ENV 6438: Physical & Chemical Processes for Drinking Water Treatment
Department of Civil & Environmental Engineering
University of South Florida

Cunningham **Spring 2020**

Homework #5 Sedimentation Due Wed., March 4

Assignment for 2020: Problem 1 ***or*** 2, then ***any three*** of the remaining problems

1. (55 pts) *This problem is adapted from one written by Paul Roberts of Stanford University.* **A spreadsheet or computer program is strongly recommended for this problem! Do not try it by hand unless you have a lot of extra time and you have been wondering how to fill it.**
Consider the following particle-size distribution for a suspension of particles. You can assume that the particles are spherical and have a density of 2.60 g/cm^3 =2600 kg/m^3.

particle size range (μm)	geometric mean diameter (μm)	number concentration ΔN (particles/m^3)
0.5 – 1	0.71	1.5×10^{12}
1 – 2	1.4	3.75×10^{11}
2 – 4	2.8	9.4×10^{10}
4 – 8	5.7	2.3×10^{10}
8 – 16	11.	5.9×10^{9}
16 – 32	23.	2.9×10^{9}
32 – 64	45.	7.3×10^{8}
64 – 128	91.	9.2×10^{7}

a. For each size class, calculate the particle mass concentration (g/m^3), the particle surface area concentration (m^2/m^3), and the particle volume fraction (m^3/m^3). Sum these to calculate the overall mass concentration, surface area concentration, and volume fraction.
b. Which particle size class dominates the number concentration? the mass concentration? the surface area concentration? For each of these, report the percentage contributed by the dominant size class.

problem 1 continues →

1. continued
   c. Suppose you treated this water by gravity sedimentation in a rectangular sedimentation basin with an overflow rate of 1.5 m/hr. The water temperature is 20 °C. Calculate the percentage reduction in the (overall) number concentration, mass concentration, and surface area concentration.
   d. Explain the results you found in part (c). Why was one of the concentrations reduced much more efficiently than another? How is this related to the way in which gravity sedimentation works?
   e. Repeat the calculations in part (c) if the density of the particles is 1.2 g/cm^3 (as might be expected for "fluffy" flocs leaving a flocculator for the sedimentation basin). How does the density change affect the results? Explain briefly.
   f. Repeat the calculations in part (c) if the water temperature is 5 °C instead of 20 °C. Use the original particle density of 2.6 g/cm^3. How does the change in water temperature affect the results? Explain briefly.
   g. Repeat the calculations in part (c) for the following overflow rates: 0.15 m/hr, 0.50 m/hr, 5.0 m/hr, 15 m/hr. Then make three graphs showing how the reduction in concentration depends upon the overflow rate. Each graph should have five data points on it corresponding to the five values of overflow rate that you considered. Briefly explain how the reduction in concentration depends upon overflow rate, and why.
   h. Suppose you needed to treat this water at a rate of 10 million gallons per day (pretty typical for centralized water treatment -- not too big, not too small). You have to design the sedimenation basins for this treatment. You want at least two sedimentation basins in parallel -- more than two is OK. How long and how wide would you make the sedimentation basins, and how many would you make? Briefly explain your reasoning. Hint: make use of your results from part (g) along with practical design considerations.

2. (55 pts) Consider water at 25 °C that has just exited the flocculation process and is *entering* a conventional sedimentation basin. There is a suspension of flocs in the water. Let's assume that all the flocs in the water are spheres. Also, the particle size distribution function is

$$\frac{dN}{dx} = Ae^{-bx}$$

where x indicates the diameter of the sphere, N indicates number concentration, and A and b are parameters of the distribution. You might remember (from homework #3) the meaning of this function: the number concentration ΔN of flocs between diameter x_1 and diameter x_2 is given by the following.

$$\Delta N = \int_{x_1}^{x_2} \left(\frac{dN}{dx}\right) dx$$

Suppose that for this particular distribution of flocs, $A = 2.3\times10^9$ particles/m^4 and $b = 2100$ m^{-1}.

a. (5 pts) Graph *dN/dx* vs *x*. Let x go from 0.1 μm (10^{-7} m) up to 1.0 cm (10^{-2} m). Present the graph three ways: once with both axes drawn on a linear scale, once with the abscissa (y-axis) drawn on a logarithmic scale, and once on a log-log scale.

b. (5 pts) Estimate/calculate the *total* number concentration of flocs in the water, N_T. This means you have to integrate over all possible values of x, i.e., from $x = 0$ to $x = \infty$. Report your answer in units of particles/m^3. (The integral is not very difficult.)

