CGN 6933-002, Transport in Porous Media

Midterm Exam, solution key

Spring 2005, University of South Florida

1. Estimate/calculate the head loss Δh across the column.

Darcy's Law: $\frac{Q}{A} = K \left| \frac{\Delta h}{L} \right|$

Given Q, A, K, L

Easy to calculate Δh!

$$\Delta h = \frac{Q L}{K A} = \frac{(100\ cm^3/day)(50\ cm)}{(10\ cm/day)(30\ cm^2)} = 16.67\ cm$$

$\boxed{\Delta h = 17\ cm}$ to two significant digits.

2. Write the PDE for transport in the column

Conservative tracer ⇒ no sorption, no reaction

Assume 1-D transport because it is a column

Then

$$\underbrace{n \frac{\partial C(x,t)}{\partial t}}_{\text{Accumulation}} = \underbrace{nD \frac{\partial^2 C(x,t)}{\partial x^2}}_{\text{Dispersion}} - \underbrace{nv \frac{\partial C(x,t)}{\partial x}}_{\text{Advection}}$$

$C(x,t)$ = bromide concentration

D = dispersion coefficient

v = pore velocity

n = porosity

3. BONUS QUESTION

Initial condition: $C(x, t=0) = 0$

Boundary condition: $C(x=0, t) = C_0$

or

$$v\,C(x=0,t) - D\,\frac{\partial C(x=0,t)}{\partial x} = v \cdot C_0$$

Boundary condition: $$D\,\frac{\partial C(x=L,t)}{\partial x} = 0$$

4. At what time does concentration profile look like that shown?

The concentration front has penetrated about 30 cm into the column. So, how long does it take to travel 30 cm by advection?

$$t = \frac{30 \text{ cm}}{v}$$

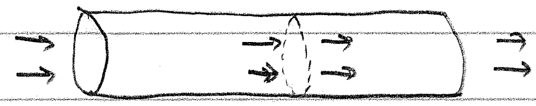
But $$v = \frac{q}{n} = \frac{Q/A}{n} = \frac{Q}{nA}$$

$$v = \frac{100 \text{ cm}^3/\text{day}}{(0.33)(30 \text{ cm}^2)} = 10. \text{ cm/day}$$

So $$t = \frac{30 \text{ cm}}{10 \text{ cm/day}} = 3.0 \text{ days}$$

$t = 3.0$ days

5. Estimate advective flux of bromide at x = 30 cm.



Flux = mass per area per time

How much bromide moves across the plane x = 30 cm, due to advection?

$$\text{Advective flux} = qC = nvC = \frac{Q}{A}C$$

$$= \frac{(100\ \text{cm}^3/\text{day})(55\ \text{mg/L})}{30\ \text{cm}^2}$$

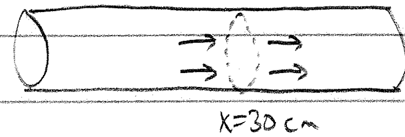
$$= 183\ \frac{\text{cm} \cdot \text{mg}}{\text{L} \cdot \text{day}}$$

But 1 L = 1000 cm^3

$$\boxed{\text{Advective flux} = 0.18\ \frac{\text{mg}}{\text{cm}^2 \cdot \text{day}}}$$

Flux is from left to right. Why? Because the flow is from left to right, and advection is the transport of bromide due to the fluid flow.

6. Estimate dispersive flux at $x = 30$ cm.



How much bromide moves across $x = 30$ cm due to dispersion?

Fick's law: $J = -D \frac{dc}{dx}$

But we have a porous medium, so fix Fick's Law:

$$J = -n D \frac{dc}{dx}$$

$n = 0.33$

$D = \alpha_L V = (0.4 \text{ cm})(10 \text{ cm/day}) = 4.0 \text{ cm}^2\text{/day}$

$\frac{dc}{dx}$ = slope of the concentration profile at $x = 30$ cm

$$\frac{dc}{dx} \approx \frac{\Delta c}{\Delta x} = \frac{20 \text{ mg/L} - 88 \text{ mg/L}}{35 \text{ cm} - 25 \text{ cm}} = -6.8 \frac{\text{mg}}{\text{cm} \cdot \text{L}}$$

$$J = -(0.33)(4.0 \text{ cm}^2\text{/day})\left(-6.8 \frac{\text{mg}}{\text{cm} \cdot \text{L}}\right) = +9.0 \frac{\text{mg} \cdot \text{cm}}{\text{L} \cdot \text{day}}$$

But $1 \text{ L} = 1000 \text{ cm}^3$

Dispersive flux $= 0.009 \frac{\text{mg}}{\text{cm}^2 \cdot \text{day}}$

Dispersive flux is from left to right. Why? Because dispersion moves mass from high concentration to low concentration.

Also notice that the advective flux is much greater than the dispersive flux, as we'd expect.

7. Graph the TCE concentration profile.

TCE will be affected by <u>sorption</u>.

Both advection and dispersion will be <u>retarded</u>.

Retarded by how much?

