

1. Water Quality in Grant City

(a) Concentrations of major ions

$$Ca^{2+} : 76 \frac{mg}{L} * \frac{1 \text{ mmol}}{40.08 \text{ mg}} = \underline{\underline{1.9 \frac{mmol}{L}}} * \frac{2 \text{ meq}}{1 \text{ mmol}} = \underline{\underline{3.8 \text{ meq/L}}}$$

$$Mg^{2+} : 34 \frac{mg}{L} * \frac{1 \text{ mmol}}{24.31 \text{ mg}} = \underline{\underline{1.4 \frac{mmol}{L}}} * \frac{2 \text{ meq}}{1 \text{ mmol}} = \underline{\underline{2.8 \text{ meq/L}}}$$

$$Na^+ : 6.9 \frac{mg}{L} * \frac{1 \text{ mmol}}{22.99 \text{ mg}} = \underline{\underline{0.30 \frac{mmol}{L}}} * \frac{1 \text{ meq}}{1 \text{ mmol}} = \underline{\underline{0.3 \text{ meq/L}}}$$

$$HCO_3^- : 366 \frac{mg}{L} * \frac{1 \text{ mmol}}{61 \text{ mg}} = \underline{\underline{6.0 \frac{mmol}{L}}} * \frac{1 \text{ meq}}{1 \text{ mmol}} = \underline{\underline{6.0 \text{ meq/L}}}$$

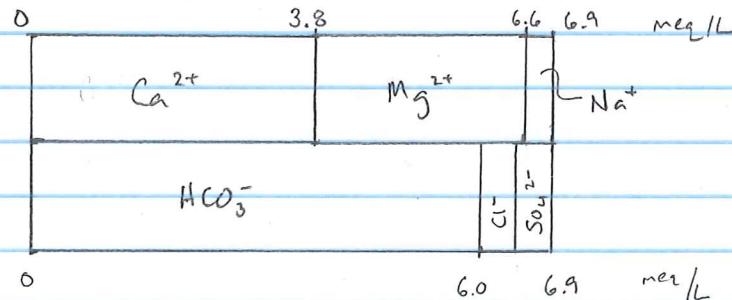
$$Cl^- : 14 \frac{mg}{L} * \frac{1 \text{ mmol}}{35.45 \text{ mg}} = \underline{\underline{0.40 \frac{mmol}{L}}} * \frac{1 \text{ meq}}{1 \text{ mmol}} = \underline{\underline{0.4 \text{ meq/L}}}$$

$$SO_4^{2-} : 24 \frac{mg}{L} * \frac{1 \text{ mmol}}{96.06 \text{ mg}} = \underline{\underline{0.25 \frac{mmol}{L}}} * \frac{2 \text{ meq}}{1 \text{ mmol}} = \underline{\underline{0.5 \text{ meq/L}}}$$

Charge balance : Cations $3.8 \text{ meq/L} + 2.8 \text{ meq/L} + 0.3 \text{ meq/L} = 6.9 \text{ meq/L}$

Anions $6.0 \text{ meq/L} + 0.4 \text{ meq/L} + 0.5 \text{ meq/L} = 6.9 \text{ meq/L}$

The balance is good. ✓



(b) Hardness

$$[Ca^{2+}] + [Mg^{2+}] = 6.6 \text{ meq/L}$$

$$6.6 \frac{\text{meq}}{\text{L}} * \frac{100 \text{ mg CaCO}_3}{2 \text{ meq}} = \underline{\underline{330 \text{ mg/L as CaCO}_3}}$$