c. (5 pts) The volume of a sphere of diameter x is $\pi x^3/6$. Therefore the volume concentration of spheres in the water (i.e., volume of flocs per volume of water) is given by

$$\int_0^\infty \left(\frac{\pi x^3}{6}\right)\left(\frac{dN}{dx}\right) dx$$

Estimate the volume concentration (volume fraction) Ω for the floc suspension in question. Hint: you will end up with an integral that you probably want to look up in an integral table – you can evaluate it yourself using integration by parts multiple times, but that is a pain – it is probably easier to just look it up.

d. (5 pts) If the flocs have a density of 1500 kg/m^3 = 1500 g/L, estimate/calculate the mass concentration of flocs in the water, in units of mg/L.

problem 2 continues →

2. continued

e. (5 pts) Suppose the overflow rate in the sedimentation basin is 1.5 m/hr. Estimate/calculate the diameter of the smallest flocs that will be 100% removed by sedimentation. In other words, flocs with diameter greater than or equal to a diameter of x_{crit} will be 100% removed; flocs with diameter smaller than x_{crit} will be partially removed; find x_{crit}. If you use Stokes' law, double-check that the Reynolds number is in the proper range.

f. (5 pts) Now let's think about the flocs *exiting* the sedimentation basin. For a floc of size x, argue or show that the fraction of flocs remaining in the water after sedimentation is

$$f_{\text{remaining}} = 0 \quad \text{if} \quad x \geq x_{\text{crit}},$$

$$f_{\text{remaining}} = 1 - c\,x^2 \quad \text{if} \quad x < x_{\text{crit}}.$$

Give the formula for the parameter c in terms of the other parameters of the problem.

g. (5 pts) Plot $f_{\text{remaining}}$ vs x, for x going from 0 to 100 μm. Use the formula from part (f).

h. (5 pts) The particle size distribution for flocs *exiting* the sedimentation basin is given by

$$\left(\frac{dN}{dx}\right)_{\text{exiting}} = \left(f_{\text{remaining}}\right)\left(\frac{dN}{dx}\right)_{\text{entering}} = \left(f_{\text{remaining}}\right)\left(Ae^{-bx}\right)$$

Plot this function vs x, for x going from 0 to 100 μm. Make both axes linear (not logarithmic). On the same graph, plot dN/dx for the *entering* flocs, i.e., the same function as in part (a), but only going from 0 to 100 μm. Based on the graph, does sedimentation remove a lot of particles?

i. (10 pts) Now let's put it all together. Find the total *number concentration* of flocs *exiting* the sedimentation basin. That is, repeat the calculation of part (b), but using the particle size distribution for the *effluent* rather than the influent. You can do it either analytically or numerically. I am not sure yet which way is easier, analytically or numerically – I plan to do it analytically. NOTE: I thought about asking for the effluent mass concentration too, but I decided that would be too much math for now. Maybe next year I will add that on.

j. (5 pts) How well did the sedimentation basin do? Did you see a significant removal of particles? Do you think the water is now particle-free, or will filtration still be necessary?

3. (15 pts)
   a. Suppose you were interested in deriving an equation for the terminal settling velocity of particles during "type 1" settling, for cases where the Reynolds number is between 1 and 100. One estimate for the relationship between Re and drag coefficient (C_D) in this regime is $C_D = 18.5\ Re^{-0.6}$. Using this relationship, derive a closed-form expression for the settling velocity of the particle. Show your work. How does the settling velocity depend upon the particle diameter in this regime?
   b. Repeat your derivation for the range $5 < Re < 1000$, using the estimated relationship $C_D = 13\ Re^{-0.5}$. How does settling velocity depend on particle diameter according to this relationship?

4. (15 pts)
   a. Answer questions 10-1A and 10-1C in the text book.
   b. Which particle settles faster? What is the dominant reason why that one settles faster?

5. (15 pts) We know that for sufficiently low values of Reynolds number (Re), the drag coefficient $C_D = 24/Re$. However, for $Re > 1$ (or so), this relationship does not hold. A number of empirical relationships have been used to estimate how C_D depends on Re in the "transition region" where $1 < Re < 1000$. In class, I gave you four such relationships. Graph C_D as a function of Re for the range $1 < Re < 1000$, using all four relationships. Make both axes logarithmic. Make sure all the lines are distinguishable from each other. Do all four relationships agree reasonably well? If so, then it does not matter which one we choose, but if not, then we might need to explore further to determine if one of these relationships is better or worse than the others. Do any of the four curves deviate strongly from the other three? – if so, perhaps we can eliminate that curve from consideration.

