SOLUTIONS MANUAL

MICROWAVE TRANSISTOR AMPLIFIERS

Analysis and Design

SECOND EDITION

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PRENTICE HALL
Upper Saddle River, NJ 07458

Acquisitions Editor: *Eric Svendsen*
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Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-271255-5

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
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CHAPTER 1

1.1) IN (1.3.41) LET $\Gamma_o = |\Gamma_o| e^{j\psi_o}$:

$$|V(d)| = |A_1| \, |1 + |\Gamma_o| e^{j(\psi_o - 2\beta d)}|$$

THE MAXIMUM VALUE OF $|V(d)|$ OCCURS WHEN $e^{j(\psi_o - 2\beta d)} = 1$, AND THE MINIMUM VALUE OCCURS WHEN $e^{j(\psi_o - 2\beta d)} = -1$. HENCE,

$$|V(d)|_{max} = |A_1| \, |1 + |\Gamma_o|| = |A_1| (1 + |\Gamma_o|) \quad \text{AND} \quad |V(d)|_{min} = |A_1| \, |1 - |\Gamma_o|| = |A_1| (1 - |\Gamma_o|)$$

1.2) $v(d,t) = \operatorname{Re}[V(d) e^{j\omega t}]$

$$V(d) = 3.95 \, e^{j(\beta d - 63.44^\circ)} + 1.77 \, e^{-j\beta d}$$

$$\therefore \; v(d,t) = 3.95 \cos(\omega t + \beta d - 63.44^\circ) + 1.77 \cos(\omega t - \beta d)$$

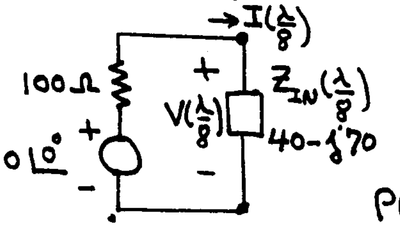
$$\text{AND} \quad i(d,t) = \frac{3.95}{50} \cos(\omega t + \beta d - 63.44^\circ) - \frac{1.77}{50} \cos(\omega t - \beta d)$$

1.3) (a) $\Gamma_o = \dfrac{Z_L - Z_o}{Z_L + Z_o} = \dfrac{(100 + j100) - 50}{(100 + j100) + 50} = 0.62 \angle 29.7^\circ$

$$Z_{IN}\left(\tfrac{\lambda}{8}\right) = 50 \, \frac{(100 + j100)\cos 45^\circ + j50 \sin 45^\circ}{50 \cos 45^\circ + j(100 + j100)\sin 45^\circ} = 40 - j70 \; \Omega$$

$$\text{VSWR} = \frac{1 + |\Gamma_o|}{1 - |\Gamma_o|} = \frac{1 + 0.62}{1 - 0.62} = 4.26$$

(b)



$$V\left(\tfrac{\lambda}{8}\right) = \frac{10\angle 0^\circ (40 - j70)}{40 - j70 + 100} = 5.15 \angle -33.7^\circ$$

$$I\left(\tfrac{\lambda}{8}\right) = V\left(\tfrac{\lambda}{8}\right) / Z_{IN}\left(\tfrac{\lambda}{8}\right) = 0.064 \angle 26.6^\circ$$

$$P\left(\tfrac{\lambda}{8}\right) = \tfrac{1}{2} \operatorname{Re}\left[V\left(\tfrac{\lambda}{8}\right) I^*\left(\tfrac{\lambda}{8}\right)\right] = 82 \text{ mW}$$

$$V\left(\tfrac{\lambda}{8}\right) = 5.15 \angle -33.7 = A_1 e^{j\frac{\pi}{4}} \left[1 + 0.62 \angle 29.7^\circ e^{-j\frac{\pi}{2}}\right]$$

$$\therefore \; A_1 = 3.643 \angle -56.31^\circ$$

$$V(d) = 3.643\angle{-56.31^\circ}\, e^{j\beta d}\left[1 + 0.62\angle{29.7^\circ}\, e^{-j2\beta d}\right]$$

$$V(0) = 3.643\angle{-56.31^\circ}\left[1 + 0.62\angle{29.7^\circ}\right] = 5.72\angle{-45.02^\circ}$$

$$I(0) = \frac{5.72\angle{-45.02^\circ}}{100 + j100} = 0.04\angle{-90^\circ}$$

$$P(0) = \frac{1}{2}\,\mathrm{Re}\left[V(0) I^*(0)\right] = 82 \text{ mW}$$

AS EXPECTED: $P\left(\frac{\lambda}{8}\right) = P(0)$

1.4) $$b_1 = S_{11} a_1 + S_{12} a_2$$
$$b_2 = S_{21} a_1 + S_{22} a_2$$

LETTING $Z_o = Z_{01} = Z_{02}$, THEN $b_1 = \frac{V_1^-}{\sqrt{Z_o}}$, $b_2 = \frac{V_2^-}{\sqrt{Z_o}}$, $a_1 = \frac{V_1^+}{\sqrt{Z_o}}$, AND $a_2 = \frac{V_2^+}{\sqrt{Z_o}}$.

$$\therefore \frac{V_1^-}{\sqrt{Z_o}} = S_{11}\frac{V_1^+}{\sqrt{Z_o}} + S_{12}\frac{V_2^+}{\sqrt{Z_o}} \Rightarrow V_1^- = S_{11} V_1^+ + S_{12} V_2^+$$

$$\frac{V_2^-}{\sqrt{Z_o}} = S_{21}\frac{V_1^+}{\sqrt{Z_o}} + S_{22}\frac{V_2^+}{\sqrt{Z_o}} \Rightarrow V_2^- = S_{21} V_1^+ + S_{22} V_2^+$$

WITH $b_1 = \sqrt{Z_o}\, I_1^-$, $b_2 = \sqrt{Z_o}\, I_2^-$, $a_1 = \sqrt{Z_o}\, I_1^+$, $a_2 = \sqrt{Z_o}\, I_2^+$

WE OBTAIN: $$I_1^- = S_{11} I_1^+ + S_{12} I_2^+$$
$$I_2^- = S_{21} I_1^+ + S_{22} I_2^+$$

1.5) $$\begin{cases} a_1 = T_{11} b_2 + T_{12} a_2 & (1) \\ b_1 = T_{21} b_2 + T_{22} a_2 & (2) \end{cases} \qquad \begin{cases} b_1 = S_{11} a_1 + S_{12} a_2 & (3) \\ b_2 = S_{21} a_1 + S_{22} a_2 & (4) \end{cases}$$

FROM (3) AND (4):

$$a_1 = \frac{1}{S_{11}} b_1 - \frac{S_{12}}{S_{11}} a_2 \quad (5)$$

$$a_1 = \frac{1}{S_{21}} b_2 - \frac{S_{22}}{S_{21}} a_2 \quad (6)$$

EQUATING (5) AND (6):

$$\frac{1}{S_{11}} b_1 - \frac{S_{12}}{S_{11}} a_2 = \frac{1}{S_{21}} b_2 - \frac{S_{22}}{S_{21}} a_2$$

$$\therefore \quad b_1 = \frac{S_{11}}{S_{21}} b_2 + \left[S_{12} - \frac{S_{22}S_{11}}{S_{21}}\right] a_2 \quad (7)$$

COMPARING (2) AND (7):

$$T_{21} = \frac{S_{11}}{S_{21}} \quad \text{AND} \quad T_{22} = S_{12} - \frac{S_{11}S_{22}}{S_{21}}$$

