Error cancellation modeling and its application to machining process control

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The product quality in a machining process can be affected by datum surface imperfections, fixture locator errors, and machine tool errors. It has been previously observed that these effects can cancel out one another for certain features. The mathematical modeling and analysis of this phenomenon is currently an open issue. We use the concept of an Equivalent Fixture Error (EFE) embedded into a modeling methodology to obtain insights into this fundamental phenomenon and achieve an improved process control. Based on our process fault model we develop a sequential root-cause identification procedure and EFE compensation methodology. A case study is presented to demonstrate the proposed diagnostic procedure. A simulation study is also performed to illustrate the error compensation procedure.

1. Introduction

In a machining process, product quality is mainly affected by fixture, datum, and machine tool errors. A fixture is a device used to locate, clamp, and support a workpiece during machining, assembly, or inspection. A fixture error is considered to be a significant fixture deviation of a locator from its specified position. Machining datum are those part features that are in direct contact with the fixture locators. Datum error is deemed to be the significant deviation of datum surfaces and is mainly induced by imperfections in raw workpieces or faulty operations in previous stages. Together the fixture and datum surfaces provide a reference system for accurate cutting operations using machine tools. Machine tool error is modeled in terms of significant tool path deviations from its intended route. In this paper, we mainly focus on kinematic aspects of these three error types.

It has been widely noted that fixture, datum, and machine tool errors may cancel out one another, i.e., their combined effect will reduce deviations in part features. This phenomenon may have the drawback that it is possible for it to conceal the fact that multiple errors have occurred in the process, however, there is the opportunity for us to purposefully use one type of error to counteract or compensate another error and thereby reduce variation.

To our knowledge, no study has been performed that models and explores this cancellation effect among different types of error sources in order to create quality control in machining processes. Most research has been focused on fixture design and machine tool error modeling.

Fixture error is generally considered to be one of the crucial factors in optimal fixture design and analysis. Shawki and Abdel-Aal (1965) experimentally studied the impact of fixture wear on the positional accuracy of a workpiece. Asada and By (1985) performed the kinematic modeling, analysis, and characterization of adaptable fixturing. Screw theory was developed as an attempt to estimate locating accuracy under a rigid body assumption (Ohwovoriele and Roth, 1981; Ball, 1990). Weill et al. (1991) have developed several optimization approaches to minimize workpiece positioning errors. A robust fixture design was proposed by Cai et al. (1997) that was able to minimize positional errors. Marin and Ferreira (2003) analyzed the influence of dimensional locator errors on the tolerance allocation problem. Researchers have also considered the effect of the geometry of the datum surface on the fixture design. The optimization of the locating setup proposed by Weill et al. (1991) was based on a locally linearized part geometry. Choudhuri and De Meter (1999) considered the contact geometry between the locators and workpiece in investigating the impact of fixture locator tolerance on the geometric error of a feature.

Machine tool errors can be caused by thermal effects, cutting forces, and geometric errors in the machine tool. Various approaches have been proposed for machine tool error modeling and compensation. A volumetric error model of a three-axis jig boring machine was developed using a vector chain expression by Schultschik (1977). Ferreira and Liu (1986) developed a model to study the geometric error of a three-axis machine using a homogeneous coordinate
2. Error cancellation modeling

Wang et al. (2005) were the first to propose the EFE concept. The equivalent amount of locator errors that can generate the same feature deviation as that due to datum or machine tool errors was defined as being the EFE. Section 2.1 briefly introduces our notation and also reviews the EFE concept. Section 2.2 then models error cancellation using the EFE model.

2.1. Notation and a review of the EFE concept

In a 3-2-1 locating scheme (Fig. 1), a fixture locates the workpiece through three datum surfaces, which are known as the primary, secondary, and tertiary datum surfaces, respectively. Let \( f_i = (f_{i1}, f_{i2}, f_{i3}) \) be a point on top of locator \( i, i = 1, \ldots, 6 \). Then the fixture error can be represented by the deviations of six locators along their axial directions:

\[
\Delta f = (\Delta f_{1x}, \Delta f_{2z}, \Delta f_{3z}, \Delta f_{4y}, \Delta f_{5y}, \Delta f_{6x})^T.
\]  

(1)

Each surface \( X_j \) is represented by its surface orientation \( v_j \) and position \( p_j \) (Huang et al., 2003), \( j = 1, 2 \ldots M \), where \( M \) is the number of part surfaces. Deviations in \( X_j \) are composed of deviations in \( v_j \) and \( p_j \), i.e., \( x_j = (\Delta v_j^T \Delta p_j^T) = (\Delta \gamma_j, \Delta \phi_j, \Delta \delta_j, \Delta \xi_j, \Delta \eta_j, \Delta \zeta_j)^T \). The datum error is expressed as the deviations in the three datum surfaces, \( x_1, x_{II}, x_{III} \).

