ENV 6002: Physical and Chemical Principles of Environmental Engineering

Fall 2008	University of South Florida
Homework #8	Civil & Environmental Eng.
Due date: Thurs., Nov. 6 -- ***start now***!	J.A. Cunningham

1. (10 pts) A well-mixed rectangular reservoir has the following dimensions: depth 5 m; width 25 m; length 40 m. Water enters in a stream having a flow rate of $Q = 2.5$ m^3/min, and leaves at the opposite end at the same flow rate. At time $t = 0$, a 50 kg slug of a conservative tracer (chloride) is added instantaneously to the reservoir at the influent. Before the tracer is added, the chloride concentration in the reservoir, as well as in the influent, is zero; the influent concentration remains constant at zero for $t > 0$.
   a. Calculate the average residence time, $\bar{t}$ [min], of the water in the reservoir.
   b. Estimate the concentration of tracer in the reservoir, C_0, immediately after the tracer is added.
   c. Derive an expression for the concentration of tracer in the effluent as a function of elapsed time after the tracer addition. For this problem, you may assume that the concentration in the influent stream is zero except for the instant when the tracer is added. In a later problem, you will consider the more general case where perhaps the influent concentration is non-zero.
   d. What is the tracer concentration in the reservoir at a time equal to the average residence time, $\bar{t}$, under the conditions specified above?
   e. Draw a graph expressing how the dimensionless effluent concentration, C_E/C_0, depends on dimensionless time, $t/\bar{t}$, starting at $t = 0$ and continuing to five times the average residence time. What is the relationship between your graph and the *residence time distribution* of the reservoir?
   f. How long will it take for 90% of the added tracer to be washed out of the reservoir? For 99%? How do these times compare to the average residence time?

2. (10 pts) Under the same well-mixed conditions described above, 50 kg of a *degradable waste* is spilled instantaneously into the reservoir. Assume that the influent concentration, C_I, is zero for all t, apart from the pulse addition at $t = 0$.
   a. Assuming the reservoir to be rapidly mixed, estimate C_0 [g/m^3].
   b. Assuming that the degradation rate is first order with respect to the concentration of the waste, derive an equation for the effluent concentration as a function of time.

problem 2 continues →

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   c. Given that the degradation rate constant is 0.001/min, calculate and plot the dependence of the effluent concentration on elapsed time. Plot it as C_E/C_0 versus time. Include on the same graph a plot of C_E/C_0 for the conservative tracer from problem 1. Compare the two cases; explain any differences between the tracer and the degradable waste.
   d. What fraction of the original mass remains in the reservoir at $t = 0.1\ \bar{t}$? At $t = \bar{t}$? At $t = 10\ \bar{t}$?
   e. What fraction of the mass that has disappeared at $t = \bar{t}$ is accounted for by degradation? What fraction of the disappearance is accounted for by advection (washing out of the reservoir)?
   f. What is the average residence time of the waste in the reservoir? Compare to that of the tracer, and explain (by inspecting the solutions, or by explicit derivation) the relative magnitudes.

3. (10 pts) Given the same flow rate (2.5 m^3/min) as in Questions 1 and 2, now consider a channel segment 0.5 m deep, 2.0 m wide, and 5000 m long. You may assume that the conditions for ideal plug flow are satisfied. Assume also that this segment is one part of a much longer channel that extends indefinitely up- and down-stream. Chloride tracer (M_0 = 50 kg) is added at $x = 0$ over the 10 min time span $-5 < t < 5$ min, i.e., as a finite pulse.
   a. Calculate a concentration C_0, which we will define as follows: the mass injected into the channel segment divided by the volume of the channel segment.
   b. Calculate the concentration of the tracer in the "parcel" of water into which it was injected. Hint #1: this is different from part (a), because the tracer was injected into only a portion of the fluid, not into the entire channel segment. Hint #2: how much water flows past the injection point in a 10-minute time span?
   c. Calculate $\bar{t}$ for the channel segment of interest.
   d. Sketch the tracer concentration spatial distribution (C vs x) at $t = 1000$ min and at $t = 2000$ min. Put both lines on the same graph.
   e. Sketch the tracer concentration history at the exit of the channel segment. Plot in dimensionless form as C_E/C_0 vs. $t/\bar{t}$. For C_0 use your answer from part (a).
   f. Based on your answers to Questions 1(e) and 3(e), discuss the differences in behavior between the CMFR and PFR cases, particularly with regard to residence time distributions.

4. (5 pts) Assume the same conditions as in Question 3. This time, however, assume that a finite pulse of degradable waste is added (the same degradable waste as in Question 2). The mass of waste is $M_0 = 50$ kg, added over a 10 minute interval, $-5 < t < 5$ min. The degradation rate constant is $k = 0.001$/min.
   a. Calculate the fraction of mass remaining in the channel segment at time $t = 1000$ min, $t = 2000$ min, and $t = 3000$ min. Compare the results to the CMFR case from problem (2): how much of the waste remains in the well-mixed reservoir after $t = 1000$ min, $t = 2000$ min, and $t = 3000$ min? Give a brief discussion comparing the results from the PFR and the CMFR.
   b. Sketch the spatial concentration distribution, C vs x, of the waste, at times $t = 1000$ min and at $t = 2000$ min. Put both lines on the same graph. Compare the result to that of problem 3(d).

5. (10 pts) Now we'll go back to the CMFR and consider what happens if we have a non-zero influent concentration. Assume the same conditions as in Question 1 -- non-reacting tracer transport in a well-mixed reservoir with $Q = 2.5$ m^3/min and $M_0 = 50$ kg -- but this time assume a constant input of $C_I = 2$ g/m^3 for $-\infty < t < \infty$, in addition to the tracer pulse input at $t = 0$. In other words, tracer has been flowing into the reactor for a very long time before the tracer is added, at an inlet concentration $C_I = 2$ g/m^3.
   a. What is the concentration in the reactor immediately before the pulse is added (i.e., at a time infinitesimally smaller than $t = 0$)?
   b. What is the concentration in the reactor immediately after the pulse is added (i.e., at a time infinitesimally greater than $t = 0$)?
   c. Derive a generalized equation for the concentration of tracer in the effluent, C_E, as a function of elapsed time. Indicate in your equation the steady-state and transient parts of the solution.
   d. Plot the effluent concentration, C_E, vs. time, t. Indicate on your graph what portion of the effluent concentration comprises the steady state, and what portion represents the transient part.
   e. How does the presence of a steady input influence your estimate of the *average residence time of the tracer* in the reactor?

6. (5 pts) Assume the same well-mixed reservoir as in Question 1, but now we'll consider a step change of C_I. Before time $t = 0$, assume that the influent concentration is zero. Then, at time $t = 0$, the influent concentration of tracer rapidly increases to $C_I = 10$ g/m^3. The influent concentration remains steady at $C_I = 10$ g/m^3 for all time after $t = 0$. In this scenario, there is no pulse addition at $t = 0$.
   a. What is the concentration in the reactor, $C(t)$, at time $t = 0$? Hint: no calculations are required, just think about it.
   b. Derive an equation for the effluent concentration, C_E, as a function of elapsed time, t. Identify the steady state and transient parts of the solution.
   c. For the conditions given above, graph the solution as C_E vs t, and as C_E/C_I vs. $t/\bar{t}$.