EQUATING (3) AND (7) GIVES

$$S_{11}a_1 + S_{12}a_2 = \frac{S_{11}}{S_{21}} b_2 + \left[S_{12} - \frac{S_{11}S_{22}}{S_{21}}\right] a_2$$

$$\therefore a_1 = \frac{1}{S_{21}} b_2 - \frac{S_{22}}{S_{21}} a_2 \quad (8)$$

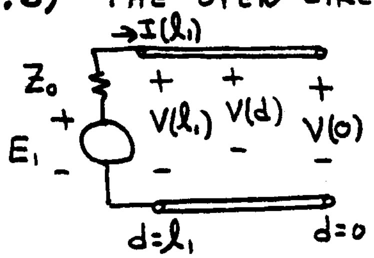
COMPARING (1) AND (8):

$$T_{11} = \frac{1}{S_{21}} \quad \text{AND} \quad T_{12} = -\frac{S_{22}}{S_{21}}$$

SIMILARLY, STARTING WITH (1) AND (2), IT FOLLOWS THAT:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \frac{T_{21}}{T_{11}} & T_{22} - \frac{T_{12}T_{21}}{T_{11}} \\ \frac{1}{T_{11}} & -\frac{T_{12}}{T_{11}} \end{bmatrix}$$

1.6) THE OPEN-CIRCUIT VOLTAGE $E_{1,TH}$ IS OBTAINED AS FOLLOWS:



$\Gamma_0 = 1 \qquad V(d) = A(e^{j\beta d} + e^{-j\beta d}) = 2A\cos\beta d$

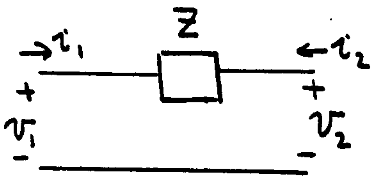
$I(d) = j2\frac{A}{Z_0}\sin\beta d$

$V(\ell_1) = 2A\cos\beta\ell_1 = E_1 - I(\ell_1)Z_0$

$= E_1 - j2A\sin\beta d$

$$\therefore A = \frac{E_1}{2(\cos\beta\ell_1 + j\sin\beta\ell_1)} = \frac{E_1}{2} e^{-j\beta\ell_1}$$

HENCE: $E_{1,TH} = V(0) = 2A = E_1 e^{-j\beta\ell_1}$

1.7) (a) 

$$v_1 = A v_2 - B i_2$$
$$i_1 = C v_2 - D i_2$$

$A = \left.\dfrac{v_1}{v_2}\right|_{i_2=0}$ WITH $i_2=0$: $v_1 = v_2$

$\therefore A = 1$

$B = -\left.\dfrac{v_1}{i_2}\right|_{v_2=0}$ WITH $v_2=0$: $v_1 = i_1 Z = -i_2 Z$

$\therefore B = -\dfrac{(-i_2 Z)}{i_2} = Z$

$C = \left.\dfrac{i_1}{v_2}\right|_{i_2=0}$ WITH $i_2=0$: $i_1 = -i_2 = 0$

$\therefore C = 0$

$D = \left.\dfrac{i_1}{-i_2}\right|_{v_2=0}$ WITH $v_2=0$: $i_1 = -i_2$

$\therefore D = 1$

(b) FROM FIG. 1.8.1:

$$S_{11} = \frac{A'+B'-C'-D'}{A'+B'+C'+D'} = \frac{1+\frac{Z}{Z_0}-0-1}{1+\frac{Z}{Z_0}+0+1} = \frac{Z}{Z+2Z_0}$$

$$S_{12} = \frac{2(A'D'-B'C')}{A'+B'+C'+D'} = \frac{2(1-0)}{1+\frac{Z}{Z_0}+0+1} = \frac{2Z_0}{Z+2Z_0}$$

$$S_{21} = \frac{2}{A'+B'+C'+D'} = \frac{2}{1+\frac{Z}{Z_0}+0+1} = \frac{2Z_0}{Z+2Z_0}$$

$$S_{22} = \frac{-A'+B'-C'+D'}{A'+B'+C'+D'} = \frac{-1+\frac{Z}{Z_0}-0+1}{1+\frac{Z}{Z_0}+0+1} = \frac{Z}{Z+2Z_0}$$

FOR THE SHUNT ADMITTANCE Y THE ABCD MATRIX IS :

$A = 1$, $B = 0$, $C = Y$, AND $D = 1$

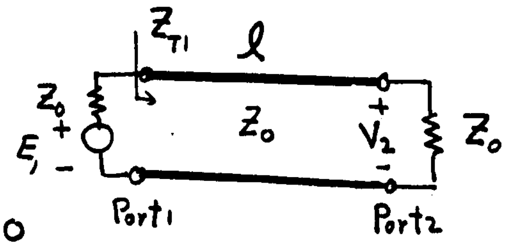
AND

$$S_{11} = \frac{A'+B'-C'-D'}{A'+B'+C'+D'} = \frac{1+0-YZ_0-1}{1+0+YZ_0+1} = \frac{-YZ_0}{2+YZ_0}$$

$$S_{12} = \frac{2(A'D'-B'C')}{A'+B'+C'+D'} = \frac{2(1-0)}{1+0+YZ_0+1} = \frac{2}{2+YZ_0}$$

ALSO: $S_{22} = S_{11}$ AND $S_{21} = S_{12}$

1.8) IN A Z_0 SYSTEM:



$$Z_{T1} = Z_0$$

$$S_{11} = \frac{Z_{T1} - Z_0}{Z_{T1} + Z_0} = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0$$

ALSO (FROM SYMMETRY): $S_{22} = 0$

$$S_{21} = 2\frac{\sqrt{Z_0}}{\sqrt{Z_0}}\frac{V_2}{E_1} = 2\frac{V_2}{E_1}$$

SINCE $V(x) = \frac{E_1}{2} e^{-j\beta x}$

$$V_2 = V(\ell) = \frac{E_1}{2} e^{-j\beta\ell}$$

$$\therefore\ S_{21} = \frac{2\left(\frac{E_1}{2} e^{-j\beta\ell}\right)}{E_1} = e^{-j\beta\ell}$$

ALSO (FROM SYMMETRY): $S_{12} = e^{-j\beta\ell}$

AN ALTERNATE WAY OF DERIVING S_{21} IS:

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

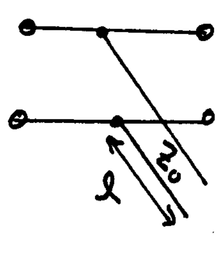
SINCE b_2 IS THE WAVE AT PORT 2 AND a_1 IS THE WAVE AT PORT 1, WE HAVE:

$$b_2 = a_1 e^{-j\beta\ell} \quad (a_2 = 0)$$

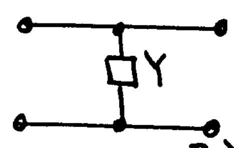
$$\therefore\ S_{21} = \frac{b_2}{a_1} = e^{-j\beta\ell}$$

$$T = \begin{bmatrix} \frac{1}{S_{21}} & -\frac{S_{22}}{S_{21}} \\ \frac{S_{11}}{S_{21}} & S_{12} - \frac{S_{11}S_{22}}{S_{21}} \end{bmatrix} = \begin{bmatrix} e^{j\beta\ell} & 0 \\ 0 & e^{-j\beta\ell} \end{bmatrix}$$

1.9)



$\Rightarrow$



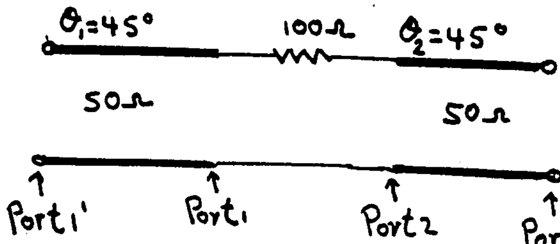
$$Y = \frac{1}{Z_{oc}} = \frac{1}{-jZ_0 \cot\beta\ell} = jY_0 \tan\beta\ell$$

$$S_{11} = S_{22} = \frac{-Z_0 Y}{2 + Z_0 Y} = \frac{-1}{1 - j2\cot\beta\ell}$$

$$S_{12} = S_{21} = \frac{2}{2 + Z_0 Y} = \frac{2}{2 + j\tan\beta\ell}$$

USE (1.4.11) TO EVALUATE THE T PARAMETERS.