Machine tool error is modeled as the deviation in the cutting tool path (Huang and Shi, 2003), and includes the displacement error \( (x_m, y_m, z_m) \) and rotational error \( (\alpha_m, \beta_m, \gamma_m) \). Using the same notation, we represent machine tool error by \( \delta q_m = (x_m, y_m, z_m, \alpha_m, \beta_m, \gamma_m)^T \), which is invariant for all machined surfaces in an operation.

For a milling example, Fig. 2 shows the concept that the machine tool, datum, and fixture errors could generate the same error pattern. A mathematical derivation of the EFE concept is given in Appendix A.

Following Equation (1), we use \( \Delta d = (\Delta d_{1x}, \Delta d_{2z}, \Delta d_{3z}, \Delta d_{4y}, \Delta d_{5y}, \Delta d_{6x})^T \) and \( \Delta m = (\Delta m_{1z}, \Delta m_{2z}, \Delta m_{3z}, \Delta m_{4y}, \Delta m_{5y}, \Delta m_{6x})^T \) to represent the EFEs caused by datum and machine tool errors, respectively. Using the EFE concept allows us to transform the error sources in the machining process into fixture deviations, i.e., \( \Delta f, \Delta d, \) and \( \Delta m \). The relationship between the EFE and feature deviation can then be derived as (see more details in Appendix B)

\[
x = (\Gamma_u, \Gamma_v, \Gamma_u)(\Delta f^T, \Delta d^T, \Delta m^T) + \varepsilon,
\]  

(2)

where \( x \) is the feature deviation vector (e.g., it can be \( [x_1^T x_2^T \cdots x_M^T]^T \)). \( \Gamma_u = [\Gamma_1^T \Gamma_2^T \cdots \Gamma_M^T]^T \) is the mapping
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Figure 2. The EFE concept: (a) a machining process with a datum error; (b) a machining process with a machine tool error; and (c) a machining process with a fixture error.

Matrix that relates the EFE to the feature deviation. Matrix \( \Gamma_j, j = 1, 2, \ldots M \), is the block matrix that corresponds to machined surface \( j \). The results in Appendix B show that the \( \Gamma_j \) for each type of error is the same and thus the three block matrices in Equation (2) are identical. This is consistent with the phenomenon that the three error types can generate the same feature deviation.

2.2. Modeling of error cancellation

The EFE concept can be used to model error cancellation and the impact of errors on feature deviations. If we group the errors in Equation (2), we have that:

\[
x = \Gamma_u(\Delta d + \Delta f + \Delta m) + \varepsilon.
\]

(3)

Therefore, the cancellation effect of the three error types can be modeled as a linear combination of the mean shift in the EFEs and fixture error. Their impact on feature deviations can be described by using the mapping matrix \( \Gamma_u \) in Equation (3). For the special case where the three error types completely cancel one another, i.e., \( E(\Delta d + \Delta f + \Delta m) \) is statistically insignificant, the mean of the process output is within control, where \( E(\cdot) \) represents the expectation of the random variables in the parentheses. It should be noted that the variances caused by the three error types cannot be cancelled.

In this paper, \( \Delta d \), \( \Delta f \), and \( \Delta m \) are assumed to be independent random vectors that follow a multivariate normal distribution. \( \varepsilon \) is a random vector that follows a normal distribution \( N(0, \sigma^2 \mathbf{I}) \). \( \varepsilon \) can be considered as the aggregated effect due to measurement noise and inherent unmodeled terms in the machining process.