(c) Osmotic pressure

$$\Pi = \phi CRT$$

C = sum of all ion concentrations

$$= 1.9 \frac{\text{mol}}{\text{m}^3} + 1.4 \frac{\text{mol}}{\text{m}^3} + 0.3 \frac{\text{mol}}{\text{m}^3} + 6.0 \frac{\text{mol}}{\text{m}^3} + 0.4 \frac{\text{mol}}{\text{m}^3} + 0.25 \frac{\text{mol}}{\text{m}^3} = 10.25 \frac{\text{mol}}{\text{m}^3}$$

$$\begin{aligned} \Pi &= (0.95)(10.25 \text{ mol/m}^3)(8.314 \text{ Pa} \cdot \text{m}^3/\text{mol} \cdot \text{K})(273 + 15 \text{ K}) = \underline{\underline{23,300 \text{ Pa}}} \\ &= \underline{\underline{0.23 \text{ bar}}} \end{aligned}$$

2. Disinfection with chlorine

What concentration of chlorine residue is required?

Short method:

When $Ct = (4.0 \frac{\text{mg}}{\text{L}})(5 \text{ min}) = 20$, we get two nines removal

When $Ct = (3.0 \frac{\text{mg}}{\text{L}})(10 \text{ min}) = 30$, we get three nines removal

So for four nines removal, we need $Ct = 40$

$$(C)(15 \text{ min}) = 40 \frac{\text{mg}}{\text{L}} \cdot \text{min} \Rightarrow C = \underline{\underline{2.7 \frac{\text{mg}}{\text{L}}}}$$

Long method:

$$\text{Batch reactor } N(t) = N_0 e^{-kt} \Rightarrow \frac{N}{N_0} = e^{-kt}$$

Chick-Watson rule $k = \lambda C$ where $C = \text{conc. of disinfectant}$

$$\therefore \frac{N}{N_0} = e^{-\lambda Ct}$$

Given 99% inactivation when $C = 4.0 \frac{\text{mg}}{\text{L}}$ and $t = 5 \text{ min}$

$$99\% \text{ inactivation} \Rightarrow \frac{N}{N_0} = 0.01$$

$$0.01 = \exp(-\lambda Ct)$$

$$0.01 = \exp[-\lambda (4.0 \frac{\text{mg}}{\text{L}})(5 \text{ min})]$$

$$\ln(0.01) = -\lambda (4.0 \frac{\text{mg}}{\text{L}})(5 \text{ min})$$

$$\lambda = \frac{\ln(0.01)}{-(4.0 \frac{\text{mg}}{\text{L}})(5 \text{ min})} = 0.23 \frac{\text{L}}{\text{mg} \cdot \text{min}}$$

Check the other data point given ... $C = 3.0 \frac{\text{mg}}{\text{L}}$ $t = 10 \text{ min}$

$$\frac{N}{N_0} = \exp\left[-(0.23 \frac{\text{L}}{\text{mg} \cdot \text{min}})(3.0 \frac{\text{mg}}{\text{L}})(10 \text{ min})\right] = 0.001$$

$$\frac{N}{N_0} = 0.001 \text{ so } 99.9\% \text{ inactivation } \checkmark \text{ It is correct.}$$

So now in our PFR:

$$N_E = N_I e^{-k_I \theta} \Rightarrow \frac{N_E}{N_I} = \exp(-k_I \theta) = \exp[-\lambda C \theta]$$

$$99.99\% \text{ inactivation} \Rightarrow \frac{N_E}{N_I} = 0.0001$$

$$0.0001 = \exp\left[-(0.23 \frac{\text{L}}{\text{mg} \cdot \text{min}}) C (15 \text{ min})\right]$$

$$C = \frac{\ln(0.0001)}{-(0.23 \frac{\text{L}}{\text{mg} \cdot \text{min}})(15 \text{ min})} = \underline{\underline{2.7 \frac{\text{mg}}{\text{L}}}}$$

Both methods work, but if you see the short cut, it is a lot quicker!

3. Lime softening for Grant City

(a) Is soda ash necessary?

Currently the hardness is 330 mg/L as CaCO_3

That is equivalent to 6.6 meq/L

We must soften to 50 mg/L , which is 1.0 meq/L

So we must remove 280 mg/L of hardness, or 5.6 meq/L hardness.