1.10)



(Z_0 SYSTEM)

THE S PARAMETERS AT PORTS 1 AND 2 ARE:

$$S_{11} = \frac{Z}{Z + 2Z_0} = \frac{100}{100 + 2(50)} = 0.5$$

$$S_{22} = S_{11} = 0.5$$

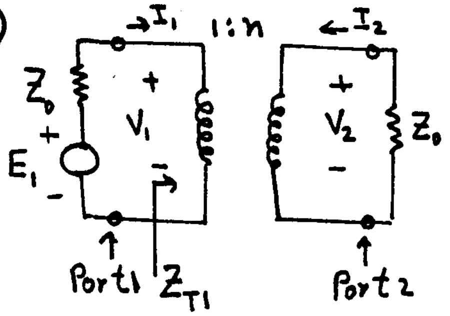
$$S_{12} = S_{21} = \frac{2Z_0}{Z + 2Z_0} = 0.5$$

THE S PARAMETERS AT PORTS 1' AND 2' ARE OBTAINED USING (1.5.4):

$$S_{11}' = S_{22}' = S_{11} e^{-j2\theta_1} = 0.5\,\underline{/-90°} = -j0.5$$

$$S_{21}' = S_{12}' = S_{21} e^{-j(\theta_1 + \theta_2)} = 0.5\,\underline{/-90°} = -j0.5$$

1.11)



IN THE Z_0 SYSTEM SHOWN, THE PARAMETERS S_{11} AND S_{21} ARE CALCULATED AS FOLLOWS:

$$I_2 = \frac{I_1}{n}, \quad V_2 = V_1 n$$

$$Z_{T1} = \frac{V_1}{I_1} = \frac{V_2}{I_2 n^2} = \frac{Z_0}{n^2}$$

$$S_{11} = \frac{b_1}{a_1}\bigg|_{a_2=0} = \frac{Z_{T1} - Z_0}{Z_{T1} + Z_0} = \frac{Z_0/n^2 - Z_0}{\frac{Z_0}{n^2} + Z_0} = \frac{1 - n^2}{1 + n^2}$$

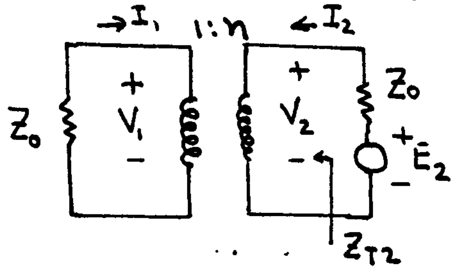
$$S_{21} = 2\sqrt{\frac{Z_{02}}{Z_{01}}}\frac{V_2}{E_1} = 2\frac{V_2}{E_1} \quad (1)$$

$$V_1 = \frac{E_1 Z_{T1}}{Z_{T1} + Z_0} = \frac{E_1 Z_0/n^2}{\frac{Z_0}{n^2} + Z_0} = \frac{E_1}{n^2 + 1}$$

$$V_2 = nV_1 = \frac{nE_1}{n^2 + 1} \quad \text{OR} \quad \frac{V_2}{E_1} = \frac{n}{n^2 + 1} \quad (2)$$

(2) INTO (1):

$$S_{21} = \frac{2n}{n^2 + 1}$$



THE PARAMETERS S_{22} AND S_{12} ARE CALCULATED AS FOLLOWS:

$$Z_{T2} = Z_0 n^2$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} = \frac{Z_{T2}-Z_0}{Z_{T2}+Z_0} = \frac{n^2 Z_0 - Z_0}{n^2 Z_0 + Z_0} = \frac{n^2-1}{n^2+1}$$

$$S_{12} = 2\frac{V_1}{E_2} \quad (3)$$

$$V_2 = \frac{E_2 Z_{T2}}{Z_{T2}+Z_0} = \frac{E_2 Z_0 n^2}{Z_0 n^2 + Z_0} = \frac{E_2 n^2}{n^2+1}$$

$$V_1 = \frac{V_2}{n} = \frac{E_2 n}{n^2+1} \quad \text{OR} \quad \frac{V_1}{E_2} = \frac{n}{n^2+1} \quad (4)$$

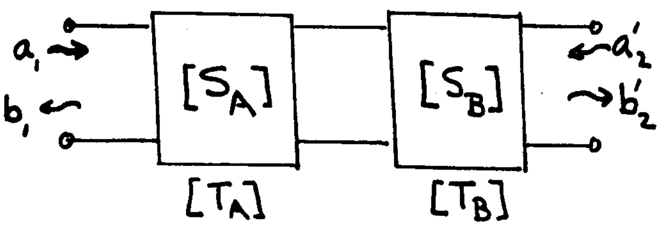
(4) INTO (3):

$$S_{12} = \frac{2n}{n^2+1}$$

AT PORTS 1' AND 2', WITH $\theta = \theta_1 = \theta_2$, WE OBTAIN:

$$[S'] = \begin{bmatrix} S_{11}e^{-j2\theta} & S_{12}e^{-j2\theta} \\ S_{21}e^{-j2\theta} & S_{22}e^{-j2\theta} \end{bmatrix} = e^{-j2\theta}\begin{bmatrix} \frac{1-n^2}{1+n^2} & \frac{2n}{n^2+1} \\ \frac{2n}{n^2+1} & \frac{n^2-1}{n^2+1} \end{bmatrix}$$

1.12)



[T] AND [S] PARAMETERS ARE RELATED BY (1.4.11)

FROM (1.4.13):

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} T_{11}^A & T_{12}^A \\ T_{21}^A & T_{22}^A \end{bmatrix}\begin{bmatrix} T_{11}^B & T_{12}^B \\ T_{21}^B & T_{22}^B \end{bmatrix}\begin{bmatrix} b_2' \\ a_2' \end{bmatrix}$$

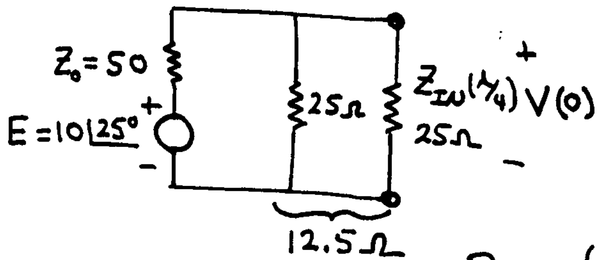
OVERALL T_{11} IS: $\quad T_{11} = T_{11}^A T_{11}^B + T_{12}^A T_{21}^B$

$$T_{11} = \frac{1}{S_{21}^A}\frac{1}{S_{21}^B} + \left(-\frac{S_{22}^A}{S_{21}^A}\right)\left(\frac{S_{11}^B}{S_{21}^B}\right)$$