The modeling approach presented in the Appendices can be applied to cases in which the datum surfaces are all planes. When the surface is not planar, we should use the tangential plane to the surface at each locator point as the datum surface. Figure 3 shows the setup of a 2-D part with nonplanar datum surfaces. The datum surfaces are tangential planes \( T_1, T_2, \) and \( T_3 \). The corresponding normal vectors are \( \mathbf{n}_1, \mathbf{n}_2, \) and \( \mathbf{n}_3 \), respectively. If the implicit form surface equation is represented by \( f_j(x_j, y_j, z_j) = 0 \), \( \mathbf{n}_j \) and \( \mathbf{p}_j \) are determined by:

\[
\mathbf{n}_j = \left( \frac{\partial f_j}{\partial x}, \frac{\partial f_j}{\partial y}, \frac{\partial f_j}{\partial z} \right)^T, \quad f_j(p_{jx}, p_{jy}, p_{jz}) = 0, \quad j = I, II, \ldots, VI.
\]

(4)

Then we substitute Equation (4) into the following to compute the EFEs (\( \Delta d \) and \( \Delta m \)).

\[
\Delta d_i (\text{or} \Delta m_{iz}) = -[n_{jx}(f_{ix} - p_{jx}) + n_{jy}(f_{iy} - p_{jy})]/n_{jz} + p_{jz} - f_{iz}, \quad i = 1, 2, 3, \quad j = I,
\]

\[
\Delta d_{i+3} (\text{or} \Delta m_{iy}) = -[n_{jx}(f_{ix} - p_{jx}) + n_{jz}(f_{iz} - p_{jz})]/n_{jy} + p_{jy} - f_{iy}, \quad i = 4, 5, \quad j = II,
\]

\[
\Delta d_{i+6} (\text{or} \Delta m_{ix}) = -[n_{jy}(f_{iy} - p_{jy}) + n_{jz}(f_{iz} - p_{jz})]/n_{jx} + p_{jx} - f_{ix}, \quad i = 6, \quad j = III.
\]

Figure 3. Nonplanar datum surfaces.
where \((\Delta u_j, \Delta v_j, \Delta p_j, \Delta p_j)\) are the deviated datum surfaces, and \(j = 1, II, III\) represent three datum surfaces. Equation (5) is determined by the distance between the two points at which the locators intersect the nominal datum \(X_j^0 = (v_j^0, v_j^0, p_j^0, p_j^0)\) and the deviated datum surfaces \(X_j = (u_j, v_j, v_j, p_j, p_j)\).  

### 3. Theoretical implications

The modeling of error cancellation and errors generating the same feature deviation has many implications for machining process control. Wang et al. (2005) found that EFE modeling could potentially lead to a reduction in the number of measurements in multistage machining processes. In this paper, we further discuss the implications on three issues: diagnosability analysis, root-cause identification, and error compensation.

#### 3.1. Diagnosability analysis

This paper studies the diagnosability of a process that is governed by a general linear fault model that relates the errors to the part feature deviation \(x\) using:

\[
x = \mathbf{\Gamma} (x_0^T \Delta f^T \delta q_{m0})^T + \varepsilon.
\]  

where matrix \(\mathbf{\Gamma}\) is determined by the part design. Its relationship with \(\mathbf{\Gamma}_u\) will be discussed in Proposition 1. \(x_D = (x_1^T x_II^T x_{III}^T)^T\) is the error vector of the three datum surfaces of the raw workpiece.

If the process is diagnosable, then a Least Squares Estimation (LSE) can be performed, i.e.,

\[
(x_D^T \Delta f^T \delta q_{m0})^T = (\mathbf{\Gamma}^T \mathbf{\Gamma})^{-1} \mathbf{\Gamma}x.
\]  

(7)

The diagnosability depends on the rank of \(\mathbf{\Gamma}\) (Zhou, Ding, Chen and Shi, 2003). We can see that Equation (7) requires \(\mathbf{\Gamma}^T \mathbf{\Gamma}\) to be full rank, or equivalently, all the columns in \(\mathbf{\Gamma}\) to be independent. Proposition 1 addresses the structure of \(\mathbf{\Gamma}\) for a machining process.

**Proposition 1.** In Equation (6) the block matrices in matrix \(\mathbf{\Gamma}\) that correspond to the three error type are dependent and matrix \(\mathbf{\Gamma}^T \mathbf{\Gamma}\) is always not full rank, i.e., the fixture, datum, and machine tool errors cannot be distinguished by solely measuring the part features.