But from problem 1, we have 6.0 meq/L of HCO_3^-

Therefore we have sufficient HCO_3^- in the water --- $6.0 > 5.6$

We do not need soda ash

(b) Rate of CaO addition

To remove 3.8 meq/L of Ca^{2+} which is 1.9 mmol/L Ca^{2+} :



So we need 1.9 mmol/L of lime to remove 1.9 mmol/L Ca^{2+}

To remove 1.8 meq/L of Mg^{2+} which is 0.9 mmol/L Mg^{2+} :



So we need 0.9 mmol/L of lime to remove 0.9 mmol/L Mg^{2+}

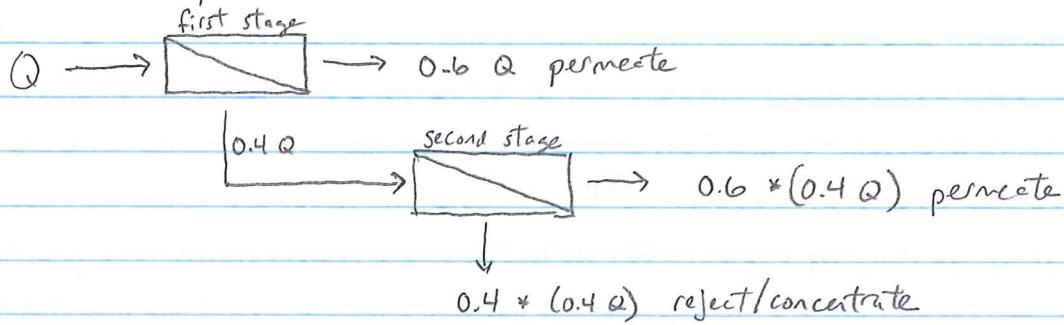
Thus overall we need $1.9 \frac{\text{mmol}}{\text{L}} + 0.9 \frac{\text{mmol}}{\text{L}} = 3.7 \frac{\text{mmol}}{\text{L}}$ lime

$$\begin{aligned} (950 \frac{\text{m}^3}{\text{hr}}) \left(\frac{24 \text{ hr}}{1 \text{ d}} \right) \left(\frac{1000 \text{ L}}{1 \text{ m}^3} \right) \left(3.7 \times 10^{-3} \frac{\text{mol CaO}}{\text{L}} \right) \left(\frac{56 \text{ g CaO}}{1 \text{ mol CaO}} \right) \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right) \\ = \underline{\underline{4700 \text{ kg/d CaO}}} \\ = 4.7 \text{ tons/d} \end{aligned}$$

That is a lot of lime for Grant City!

4. Reverse osmosis for Grant City

② Overall recovery?



$$\text{Overall recovery} = 0.6 + 0.6 * 0.4 = \underline{\underline{0.84}} = \underline{\underline{84\%}}$$

③ How much groundwater needed?

$$950 \text{ m}^3/\text{hr} = (0.84)(Q_{\text{tot}}) \Rightarrow Q_{\text{tot}} = \frac{950 \text{ m}^3/\text{hr}}{0.84} = \underline{\underline{1130 \text{ m}^3/\text{hr}}}$$

$$\text{So split among 30 pressure vessels, } \frac{1130 \text{ m}^3/\text{hr}}{30} = \underline{\underline{38 \text{ m}^3/\text{hr}}} \text{ to each pressure vessel}$$

④ What mass transfer coefficient is needed?

$$\text{Feed to the pressure vessel} = 37.7 \text{ m}^3/\text{hr}$$

$$\begin{aligned} 12\% \text{ recovery} &= (0.12)(37.7 \text{ m}^3/\text{hr}) = 4.52 \text{ m}^3/\text{hr} \text{ from the first element} \\ &= 4520 \text{ L/hr} \end{aligned}$$

$$(4520 \frac{\text{L}}{\text{hr}}) = k_w (35 \text{ m}^2) (\Delta P - \Delta \pi)$$