SINCE $T_{11} = \dfrac{1}{S_{21}} = \dfrac{1-S_{22}^A S_{11}^B}{S_{21}^A S_{21}^B} \Rightarrow S_{21} = \dfrac{S_{21}^A S_{21}^B}{1-S_{22}^A S_{11}^B}$

1.13) (a) $\Gamma_0 = \dfrac{100-50}{100+50} = \dfrac{1}{3}$, $VSWR = \dfrac{1+\frac{1}{3}}{1-\frac{1}{3}} = 2$

(b) $Z_{IN}(\lambda/4) = \dfrac{Z_0^2}{Z_L} = \dfrac{50^2}{100} = 25\,\Omega$



$V(0) = \dfrac{10\angle 25°\,(12.5)}{12.5+50} = 2\angle 25°$

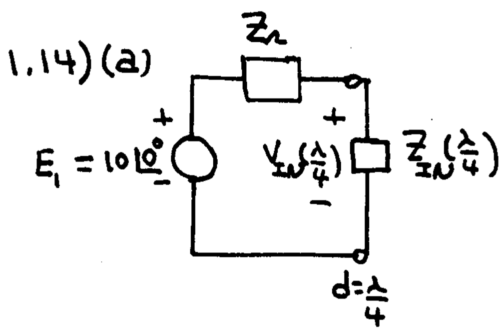
$P_{AVS} = \dfrac{|E|^2}{8\,Z_0} = \dfrac{10^2}{8(50)} = 0.25\text{ W}$

THE INPUT POWER IS : $P_{IN} = \dfrac{(V(0)_{rms})^2}{12.5} = \dfrac{\left(\frac{2}{\sqrt{2}}\right)^2}{12.5} = 0.16\text{ W}$

SINCE THE TRANSMISSION LINE IS LOSSLESS, THE POWER DELIVERED TO THE LOAD (P_L) IS THE SAME AS P_{IN}.

$\therefore\ P_L = P_{IN} = 0.16\text{ W}$

1.14) (a)



$Z_{IN}(\frac{\lambda}{4}) = \dfrac{Z_0^2}{Z_L} = \dfrac{50^2}{50+j50} = 25-j25$

NOTE: IF Z_s IS GIVEN, THE VALUE OF Z_{IN} FOR MAXIMUM POWER IS $Z_{IN} = Z_s^*$. HOWEVER, IN THIS PROBLEM Z_{IN} IS GIVEN. HENCE, THE VALUE OF Z_s FOR MAX. POWER DELIVERED TO Z_{IN} IS: $Z_s = -\,\text{Im}[Z_{IN}]$

$\therefore\ Z_s = j25$

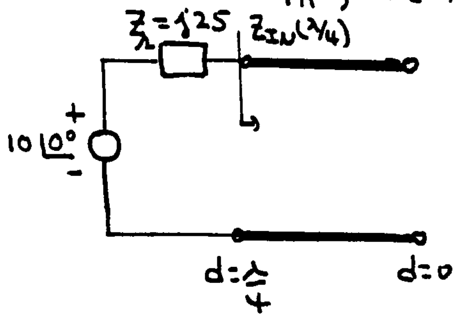
$V_{IN}(\frac{\lambda}{4}) = \dfrac{10\angle 0°\,(25-j25)}{j25+(25-j25)} = 10-j10 = 14.14\angle -45°$

$I_{IN}(\frac{\lambda}{4}) = \dfrac{14.14\angle -45°}{25-j25} = 0.4$

$P_{IN} = \frac{1}{2}\,\text{Re}\left[14.14\angle -45°\ (0.4)\right] = 2\text{ W}$

$P_L = P_{IN} = 2\text{ W}$

(b) TO FIND E_{TH}, WE FIND THE OPEN CIRCUIT VOLTAGE AT $d=0$:



$\Gamma_0 = 1$, $V(d) = 2A\cos\beta d$

$I(d) = j\dfrac{2A}{Z_0}\sin\beta d \quad (1)$

$Z_{IN}(\frac{\lambda}{4}) = 0$ (A SHORT CIRCUIT)

$\therefore I(\frac{\lambda}{4}) = \dfrac{10\angle 0^\circ}{j25+0} = -j0.4 \quad (2)$

FROM (1) AND (2): $-j0.4 = j\dfrac{2A}{50}\sin\dfrac{\pi}{2} \Rightarrow A = -10$

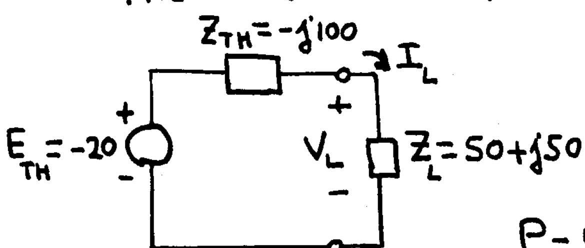
HENCE: $V(d) = 2(-10)\cos\beta d = -20\cos\beta d$

$E_{TH} = V(0) = -20$

TO FIND Z_{TH} WE SET $E_1 = 0$, THEN:

$$Z_{TH} = \frac{Z_0^2}{Z_\ell} = \frac{50^2}{j25} = -j100$$

THE THEVENIN'S EQUIVALENT CIRCUIT AT $d=0$ IS:



$I_L = \dfrac{-20}{-j100+50+j50} = 0.283\angle -135^\circ$

$V_L = 0.283\angle -135^\circ(50+j50) = 20\angle -90^\circ$

$P_L = \frac{1}{2}\text{Re}[20\angle -90^\circ(0.283\angle 135^\circ)] = 2\text{W}$

1.15) $v_2(0) = \dfrac{V_2(0)}{\sqrt{Z_0}} = a_2(0) + b_2(0) = 0 + 3.54\angle 45^\circ$

$\therefore V_2(0) = \sqrt{50}\,(3.54\angle 45^\circ) = 25.03\angle 45^\circ$

$i_2(0) = \sqrt{Z_0}\,I_2(0) = a_2(0) - b_2(0) = 0 - 3.54\angle 45^\circ$

$\therefore I_2(0) = \dfrac{-3.54\angle 45^\circ}{\sqrt{50}} = 0.501\angle -135^\circ$

$P_2(0) = \frac{1}{2}|I_2(0)|^2\,50 = \frac{1}{2}(0.501)^2\,50 = 6.27\text{ W}$

$P_2(0) = \dfrac{1}{2}\dfrac{|V_2(0)|^2}{50} = \dfrac{1}{2}\dfrac{(25.03)^2}{50} = 6.27\text{ W}$

ALSO: $P_2(0) = \frac{1}{2}|a_2(0)|^2 - \frac{1}{2}|b_2(0)|^2 = \frac{1}{2}(3.54)^2 = 6.27\text{ W}$

1.16) (a) $Z_{IN}(0) = Z_{IN}(d)\Big|_{d=\frac{\lambda}{8}} = 50\,\dfrac{(150+j150)+j50\tan 45^\circ}{50+j(150+j150)\tan 45^\circ} = 23-j65\ \Omega$

(b) $a_1(0) = \dfrac{1}{2\sqrt{Z_o}}\left[V_1(0)+Z_oI_1(0)\right]$, $V_1(0) = E_1 - Z_oI_1(0)$

$$\therefore\ a_1(0) = \frac{E_1}{2\sqrt{Z_o}} = \frac{10}{2\sqrt{50}} = 0.707$$

$$a_1(\tfrac{\lambda}{8}) = a_1(0)\,e^{-j\pi/4} = 0.707\,\underline{\lfloor -45^\circ}$$

$$b_1(0) = \frac{1}{2\sqrt{50}}\left[V_1(0)-50I_1(0)\right] = \frac{1}{2\sqrt{50}}\left[10-50I_1(0)-50I_1(0)\right] \quad (1)$$

$$I_1(0) = \frac{10\,\underline{\lfloor 0^\circ}}{50+23-j65} = 0.102\,\underline{\lfloor 41.68^\circ} \quad (2)$$