**Proof.** If we use the transformation matrices \(\mathbf{K}_1\) (Equation (A.1)) and \(\mathbf{K}_2\) (Equation (A.2)) to map the datum error \(x_D\) to \(\Delta \mathbf{d}\) and the machine tool error \(\delta q_{m0}\) to \(\Delta \mathbf{m}\), respectively, Equation (2) becomes:

\[
x = [\mathbf{\Gamma}_u \mathbf{K}_1 | \mathbf{\Gamma}_u | \mathbf{\Gamma}_u \mathbf{K}_2] (x_0^T | \Delta f^T | \delta q_{m0})^T + \varepsilon.
\]  

(8)

Comparing Equation (8) with Equation (6), we get that

\[
\mathbf{\Gamma} = [\mathbf{\Gamma}_u \mathbf{K}_1 | \mathbf{\Gamma}_u | \mathbf{\Gamma}_u \mathbf{K}_2].
\]

However, the columns that correspond to the fixture and machine tool errors in matrix \(\mathbf{\Gamma}\) are linearly dependent because the columns of \(\mathbf{\Gamma}_u \mathbf{K}_1\) and \(\mathbf{\Gamma}_u \mathbf{K}_2\) are linear combination of the columns of \(\mathbf{\Gamma}_u\). Therefore, the rank of \(\mathbf{\Gamma}\) is equal to the rank of \(\mathbf{\Gamma}_u\). This also implies that the system is not diagnosable.

An implication of this proposition is that a LSE cannot be obtained. However, the fault model of Equation (3) with the errors grouped eliminates the linearly dependent columns in matrix \(\mathbf{\Gamma}\). This fact leads to sequential root-cause identification which is discussed in Section 3.2.

#### 3.2. Sequential root cause identification

Using Equation (3), the grouped errors \(\mathbf{u}\) can be estimated as:

\[
\hat{\mathbf{u}}^{(n)} = \Delta \hat{\mathbf{u}}^{(n)} + \Delta \hat{\mathbf{f}}^{(n)} + \Delta \hat{\mathbf{m}}^{(n)} = (\mathbf{\Gamma}_u^T \mathbf{\Gamma}_u)^{-1} \mathbf{\Gamma}_u^T x^{(n)}.
\]

(9)

where \(\hat{\mathbf{u}}^{(n)}\) is the LSE of \(\mathbf{u}\) for the \(n\)th replicate of the measurement. Each row of \(\mathbf{\Gamma}_u\) corresponds to an output feature whereas each column of \(\mathbf{\Gamma}_u\) corresponds to a component in the error vectors. Hence, the number of rows of \(\mathbf{\Gamma}_u\) must be larger than the number of columns to ensure that sufficient features are measured for LSE.

Denote \(\Delta f^{(n)}_i\), \(\Delta d^{(n)}_i\), and \(\Delta m^{(n)}_i\) as the \(i\)th component of the vectors \(\Delta f^{(n)}\), \(\Delta d^{(n)}\), and \(\Delta m^{(n)}\), respectively. We can develop a strategy for root cause identification that involves the following steps:

1. Necessary error information is collected first to identify the existence of error sources using Equation (9). The process error information can be analyzed by conducting a hypothesis test on \(\{\hat{\mathbf{u}}^{(n)}\}_{n=1}^N\). Since the estimated \(\mathbf{u}\) is a mixture of noise and errors, a proper test statistic should be developed to detect the faults due to process noise. Hypothesis tests on the mean and variance can then be used to find out if the faults are mean shift or large variance in nature.

2. Additional measurement on the locator deviation (\(\Delta f^{(n)}_i\)) and datum error (\(\Delta d^{(n)}_i\)) of the raw workpiece is conducted (due to Proposition 1) to distinguish between different types of errors. The mean shift of the errors can be estimated using the sample mean of \(\Delta d^{(n)}_i\), \(\Delta f^{(n)}_i\), and \(\Delta m^{(n)}_i\) as the \(i\)th component of the vectors \(\Delta f^{(n)}\), \(\Delta d^{(n)}\), and \(\Delta m^{(n)}\), respectively. The variance can then be estimated by the sample variance for \(\Delta d^{(n)}_i\), \(\Delta f^{(n)}_i\), and \(\Delta m^{(n)}_i\).

This approach can effectively identify the machine tool errors. The detailed procedures will be given in Section 4.

#### 3.3. Error compensation

We can use the error cancellation effect to compensate for process errors. With the development of an adjustable fixture whose locator length is changeable, it is feasible to compensate errors by simply changing the length of the locators. We use the index \(i\) to represent the \(i\)th adjustment period. During period \(i\), the part feature deviations \(\{x^{(n)}\}_{n=1}^N\) are measured to determine the amount of locator adjustment.
Fig. 4. Error compensation for a disturbed process.