Given $\Delta P = 40 \text{ bar}$, know $\Delta \pi \approx 0.23 \text{ bar}$ from 1c

$$4520 \frac{\text{L}}{\text{hr}} = k_w (35 \text{ m}^2) (40 \text{ bar} - 0.23 \text{ bar})$$

$$k_w = \frac{4520 \frac{\text{L}}{\text{hr}}}{(35 \text{ m}^2)(39.77 \text{ bar})} = \underline{\underline{3.25 \frac{\text{L}}{\text{m}^2 \cdot \text{hr}} * \frac{1 \text{ hr}}{3600 \text{ s}}}} = \underline{\underline{9.0 \times 10^{-4} \frac{\text{L}}{\text{m}^2 \cdot \text{bar} \cdot \text{s}}}}$$

NOTE: I did this calculation for the first membrane element in the vessel.

The second element is required to produce less water ... 12% of 88%. So this k_w should be fine for elements #2 through #7.

5. Nitrogen removal in new Sprawlville wastewater plant

- (a) How much NH_4^+ removal in anaerobic, anoxic reactors?

We would not expect any NH_4^+ removal in these two reactors.

NH_4^+ is "removed" when it is transformed to nitrate, NO_3^- , via nitrification. That happens in the oxic/aerobic reactor. So in the first two reactors, NH_4^+ should be unreactive. $\xrightarrow{\text{no oxygen}} \xrightarrow{\text{no nitrification!}}$
 (Subtle note: there is a process called "anaerobic ammonium oxidation" or ANAMMOX, which could lead to NH_4^+ removal in these reactors. But that process usually does not occur to an appreciable extent so we will ignore it here.)

- (b) Loading of N to oxic/aerobic reactor

$$(900 \frac{\text{m}^3}{\text{hr}})(28 \frac{\text{mg N}}{\text{L}})\left(\frac{1000 \text{ L}}{\text{m}^3}\right)\left(\frac{1 \text{ kg}}{10^6 \text{ mg}}\right)\left(\frac{24 \text{ hr}}{\text{d}}\right) = \underline{\underline{600 \frac{\text{kg N}}{\text{d}}}}$$

- (c) Size the aerobic reactor

$$\text{Allowable loading} = 256 \frac{\text{mg}}{\text{L} \cdot \text{d}} * 0.8 \text{ pH correction} = 205 \frac{\text{mg N}}{\text{L} \cdot \text{d}}$$

$$= 205 \frac{\text{g N}}{\text{m}^3 \cdot \text{d}} = 0.205 \frac{\text{kg N}}{\text{m}^3 \cdot \text{d}}$$

$$\text{Volume} = \frac{(600 \frac{\text{kg N}}{\text{d}})(1.5 \text{ peaking factor})}{0.205 \frac{\text{kg N}}{\text{m}^3 \cdot \text{d}}} = \underline{\underline{4400 \text{ m}^3}}$$

$$\text{Residence time} = \frac{(28 \frac{\text{mg}}{\text{L}})(1.5 \text{ peaking})}{(256 \frac{\text{mg}}{\text{L} \cdot \text{d}})(0.8 \text{ correction})} = 0.205 \text{ d} = \underline{\underline{4.9 \text{ hr}}}$$

$$\text{Check volume} = Q \cdot t = (900 \frac{\text{m}^3}{\text{hr}})(4.9 \text{ hr}) = 4410 \text{ m}^3 \checkmark. \text{ They agree.}$$

6. BOD removal in the A²O process

(a) Why is BOD₅ entering the aerobic reactor so low?

Reason #1: Some of the carbon is used for denitrification in the anoxic reactor. When the carbon is oxidized by NO₃⁻, the BOD decreases as the carbon is removed.