(2) INTO (1): $b_1(0) = \dfrac{1}{2\sqrt{50}}\left[10-2(50)(0.102\,\underline{\lfloor 41.68^\circ})\right] = 0.508\,\underline{\lfloor -70.65^\circ}$

$$b_1(\tfrac{\lambda}{8}) = b_1(0)\,e^{j\pi/4} = 0.508\,\underline{\lfloor -25.65^\circ}$$

SINCE $Z_2 = Z_o$, THE OUTPUT IS MATCHED. HENCE, $a_2(0)=0$

(c) $V_1(0) = I_1(0)\,Z_{IN}(0) = 0.102\,\underline{\lfloor 41.68^\circ}\,(23-j65) = 7.05\,\underline{\lfloor -28.8^\circ}$

OR $V_1(0) = \sqrt{Z_o}\left[a_1(0)+b_1(0)\right] = \sqrt{50}\left[0.707+0.508\,\underline{\lfloor -70.65^\circ}\right] = 7.05\,\underline{\lfloor -28.8^\circ}$

$V_1(\tfrac{\lambda}{8}) = \sqrt{Z_o}\left[a_1(\tfrac{\lambda}{8})+b_1(\tfrac{\lambda}{8})\right] = \sqrt{50}\left[0.707\,\underline{\lfloor -45^\circ}+0.508\,\underline{\lfloor -25.65^\circ}\right] = 8.47\,\underline{\lfloor -36.92^\circ}$

$I_1(\tfrac{\lambda}{8}) = \dfrac{8.47\,\underline{\lfloor -36.92^\circ}}{150+j150} = 0.04\,\underline{\lfloor -81.92^\circ}$

(d), (e) $P_1(0) = \frac{1}{2}\mathrm{Re}\left[V_1(0)I_1^*(0)\right] = 0.12\,\mathrm{W}$, $P_1(\tfrac{\lambda}{8}) = \frac{1}{2}\mathrm{Re}\left[V_1(\tfrac{\lambda}{8})I_1^*(\tfrac{\lambda}{8})\right] = 0.12\,\mathrm{W}$

ALSO: $P_1(0) = P_1(\tfrac{\lambda}{8}) = \frac{1}{2}|a_1(0)|^2 - \frac{1}{2}|b_1(0)|^2 = \frac{1}{2}|a_1(\tfrac{\lambda}{8})|^2 - \frac{1}{2}|b_1(\tfrac{\lambda}{8})|^2 = \frac{1}{2}(0.707)^2 - \frac{1}{2}(0.508)^2 = 0.12\,\mathrm{W}$

(f) $S_{11}(\tfrac{\lambda}{8}) = \dfrac{Z_{T1}-Z_o}{Z_{T1}+Z_o} = \dfrac{150+j150-50}{150+j150+50} = 0.721\,\underline{\lfloor 19.44^\circ}$

$S_{11}(0) = S_{11}(\tfrac{\lambda}{8})\,e^{-j2(\pi/4)} = 0.721\,\underline{\lfloor 19.44^\circ}\,(\underline{\lfloor -90^\circ}) = 0.721\,\underline{\lfloor -70.56^\circ}$

(g) $(VSWR)_{in} = \dfrac{1+|S_{11}|}{1-|S_{11}|} = 6.17$, $(VSWR)_{out} = 1$

(h) $\lambda = \dfrac{3\,10^8}{10^9} = 0.3\,\mathrm{m}$ (OR 30 cm) . $\ell = \dfrac{\lambda}{8} = \dfrac{30}{8} = 3.75\,\mathrm{cm}$

(i) $b_2(\tfrac{\lambda}{8}) = S_{21}\,a_1(\tfrac{\lambda}{8}) + S_{22}\,\cancel{a_2(\tfrac{\lambda}{8})}^{\,0} = 3\,\underline{\lfloor 60^\circ}\,(0.707\,\underline{\lfloor -45^\circ}) = 2.12\,\underline{\lfloor 15^\circ}$

$$P_2(0) = \tfrac{1}{2}|b_2(0)|^2 = \tfrac{1}{2}|b_2(\tfrac{\lambda}{8})|^2 = \tfrac{1}{2}(2.12)^2 = 2.25\,\mathrm{W}$$

1.17) (a) $Z_{IN}(0) = Z_{IN}(d)\Big|_{d=\frac{\lambda}{4}} = 75\,\dfrac{(150+j150)+j75\tan 45^\circ}{75+j(150+j150)\tan 45^\circ} = 60-j105\ \Omega$

(b) THE VSWR IN THE $l_2=\frac{\lambda}{4}$ LINE IS UNITY, SINCE THE LINE IS MATCHED.

IN THE $l_1=\frac{\lambda}{8}$ LINE : $\Gamma_0 = \dfrac{(150+j150)-75}{(150+j150)+75} = 0.62\,\angle 29.7^\circ$

$\therefore$ VSWR $= \dfrac{1+0.62}{1-0.62} = 4.26$

(c) $V_1(0) = \dfrac{10\angle 0^\circ\, Z_{IN}(0)}{Z_{IN}(0)+Z_1} = \dfrac{10(60-j105)}{60-j105+100} = 6.32\,\angle -26.98^\circ$

$I_1(0) = \dfrac{V_1(0)}{Z_{IN}(0)} = \dfrac{6.32\angle -26.98^\circ}{60-j105} = 0.0523\,\angle 33.27^\circ$

$a_1(0) = \dfrac{1}{2\sqrt{75}}\left[V_1(0)+75\,I_1(0)\right] = 0.516\,\angle -4.6^\circ$

$b_1(0) = \dfrac{1}{2\sqrt{75}}\left[V_1(0)-75\,I_1(0)\right] = 0.32\,\angle -64.88^\circ$

$a_1(\frac{\lambda}{8}) = a_1(0)e^{-j\pi/4} = 0.516\,\angle -49.6^\circ$

$b_1(\frac{\lambda}{8}) = b_1(0)e^{j\pi/4} = 0.32\,\angle -19.8^\circ$

$a_2(0) = 0$

(d) $P_1(0) = \frac{1}{2}|a_1(0)|^2 - \frac{1}{2}|b_1(0)|^2 = \frac{1}{2}(0.516)^2 - \frac{1}{2}(0.32)^2 = 0.082$ W

$P_1(\frac{\lambda}{8}) = \frac{1}{2}|a_1(\frac{\lambda}{8})|^2 - \frac{1}{2}|b_1(\frac{\lambda}{8})|^2 = \frac{1}{2}(0.516)^2 - \frac{1}{2}(0.32)^2 = 0.082$ W

(e) $P_{AVS} = \dfrac{|E_1|^2}{8\,\mathrm{Re}[Z_1]} = \dfrac{(10)^2}{8(100)} = 0.125$ W

$\frac{1}{2}|a_1(0)|^2 = 0.133$ W

SINCE $Z_1 \neq Z_0$, IT FOLLOWS THAT $P_{AVS} \neq \frac{1}{2}|a_1(0)|^2$

1.18) (a) $V = \dfrac{5\angle 30^\circ\,(100)}{100+(50+j50)} = 3.162\,\angle 11.565^\circ$

$I = \dfrac{V}{Z_L} = \dfrac{3.162\angle 11.565^\circ}{100} = 0.0316\,\angle 11.565^\circ$