$R = 1 + \frac{\rho_b K_d}{n}$

$\rho_b = (1-n)\rho_g = (1-0.33)(2.54\ g/cm^3) = 1.7\ g/cm^3$

$K_d = f_{oc} K_{oc} = f_{oc}\left[0.63\ \frac{mL}{g} \cdot K_{ow}\right]$ because $K_{oc} \approx (0.63\ \frac{mL}{g}) K_{ow}$

$= (0.0023)(0.63\ \frac{mL}{g})(263) = 0.38\ mL/g$

$n = 0.33$

$R = 1 + \frac{(1.7\ g/cm^3)(0.38\ mL/g)}{(0.33)} = 2.96 \approx 3.0$

Use $R = 3.0$ for simplicity.

So <u>advection</u> is retarded by a factor of 3 ⇒ the distance traveled will only be 10 cm instead of 30 cm.

Also <u>dispersion</u> is retarded by a factor of 3

But recall $\sigma_x = \sqrt{2Dt}$, so now we will use $\sigma_x = \sqrt{2 \frac{D}{R} t}$

Therefore the <u>spread</u> is reduced by a factor $\sqrt{R} = 1.7$

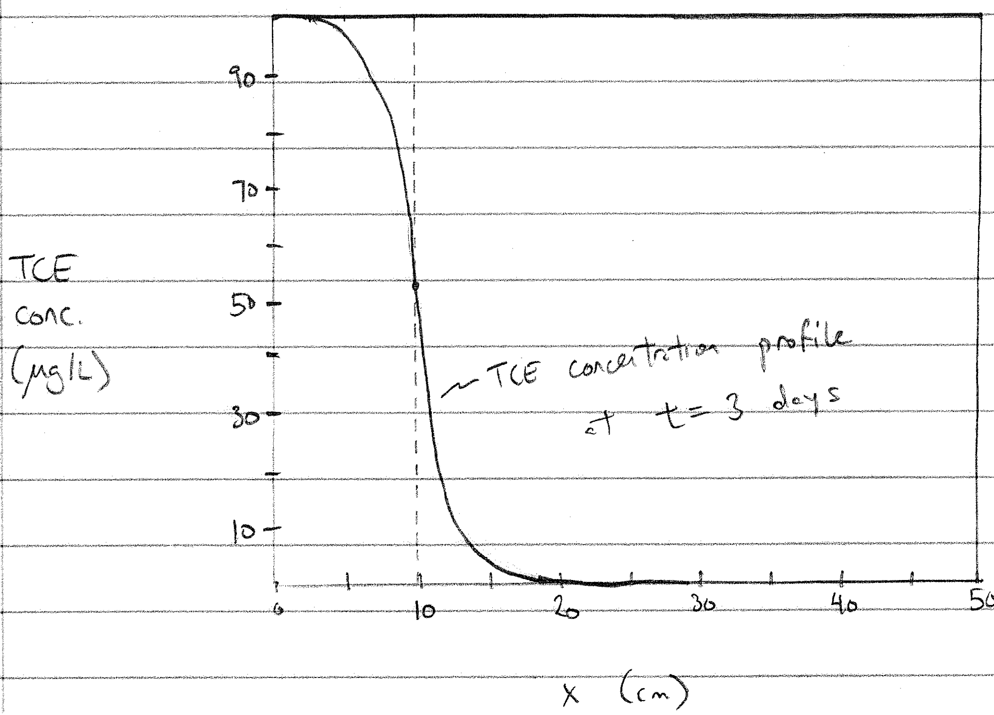
For bromide, the concentration profile spreads about 10-15 cm on either side of the center. (from graph)

For TCE, the spread will be reduced by a factor <u>1.7</u>

See graph on next page

Distance traveled by center of mass = 10 cm

Spread = 7.3 cm upgradient, 8.8 cm downgradient



By the way, the question is a bit ambiguous as to whether the sorption should suppress the C_0 concentration from 100 μg/L down to 33 μg/L. If you assume it does, I will award full points, though that is not what I intended.

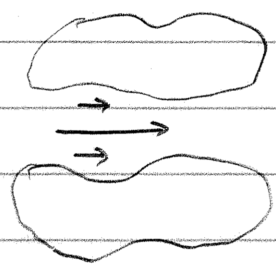
The TCE concentration profile at 3 days looks just like the bromide concentration profile after only 1 day!

8. (a) What is the relationship between hydrodynamic dispersion, mechanical dispersion, and molecular diffusion?

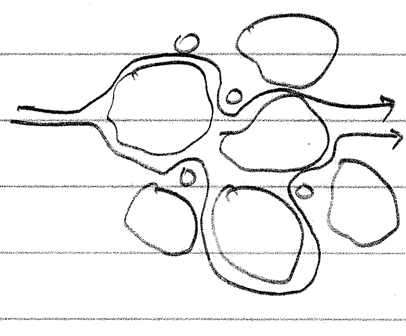
Hydrodynamic dispersion = Mechanical dispersion + Molecular diffusion

$$D^{hydrodynamic} = D^{mech} + D^{molec}$$

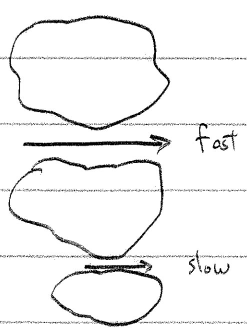
(b) What are three mechanisms of mechanical dispersion?



Taylor dispersion. Pore velocities are higher in the center of the pore.



Path lengths are different. Some paths are shorter than others.



Pore diameters are different. Flow is faster in wider pores.

(c) $D^{mech} = \alpha v$, mechanical dispersion is linear with velocity

α is called the dispersivity