Suppose we have data for Q_{in} and C_{in} at certain time intervals, like every hour.

Suppose we know the volume in the tank, V, at some time t.

We also know the concentration in the tank, C, at some time t.

We want to know V and C at time t+dt.

How do we make these estimates?

V is pretty easy.

Say $Q_{in}^{avg} = 0.5*[Q_{in}(t) + Q_{in}(t+dt)]$ …the average flow rate into the tank during the interval

Then $V(t+dt) = V(t) + (Q_{in}^{avg} – Q_{out})*dt$

That one, hopefully, is pretty easy to see.

C is much trickier.

In class we derived a differential equation for C(t), based on a mass balance.

$V*(dC/dt) = Q_{in} [C_{in}(t) – C(t)]$

But, how do we solve that differential equation to get C(t+dt) ?

Here are the steps.

(a) We already said $Q_{in}^{avg} = 0.5*[Q_{in}(t) + Q_{in}(t+dt)]$

(b) Likewise, say that $V^{avg} = 0.5*[V(t) + V(t+dt)]$ we know V(t+dt) because we found it above!

(c) Likewise, say that $C_{in}^{avg} = 0.5*[C_{in}(t) + C_{in}(t+dt)]$

(d) Then apply the following equation. This equation is not obvious. I would be happy to show you where this came from, but we'll have to do it during office hours.

$$C(t+\Delta t) = \frac{2V^{avg} - Q_{in}^{avg}\Delta t}{2V^{avg} + Q_{in}^{avg}\Delta t}\, C(t) + \frac{2Q_{in}^{avg}\Delta t}{2V^{avg} + Q_{in}^{avg}\Delta t}\, C_{in}^{avg}$$

If you apply that equation to the homework problem, assuming that the tank is 90% full at midnight, you can get V(t) and C(t) for the entire 24-hour period.