$a_p = \dfrac{1}{2\sqrt{R_\Omega}}(V+Z_\Omega I) = \dfrac{1}{2\sqrt{50}}\left(3.162\angle 11.565^\circ + (50+j50)(0.0316\angle 11.565^\circ)\right)$

$= 0.355\,\angle 30.14^\circ$

$$b_p = \frac{1}{2\sqrt{R_r}}(V - Z_r^* I) = \frac{1}{2\sqrt{50}}\left(3.162\angle 11.565^\circ - (50-j50)(0.0316\angle 11.565^\circ)\right)$$

$$= 0.158\angle 56.53^\circ$$

$$V_p^+ = \frac{E_r Z_r^*}{2R_r} = \frac{5\angle 30^\circ (50-j50)}{2(50)} = 3.536\angle -15^\circ$$

$$V_p^- = V - V_p^+ = 3.162\angle 11.565^\circ - 3.536\angle -15^\circ = 1.581\angle 101.59^\circ$$

$$I_p^+ = \frac{E_r}{2R_r} = \frac{5\angle 30^\circ}{2(50)} = 0.05\angle 30^\circ$$

$$I_p^- = I_p^+ - I = 0.05\angle 30^\circ - 0.0316\angle 11.565^\circ = 0.022\angle 56.52^\circ$$

(b)
(c) $$V_p^+ = \frac{Z_r^*}{\sqrt{R_r}} a_p = \frac{(50-j50)}{\sqrt{50}} 0.355\angle 30.14^\circ = 3.54\angle -15^\circ$$

$$V_p^- = \frac{Z_r}{\sqrt{R_r}} b_p = \frac{(50+j50)}{\sqrt{50}} 0.158\angle 56.53^\circ = 1.58\angle 101.5^\circ$$

$$V = V_p^+ + V_p^- = 3.16\angle 11.56^\circ$$

$$I_p^+ = \frac{a_p}{\sqrt{R_r}} = \frac{0.355\angle 30.14^\circ}{\sqrt{50}} = 0.05\angle 30.14^\circ$$

$$I_p^- = \frac{b_p}{\sqrt{R_r}} = \frac{0.158\angle 56.53^\circ}{\sqrt{50}} = 0.022\angle 56.53^\circ$$

$$I = I_p^+ - I_p^- = 0.0316\angle 11.56^\circ$$

1.19) IN EXAMPLE 1.71: $V = 5.59\angle -26.57^\circ$, $a_p = 0.5$, $b_p = 0$

(a) $$V_p^+ = \frac{E_r Z_r^*}{2R_r} = \frac{10(100-j50)}{2(100)} = 5.59\angle -26.57^\circ$$

$$V_p^- = V - V_p^+ = 5.59\angle -26.57^\circ - 5.59\angle -26.57^\circ = 0$$

(b) $$a_p = \frac{\sqrt{R_r}}{Z_r^*} V_p^+ = \frac{\sqrt{100}}{100-j50}(5.59\angle -26.57^\circ) = 0.5$$

$$b_p = \frac{\sqrt{R_r}}{Z_r} V_p^- = 0$$

1.20) IN EXAMPLE 1.7.2: $V=10$, $a_p=1.5$, $b_p=-0.5$

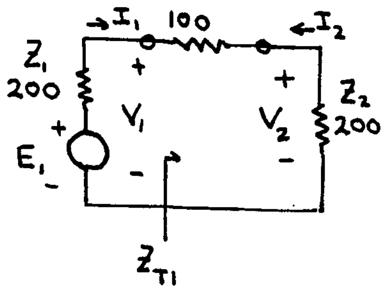
(a) $$V_p^+ = \frac{E_n Z_n^*}{2R_n} = \frac{30\,(100)}{2(100)} = 15$$

$$V_p^- = V - V_p^+ = 10 - 15 = -5$$

(b) $$a_p = \frac{\sqrt{R_n}}{Z_n^*} V_p^+ = \frac{\sqrt{100}}{100}(15) = 1.5$$

$$b_p = \frac{\sqrt{R_n}}{Z_n} V_p^- = \frac{\sqrt{100}}{100}(-5) = -0.5$$

1.21) WITH $E_2=0$: $Z_{T1} = 100+200 = 300$



$$S_{p11} = \frac{Z_{T1} - Z_1^*}{Z_{T1} + Z_1} = \frac{300-200}{300+200} = 0.2$$

$$V_1 = \frac{E_1\,300}{300+200} = \frac{3}{5}E_1 \quad , \quad I_1 = \frac{E_1}{500}$$

$$V_2 = \frac{E_1\,200}{200+300} = \frac{2}{5}E_1 \quad , \quad I_2 = \frac{-E_1}{500}$$

$$a_{p1} = \frac{1}{2\sqrt{200}}(V_1 + Z_1 I_1) = \frac{1}{2\sqrt{200}}\left(\frac{3}{5}E_1 + 200\frac{E_1}{500}\right) = \frac{E_1}{2\sqrt{200}} \quad , \quad a_{p2} = 0$$

$$b_{p2} = \frac{1}{2\sqrt{200}}(V_2 - Z_2^* I_2) = \frac{1}{2\sqrt{200}}\left(\frac{2}{5}E_1 + 200\frac{E_1}{500}\right) = \frac{2E_1}{5\sqrt{200}}$$

$$S_{p21} = \left.\frac{b_{p2}}{a_{p1}}\right|_{a_{p2}=0} = \frac{2E_1}{5\sqrt{200}}\,\frac{2\sqrt{200}}{E_1} = \frac{4}{5}$$

FROM SYMMETRY: $S_{p22} = S_{p11}$ AND $S_{p12} = S_{p21}$

THE MAGNITUDE OF S_{p21} CAN ALSO BE CALCULATED AS FOLLOWS:

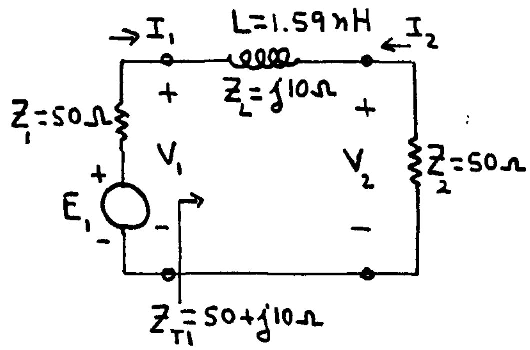
$$|S_{p21}|^2 = \frac{P_L}{P_{AVS}} \quad , \quad P_L = \frac{1}{2}\frac{|E_1(\frac{2}{5})|^2}{200} \quad , \quad P_{AVS} = \frac{|E_1|^2}{8\,(200)}$$

$$\therefore\ |S_{p21}|^2 = \frac{16}{25} \quad \text{OR} \quad |S_{p21}| = \frac{4}{5}$$

SINCE IN THIS EXAMPLE S_{p21} IS REAL, THEN $S_{p21} = \frac{4}{5}$.