6. (15 pts) All of our calculations for settling velocity are based on an assumption of spherical particles. But we know that particles are not necessarily spherical; in homework 3, we had "flaky" particles. Re-do the force balance (p 646 of the text, or notes from class #11) for the settling of a "flaky" elliptical particle like we had in homework 3. Derive a formula for the terminal settling velocity of such particles. If possible, express the settling velocity in terms of the sphericity, ψ, which we saw previously. As particles become less spherical, how does that affect the settling velocity? If we pretend that particles are spherical (when they are not), are we being overly optimistic or overly pessimistic?

7. (15 pts) Suppose a sand particle is settling in a sedimentation basin at 20 °C. The particle is roughly spherical in shape with a diameter of 0.73 mm and a density of 2.60 g/cm^3. Assume that the particle is in the "transition" regime, i.e., Re > 5 or so. Estimate the particle's settling velocity using *each* of the four different models for the relationship between drag coefficient and Reynolds number (based on your notes from class). Report the settling velocity and the Reynolds number that you estimate using each of the four models. How much variation is there between the predictions of the four models? Do you think it matters which one you choose, or do they all give you pretty much the same results? (Also verify that the Reynolds number is in the transition regime, not the laminar/Stokes regime.)

8. (15 pts) In class, we derived a formula for the terminal settling velocity of spherical particles. However, we did not determine how long it takes a particle to reach its terminal settling velocity, assuming that the particle starts at rest. If the time to reach terminal velocity is long compared to the residence time in a sedimentation basin, then a lot of our calculations are off. Hopefully, we will see that terminal velocity is reached in a time much less than the residence time of the basin – in that case, our methodologies are intact. So let's find out.

   Force balance for a spherical particle: $F_{net} = (\pi/6)*d_p^3*(\rho_p-\rho_L)*g - 0.5*C_D*\rho_L*A_{proj}*v^2$

   Drag coefficient: $C_D = 24/Re$ *assuming* that we are in the Stokes' flow regime

   Reynolds number: $Re = \rho_L * v * d_p / \mu$

   Newton's law: $F_{net} = m * dv/dt$, where $m = (\pi/6)*d_p^3*\rho_p$

   Initial condition: $v(t=0) = 0$

   Combine the first four equations to derive a differential equation $dv/dt = f(v)$. Solve that differential equation subject to the initial condition. You should get an answer that is of the following general form:

   $v(t) = v_{terminal} * [1 - \exp(-kt)]$

   where k is an apparent rate coefficient that depends on physical parameters of the problem. We can estimate that terminal velocity is reached within a time 5/k.
   For a spherical particle that has a diameter of 0.1 μm and a density of 2500 kg/m^3, how long does it take to reach terminal velocity? What if the diameter is 100 μm instead? Is terminal velocity reached rapidly enough that we can assume the velocity is constant?

9. (15 pts) Answer question 10-7D in the Crittenden text. (Only recommended if you are interested in solids thickening, which is not "type 1" settling. We did not discuss this in class. You will have to figure it out on your own.)

10. (15 pts) Suppose you are designing a circular sedimentation basin that uses center feed with radial collection (see Figure 10-13 in your text). Base your design on a flow rate of 20 million gallons per day, a water temperature of 20 °C, and a particle (floc) removal efficiency of 99%. Assume that the flocs are spherical with diameter 0.1 mm and density 1200 kg/m^3. The depth of the clarifier is 4.5 m, and the radius of the inlet zone is 1.5 m. What is the minimum outer diameter for the clarifier?

11. (15 pts) In class we derived expressions for settling velocity assuming a spherical shape for the particles. If the particles are not spherical, then you can proceed as follows. Find the diameter of an "equivalent" sphere that has the same surface-area-to-volume ratio as the particle of interest. Then find the settling velocity of that "equivalent" sphere. That will be a good estimate for the settling velocity of the particle of interest.
    a. Use this method to estimate the settling velocity of the smallest size class of "flaky particles" from homework #3. (Hint: if you're clever enough, you can use the sphericity to help save some time on this part.)
    b. Repeat for the largest size class of "flakes" from homework #3. You should notice something interesting. Explain briefly.
    c. Estimate the fractional removal of the two size classes of flaky particles in a sedimentation basin with an overflow rate of 1.5 m/hr. Is sedimentation an effective removal mechanism for these flaky particles?