay, silt, sand, and gravel. Compare your estimated values to the actual ranges of values encountered for these types of media. How closely do the values agree? (It is likely that you will need to look up some values to complete this problem, unless you are uncommonly good at remembering things like the grain size, porosity, and conductivity of silt.) What is your opinion about using this equation as a predictor of hydraulic conductivity? Would you trust the predicted values to within an order of magnitude? to within a factor of 2 or 3? to within 10–20%? How many significant digits are warranted when reporting estimates from the Kozeny-Carman equation?

(3) Suppose that you are drilling a well into a very deep aquifer. Further suppose that the water in the aquifer is at a temperature of 15 $^{\mathrm{O}}$C everywhere. What is the density of the water at the water table (i.e., at atmospheric pressure)? What is the density of the water 500 m below the water table? On a percentage basis, how much does the density of water change between the water table and the depth of the aquifer? Does this sound like a significant change in density, or do you think it is OK to ignore it?

(4) About how long (measured in hours) did it take you to complete this homework?