1.22) (a) $f = 1\text{ GHz}$

EVALUATION OF S_{p11} AND S_{p21} (LET $E_1 = 1\angle 0°$)



$$V_1 = \frac{E_1(50+j10)}{100+j10} = 0.5074\angle 5.599°$$

$$I_1 = \frac{V_1}{50+j10} = 0.01\angle -5.711°$$

$$V_2 = \frac{E_1\,50}{100+j10} = 0.4975\angle -5.711°$$

$$I_2 = -\frac{V_2}{50} = 0.01\angle 174.29°$$

$$a_{p1} = \frac{1}{2\sqrt{R_1}}(V_1 + Z_1 I_1) = \frac{1}{2\sqrt{50}}\left(0.5074\angle 5.599° + 50(0.01\angle -5.711°)\right) = 0.0709\angle -0.0143°$$

$$b_{p1} = \frac{1}{2\sqrt{R_1}}(V_1 - Z_1^* I_1) = \frac{1}{2\sqrt{50}}\left(0.5074\angle 5.599° - 50(0.01\angle -5.711°)\right) = 0.007\angle 85.70°$$

$$a_{p2} = \frac{1}{2\sqrt{R_2}}(V_2 + Z_2 I_2) = 0$$

$$b_{p2} = \frac{1}{2\sqrt{R_2}}(V_2 - Z_2^* I_2) = \frac{1}{2\sqrt{50}}\left(0.4975\angle -5.711° - 50(0.01\angle 174.29°)\right) = 0.0705\angle -5.711°$$

$$S_{p11} = \left.\frac{b_{p1}}{a_{p1}}\right|_{a_{p2}=0} = \frac{0.007\angle 85.7°}{0.0709\angle -0.0143°} = 0.099\angle 85.7°$$

S_{p11} CAN ALSO BE CALCULATED USING:

$$S_{p11} = \frac{Z_{T1} - Z_1^*}{Z_{T1} + Z_1} = \frac{50 + j10 - 50}{50 + j10 + 50} = 0.099\angle 84.3°$$

$$S_{p21} = \left.\frac{b_{p2}}{a_{p1}}\right|_{a_{p2}=0} = \frac{0.0705\angle -5.711°}{0.0709\angle -0.0143°} = 0.994\angle -5.7°$$

(b) WITH $Z_o = 50\,\Omega$, IT FOLLOWS FROM (1.6.24) AND (1.6.25) THAT

$$S_{11} = S_{22} = \frac{Z_L}{Z_L + 2Z_o} = \frac{j10}{j10 + 100} = 0.099\angle 84.3°$$

$$S_{21} = S_{12} = \frac{2Z_o}{Z_L + 2Z_o} = \frac{100}{j10 + 100} = 0.995\angle -5.7°$$

AS EXPECTED, FOR $Z_1 = Z_2 = Z_o = 50\,\Omega$ THE S_p PARAMETERS ARE IDENTICAL TO THE S PARAMETERS.

1.23) AT P: $I_1^+ = I_1^- + I_2^- + I_3^-$

AND $b_1 = S_{11} a_1 \Rightarrow \sqrt{Z_o}\, I_1^- = S_{11} \sqrt{Z_o}\, I_1^+$ OR $I_1^- = S_{11} I_1^+$

$b_2 = S_{21} a_1 \Rightarrow \sqrt{Z_o}\, I_2^- = S_{21} \sqrt{Z_o}\, I_1^+$ OR $I_2^- = S_{21} I_1^+$

$b_3 = S_{31} a_1 \Rightarrow \sqrt{Z_o}\, I_3^- = S_{31} \sqrt{Z_o}\, I_1^+$ OR $I_3^- = S_{31} I_1^+$

$\therefore \quad I_1^+ = S_{11} I_1^+ + S_{21} I_1^+ + S_{31} I_1^+$

$$S_{11} + S_{21} + S_{31} = 1$$

SIMILARLY, WITH E_2 APPLIED TO PORT 2 WITH THE OTHER PORTS MATCHED GIVES

$$S_{12} + S_{22} + S_{32} = 0$$

WITH E_3 APPLIED TO PORT 3, WITH THE OTHER PORTS MATCHED GIVES

$$S_{13} + S_{23} + S_{33} = 0$$

WITH ALL THE SOURCES EQUAL TO E_0, AS SHOWN IN FIG. 1.22b, THE CURRENTS I_1, I_2, AND I_3 ARE EQUAL TO ZERO. HENCE,

$I_1^+ = I_1^-$, $I_2^+ = I_2^-$, $I_3^+ = I_3^-$, AND $I_1^+ = I_2^+ = I_3^+$

FROM

$$b_1 = S_{11} a_1 + S_{12} a_2 + S_{13} a_3$$

OR

$$I_1^- = S_{11} I_1^+ + S_{12} I_2^+ + S_{13} I_3^+ \Rightarrow S_{11} + S_{12} + S_{13} = 1$$

SIMILARLY:

$$I_2^- = S_{21} I_1^+ + S_{22} I_2^+ + S_{23} I_3^+ \Rightarrow S_{21} + S_{22} + S_{23} = 1$$

$$I_3^- = S_{31} I_1^+ + S_{32} I_2^+ + S_{33} I_3^+ \Rightarrow S_{31} + S_{32} + S_{33} = 1$$

1.24) (a) $v_1 = z_{11} i_1 + z_{12} i_2 \quad (1) \qquad i_1 = y_{11} v_1 + y_{12} v_2 \quad (3)$

$v_2 = z_{21} i_1 + z_{22} i_2 \quad (2) \qquad i_2 = y_{21} v_1 + y_{22} v_2 \quad (4)$

FROM (2): $i_2 = \dfrac{v_2}{z_{22}} - \dfrac{z_{21}}{z_{22}} i_1 \quad (5)$

(5) INTO (1): $i_1 = \dfrac{z_{22}}{|z|} v_1 - \dfrac{z_{12}}{|z|} v_2 \quad (6)$, $|z| = z_{11} z_{22} - z_{21} z_{12}$

COMPARING (6) WITH (3) GIVES: $y_{11} = \dfrac{z_{22}}{|z|}$ AND $y_{12} = -\dfrac{z_{12}}{|z|}$

(6) INTO (2): $v_2 = z_{21}\left(\frac{z_{22}}{|z|}v_1 - \frac{z_{12}}{|z|}v_2\right) + z_{22} i_2$

OR $$i_2 = -\frac{z_{21}}{|z|}v_1 + \frac{z_{11}}{|z|}v_2 \quad (7)$$

COMPARING (7) WITH (4) GIVES: $y_{21} = -\frac{z_{21}}{|z|}$ AND $y_{22} = \frac{z_{11}}{|z|}$.

THE DERIVATION BETWEEN y AND z PARAMETERS IS SIMILAR.

(b) $$v_1 = A v_2 - B i_2 \quad (8)$$
$$i_1 = C v_2 - D i_2 \quad (9)$$

FROM (9): $$v_2 = \frac{1}{C} i_1 + \frac{D}{C} i_2 \quad (10)$$

(10) INTO (8): $$v_1 = \frac{A}{C} i_1 + \frac{AD-BC}{C} i_2 \quad (11)$$

COMPARING (11) WITH (1) GIVES: $z_{11} = \frac{A}{C}$ AND $z_{12} = \frac{AD-BC}{C}$

(11) INTO (8): $\frac{A}{C} i_1 + \frac{AD-BC}{C} i_2 = A v_2 - B i_2$

OR $$v_2 = \frac{1}{C} i_1 + \frac{D}{C} i_2 \quad (12)$$

COMPARING (12) WITH (2) GIVES: $z_{21} = \frac{1}{C}$ AND $z_{22} = \frac{D}{C}$

THE DERIVATION BETWEEN z AND ABCD IS SIMILAR.