Reason #2: The stream is diluted after it leaves the flow equalization tank. The RAS comes from the settling tank, and the NO₃⁻ return stream enters the anoxic tank. Both of these streams are low in BOD₅.

Since the original stream (80 mg/L) is being mixed with new streams at a lower conc., the overall conc. of BOD₅ decreases.

(b) Find the solids retention time.

$$\frac{1}{\theta_c} = \mu - k_d = \mu_{max} \frac{s}{s+k_s} - k_d = (2.0 \text{ d}^{-1}) \left(\frac{9 \text{ mg/L}}{9 \frac{\text{mg}}{\text{L}} + 60 \frac{\text{mg}}{\text{L}}} \right) = 0.14 \text{ d}^{-1}$$

$$\frac{1}{\theta_c} = 0.26 \text{ d}^{-1} - 0.14 \text{ d}^{-1} = 0.12 \text{ d}^{-1} \Rightarrow \underline{\underline{\theta_c = 8.3 \text{ d}}}$$

(c) Find the required volume and aeration time

$$\Theta = \frac{\theta_c Y (S_0 - S)}{X [1 + k_d \theta_c]} = \frac{(8.3 \text{ d})(0.6 \text{ mg/mg})(20 \text{ mg/L} - 9 \text{ mg/L})}{(2000 \text{ mg/L})[1 + (0.14 \text{ d}^{-1})(8.3 \text{ d})]}$$

$$\Theta = 0.0127 \text{ d} = \underline{\underline{0.30 \text{ hr}}} = 18 \text{ min ... very short!!}$$

$$V = \Theta Q = (0.30 \text{ hr})(900 \text{ m}^3/\text{hr}) = \underline{\underline{274 \text{ m}^3}}$$

Since we only need very little BOD removal ... 20 mg/L to 9 mg/L ... the reactor can be small and Θ can be short.

(d) What controls reactor size?

Need the bigger reactor so both processes can occur... 4400 m³ ... controls reactor size

denitrification

7. Sludge handling

② What is solids conc. of WAS in % by mass?

Consider 1 L of WAS.

We know it has 12,000 mg of solids (given), or 12 g solids.

We know it has a total mass of 1006 g (given).

$$\frac{12 \text{ g solids}}{1006 \text{ g sludge}} = 0.01193 = 1.19\% = \underline{\underline{1.2\% \text{ by mass}}}$$

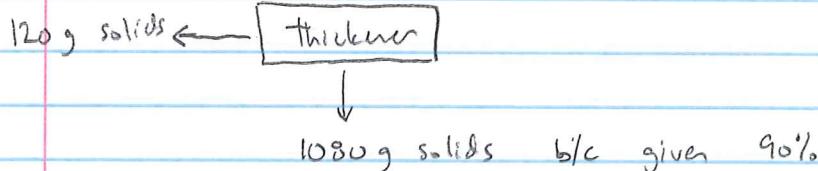
This is good, because we know $10,000 \frac{\text{mg}}{\text{L}} \approx 1\%$, so $12,000 \frac{\text{mg}}{\text{L}} \approx 1.2\% \checkmark$

③ For 100 L WAS, how much to digestion, how much returned?

100 L WAS

100,600 g WAS

1,200 g solids



Mass of thickened sludge: $\frac{1080 \text{ g solids}}{0.9} = 36,000 \text{ g thickened sludge}$

Volume of thickened sludge: $\frac{36,000 \text{ g sludge}}{1017 \text{ g/L}} = \underline{\underline{35.4 \text{ L thickened sludge to digester}}}$

Mass of liquid return: $100,600 \text{ g WAS} - 36,000 \text{ g thickened sludge}$
 $= 64,600 \text{ g of liquid return}$

Volume of liquid return: $\frac{64,600 \text{ g}}{1000 \text{ g/L}} = \underline{\underline{64.6 \text{ L liquid return}}}$

So the gravity thickener does a good job, reduces the sludge volume by approximately 65%.