

ENV 6002: Physical and Chemical Principles of Environmental Engineering

Fall 2008

Homework #10

Due date: Tuesday, Nov. 25

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Civil & Environmental Eng.

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For this assignment, use the Excel workbook PULSES.xls, which will be provided to you, probably by posting on Blackboard. The PULSES spreadsheet calculates the solution to the advection-dispersion equation (with reaction) for two sets of conditions:

A. Response to an infinitesimal (instantaneous, or Dirac) pulse:

$$C = \frac{M_0/A}{\sqrt{4\pi E_x t}} \exp\left[-\frac{(x-ut)^2}{4E_x t}\right] \exp(-kt)$$

B. Response to a finite pulse, i.e., a steady input over a short period of time relative to the time of analysis:

$$C = \frac{1}{2} C_p \left\{ \operatorname{erf}\left[\frac{0.5 u \Delta t_p - (x-ut)}{\sqrt{4 E_x t}}\right] + \operatorname{erf}\left[\frac{0.5 u \Delta t_p + (x-ut)}{\sqrt{4 E_x t}}\right] \right\} \exp(-kt)$$

where the function $\operatorname{erf}(\bullet)$ is the error function, a well-known transcendental function that can be evaluated by Excel, as long as the data analysis tool pack is installed.

The inputs for the infinitesimal pulse option are the following.

M_0 = solute mass introduced at time $t = 0$ [kg]

A = channel cross-section area [m^2]

u = average velocity [m/min]

E_x = dispersion coefficient [m^2/min]

k = first-order decay rate constant [min^{-1}]

t = elapsed time after input [min]

For the finite pulse option, you also have to put in

Δt_p = time period over which the mass is introduced [min]

Important Note: the PULSES program only works in Excel 2003 if the **data analysis tool pack** is installed. Within Excel, if you pull down from the Tools menu, you should see an option for Data Analysis. If not, then you can pull down to Tools/Add-ins, and add the Analysis ToolPack. This will allow Excel to compute error functions, which are needed for Question 2. I have not yet tried running the PULSES program in Excel 2007, so I don't know how Excel 2007 implements

the error function. If you use Excel 2007 then you might have to do a little investigating to figure out how to get it to compute the error function.

1. (15 pts) Given: the same stream segment as in Question 3 of Homework Set #8, except that we now allow for longitudinal dispersion. Assume complete mixing in the transverse (i.e., y and z) directions.

Channel Depth, $H = 0.5$ m

Channel Width, $W = 2.0$ m

Average Velocity, $u = 2.5$ m/min

Dispersion Coefficient, $E_x = 5.0$ m²/min

- a. A conservative tracer is added as a sharp pulse of infinitesimal duration (i.e., Dirac pulse) at $t = 0$ min. The mass of the pulse is $M_0 = 50$ kg. On a single graph, plot the tracer's spatial concentration distribution (C vs x) at $t_1 = 500$ min and at $t_2 = 2000$ min. (Use the "infinitesimal pulse" worksheet within PULSES). Print out your graph, label it problem 1a, and submit it as part of your assignment.
 - b. For both t_1 and t_2 , determine the peak concentration, C_{\max} , and the value of x where it occurs.
 - c. Calculate the standard deviation of the spatial distribution (C vs. x) for both t_1 and t_2 , and indicate on the graph (i.e., draw in by hand) the x -values corresponding to \pm one and two standard deviations from the mean x for both t_1 and t_2 . The peaks are relatively narrow so it is a little tricky to draw in the locations of $\pm\sigma$ -- you might want to make separate graphs for t_1 and t_2 so that you can clearly draw in these lines. Print out your graphs, label them as problem 1c, and submit them as part of your assignment.
 - d. How does the elapsed time affect the peak concentration, C_{\max} ? the peak location, $x_{C=C_{\max}}$? the spread, $(\Delta x)_{\text{plume}}$, i.e., the range of x over which $C > 0.01C_{\max}$?
2. (10 pts) Instead of a non-reacting tracer, 50 kg of a degradable waste ($k = 0.001/\text{min}$) is added as a sharp infinitesimal (i.e., Dirac) pulse at $t = 0$. Other input parameters are the same as in problem 1.
 - a. Plot the waste's spatial concentration distribution at $t_1 = 500$ min and at $t_2 = 2000$ min. (Again use the "infinitesimal pulse" option of the program.) For comparison, put these on the same graph as your non-reacting pulses. Print out the graph, label it problem 2a, and submit it as part of your assignment. Indicate which curves are for the conservative tracer and which are for the degradable waste.
 - b. Compare the degradable waste to the non-reacting tracer: how does degradation affect the *peak concentration* and the *spread of the concentration distribution*?
 - c. For the degradable waste, what fraction of the original mass is degraded by time $t = 2000$ min? Compare your answer with that for an ideal CMFR and PFR.

3. (10 pts) Given: the same stream segment as in Question 1. A conservative tracer is added as a pulse ($M_0 = 50$ kg) of finite time duration over the interval $-5 < t < 5$ min., i.e., $\Delta t_p = 10$ min. The computations required below can be carried out using the “finite pulse” worksheet of the PULSES workbook.
- Plot the tracer’s spatial concentration distribution (C vs x) at $t_1 = 500$ min and at $t_2 = 2000$ min. Print out your graph, label it as problem 3c, and submit it as part of your assignment.
 - Compare the peak height and the spread to those in the infinitesimal scenario, i.e., Question 1a, for $t_1 = 500$ min and $t_2 = 2000$ min. Does the form of the pulse (infinitesimal vs short finite) appear to make a difference?
 - Now examine what each pulse (infinitesimal and short finite) looks like at $t = 20$ min. What is the peak concentration of each? For $t = 20$ min, does the form of the pulse appear to make a difference?
 - Based on your observations from parts b and c, discuss briefly the effect of the pulse form (infinitesimal vs. short finite) on the C vs x distribution, focusing on the effect of elapsed time.
4. (5 pts) The same degradable waste as in Question 2 (degradation rate constant, $k = 0.001/\text{min}$) is added as a finite pulse from $t = -5$ to $t = 5$ min. Plot the concentration distribution (C vs x) at times $t_1 = 500$ min and $t_2 = 2000$ min. Print out your graph, label it as problem 4, and submit it as part of your assignment. Compare to your results for the infinitesimal pulse (from problem 2a).
5. (10 pts)
- (5 pts) Evaluate the validity of assuming plug flow in a stream for a tracer in a stream segment where:
 $u_x = 0.33$ m/s; $H = 2$ m; $W = 20$ m; $L = 200$ m.
 $E_x = 12$ m²/s; $E_y = E_z = 0.15$ m²/s
 - (5 pts) Repeat the assessment for the tracer with $L = 2$ km and with $L = 20$ km. How does our confidence in assuming plug flow change with increasing stream segment length? Explain in terms of the theory of advective/dispersive transport.
 - [not required for 2008]** With a reactive species, how does your judgment of the validity of ideal plug flow depend on the value of dimensionless reaction time, $k_1 \bar{t}$? Assume constant C_1 and steady state and $Pe = 100$, which approximates ideal plug flow. Explain with reference to the reader pg. 11-15a-b.