1.25) (a) FOR REAL $[Z_0]$ AND $[Y_0]$

$$[I] = [y][V]$$
$$[I^-] = [I^+] - [I]$$
$$[V^-][Y_0] = [V^+][Y_0] - [y]([V^+] + [V^-])$$
$$([Y_0] + [y])[V^-] = ([Y_0] - [y])[V^+]$$
$$[V^-] = ([Y_0] + [y])^{-1}([Y_0] - [y])[V^+]$$
$$= [S][V^+]$$
$$\therefore \ [S] = -([Y_0] + [y])^{-1}([y] - [Y_0]) \quad (1)$$

$$[Y_0]-[y]=[Y_0][S]+[y][S]$$
$$[Y_0]([I]-[S])=[y]([I]+[S])$$
$$[y]=[Y_0]([I]-[S])([I]+[S])^{-1}$$

(b) FROM (1):

$$[S]=-\left[\begin{bmatrix}Y_0 & 0\\ 0 & Y_0\end{bmatrix}+\begin{bmatrix}y_{11} & y_{12}\\ y_{21} & y_{22}\end{bmatrix}\right]^{-1}\left[\begin{bmatrix}y_{11} & y_{12}\\ y_{21} & y_{22}\end{bmatrix}-\begin{bmatrix}Y_0 & 0\\ 0 & Y_0\end{bmatrix}\right]$$

$$[S]=-\begin{bmatrix}y_{11}+Y_0 & y_{12}\\ y_{21} & y_{22}+Y_0\end{bmatrix}^{-1}\begin{bmatrix}y_{11}-Y_0 & y_{12}\\ y_{21} & y_{22}-Y_0\end{bmatrix}$$

$$\begin{bmatrix}y_{11}+Y_0 & y_{12}\\ y_{21} & y_{22}+Y_0\end{bmatrix}^{-1}=\frac{\begin{bmatrix}y_{22}+Y_0 & -y_{12}\\ -y_{21} & y_{11}+Y_0\end{bmatrix}}{(y_{11}+Y_0)(y_{22}+Y_0)-y_{12}y_{21}}$$

$$\therefore\quad [S]=-\frac{\begin{bmatrix}(y_{22}+Y_0)(y_{11}-Y_0)-y_{12}y_{21} & (y_{22}+Y_0)y_{12}-y_{12}(y_{22}-Y_0)\\ -y_{21}(y_{11}-Y_0)+y_{21}(y_{11}+Y_0) & -y_{12}y_{21}+(y_{11}+Y_0)(y_{22}-Y_0)\end{bmatrix}}{(y_{11}+Y_0)(y_{22}+Y_0)-y_{12}y_{21}}$$

$$[S]=\frac{1}{\Delta_2}\begin{bmatrix}(1-y_{11}')(1+y_{22}')+y_{12}'y_{21}' & -2y_{12}'\\ -2y_{21}' & (1+y_{11}')(1-y_{22}')+y_{12}'y_{21}'\end{bmatrix}$$

WHERE $\Delta_2=(1+y_{11}')(1+y_{22}')-y_{12}'y_{21}'$, $y_{11}'=y_{11}Z_0$, $y_{12}'=y_{12}Z_0$, $y_{21}'=y_{21}Z_0$, AND $y_{22}'=y_{22}Z_0$ ($Y_0=1/Z_0$).

1.26) STEP 1. CONVERT THE [S] PARAMETERS OF THE BJT TO [z] PARAMETERS (CALLED $[z_1]$).

STEP 2. CALCULATE THE [z] PARAMETERS OF THE INDUCTOR (CALLED $[z_2]$).

STEP 3. ADD THE [z] PARAMETERS (i.e., $[z_3]=[z_1]+[z_2]$)

STEP 4. CONVERT $[z_3]$ TO [S] PARAMETERS. THESE ARE THE [S] PARAMETERS OF THE TWO-PORT NETWORK.

1.27) CONVERT THE CE S PARAMETERS TO CE y PARAMETERS (SEE FIG.1.8.1), THAT IS:

$$y_{11,e}=0.016+j0.034 \qquad y_{12,e}=0.00141-j0.0272(10^{-3})$$
$$y_{21,e}=0.04-j0.036 \qquad y_{22,e}=0.00363+j0.0079$$

USE THE RELATIONS IN FIG. 1.8.1b TO CALCULATE THE CB AND CC y PARAMETERS. THAT IS,

$$y_{11,b}=0.061+j0.00587 \qquad y_{12,b}=-0.00503-j0.00787$$
$$y_{21,b}=-0.0436+j0.0281 \qquad y_{22,b}=0.00363+j0.0079$$

AND

$$y_{11,c}=0.016+j0.034 \qquad y_{12,c}=-0.0174-j0.034$$
$$y_{21,c}=-0.056+j0.002 \qquad y_{22,c}=0.061+j0.00587$$

CONVERT FROM CB AND CC y PARAMETERS TO CB AND CC S PARAMETERS. THAT IS,

$$[S]_{CB}=\begin{bmatrix}0.356\angle{-173.6^\circ} & 0.243\angle{35.41^\circ}\\ 1.348\angle{-54.81^\circ} & 1.198\angle{-32.7^\circ}\end{bmatrix} \text{ AND } [S]_{CC}=\begin{bmatrix}0.893\angle{-62.03^\circ} & 0.764\angle{29.68^\circ}\\ 1.12\angle{-35.26^\circ} & 0.176\angle{98.35^\circ}\end{bmatrix}$$

1.28) (a)

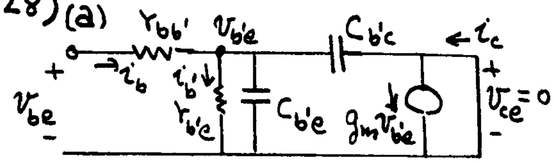


FIG. 1.11.4 WITH $v_{ce}=0$

$$i_c = g_m v_{b'e} \quad (1)$$

$$i_{b'} = i_b \frac{\left(\frac{1}{j\omega(C_{b'e}+C_{b'c})}\right)}{r_{b'e}+\left(\frac{1}{j\omega(C_{b'e}+C_{b'c})}\right)} \quad (2)$$

$$v_{b'e} = i_{b'} r_{b'e} \quad (3)$$

FROM (1), (2), AND (3): $i_c = g_m r_{b'e} i_{b'} = \dfrac{g_m r_{b'e} i_b}{1+j\omega r_{b'e}(C_{b'e}+C_{b'c})}$

DEFINE $h_{fe}(j\omega)$ AS:

$$h_{fe}(j\omega) = \left.\frac{i_c}{i_b}\right|_{v_{ce}=0} = \frac{g_m r_{b'e}}{1+j\omega r_{b'e}(C_{b'e}+C_{b'c})} = \frac{g_m r_{b'e}}{1+j\omega/\omega_\beta}$$

WHERE

$$f_\beta = \frac{\omega_\beta}{2\pi} = \frac{1}{2\pi r_{b'e}(C_{b'e}+C_{b'c})} \approx \frac{1}{2\pi r_{b'e}C_{b'e}} \qquad (\text{SINCE } C_{b'e} \gg C_{b'c})$$

f_T (THE GAIN-BANDWIDTH PRODUCT) IS THE FREQUENCY WHERE $|h_{fe}(j\omega)|=1$, OR

$$1 = \frac{g_m r_{b'e}}{\sqrt{1+(\omega_T/\omega_\beta)^2}} \approx g_m r_{b'e}\frac{\omega_\beta}{\omega_T} \Rightarrow f_T = g_m r_{b'e} f_\beta = \frac{g_m}{2\pi C_{b'e}}$$

(b) SIMILARLY, FOR THE FET IN FIG. 1.11.15 WE OBTAIN

$$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})} \approx \frac{g_m}{2\pi C_{gs}} = \frac{g_m}{2\pi C_i} \qquad (\text{SINCE } C_{gs} \text{ IS DENOTED BY } C_i \text{ IN FIG. 1.11.15})$$