Such a compensation is only implemented at the beginning of a period. Denote \( \epsilon^i \) as the accumulated value of the locator length adjustments after the \( i \)th period but before the beginning of period \( i+1 \). The compensation procedure is illustrated in Fig. 4. One can see that a nominal machining process is disturbed by errors \( \Delta d, \Delta f, \) and \( \Delta m \), and the observation noise \( \epsilon \). Error sources, noise, and the machining process constitute a disturbed process, as marked in the dashed line block. Using the feature deviation \( \dot{x}^i \) for the \( i \)th period as the input \( (x' \) can be estimated as the average of \( N \) measured parts in the period \( i \), i.e., \( \dot{x}^i = 1/N \sum_{n=1}^{N} x^{(n)} \)), a controller is introduced to generate signal \( \epsilon^i \) that manipulates adjustable fixture locators so as to counteract the errors for the \((i+1)\)th machining period. The amount of compensation at period \( i+1 \) should be \( \epsilon^i - \epsilon^{i-1} \). The error compensation model can then be written as:

\[
x^{i+1} = S^{i+1} + \Gamma_u \epsilon^i \quad \text{and} \quad S^{i+1} = \Gamma_u u^{i+1} + \epsilon^{i+1},
\]

where \( S^{i+1} \) is the output of the disturbed process for time \( i+1 \). This term represents the feature deviation measured without any compensation being made.

In this paper, we focus on static errors because they account for the majority of machining errors (Zhou, Huang, and Shi, 2003). A negative value for predicted EFEs means that they can be used to adjust the locators. Thus, we derive an integral control that can minimize the Mean Square Error (MSE) of the feature deviation, i.e.,

\[
\epsilon^i = (\Gamma_u^T \Gamma_u)^{-1} \Gamma_u^T \sum_{i=1}^{i} x' = - \sum_{i=1}^{i} (\Delta \hat{d}' + \Delta \hat{f}' + \Delta \hat{m}').
\]

Equation (11) shows that the accumulated amount of compensation for the next period is equal to the sum of the LSE of the EFEs of the current and all the previous time periods of the machining process. The accumulated compensation \( \epsilon^i \) can be used to evaluate the controller performance in stability and robustness analyses. The amount of compensation for the \((i+1)\)th period is \( \epsilon^i - \epsilon^{i-1} \):

\[
\epsilon^i - \epsilon^{i-1} = -(\Gamma_u^T \Gamma_u)^{-1} \Gamma_u^T x'.
\]

The compensation accuracy can be estimated by \( \Gamma_u u - (\Gamma_u^T \Gamma_u)^{-1} \Gamma_u^T x' \), i.e., the difference between \( x' \) and its LSE. Denote the range space of \( \Gamma_u \) as \( R(\Gamma_u) \) and the null space of \( \Gamma_u^T \) as \( N(\Gamma_u^T) \). Spaces \( R(\Gamma_u) \) and \( N(\Gamma_u^T) \) are orthogonal and constitute the whole vector space \( \mathbb{R}^{q \times 1} \), where \( q \) is the number of rows in \( x' \) (or \( \Gamma_u \)). By the LSE property we know that the estimation error vector \( x' - \Gamma_u (\Gamma_u^T \Gamma_u)^{-1} \Gamma_u^T x' \) is orthogonal to \( R(\Gamma_u) \). Therefore, the compensation accuracy of Equation (12) can be estimated by the projection of the observation (feature deviation) vector \( x' \) onto \( N(\Gamma_u^T) \). This conclusion also shows the components of the observation that can be compensated. The projection of the observation vector \( x' \) onto space \( R(\Gamma_u) \) can be fully compensated by using Equation (12) whereas the projection onto \( N(\Gamma_u^T) \) cannot be compensated.

In practice, the accuracy that the adjustable locator can achieve must be considered. Suppose that the standard deviation of a locator’s movement is \( \sigma_f \). We can set the stopping region for applying error compensation with a 99.73% confidence level as:

\[
-3\sigma_f \leq \epsilon^i - \epsilon^{i-1} \leq 3\sigma_f.
\]

4. Case studies

The discussion in Section 3 has highlighted the application of the EFE concept to sequential root-cause identification and error compensation. The diagnostic algorithm for these effects is proposed in this section and demonstrated by its application to a machining experiment. EFE compensation for process control is illustrated by a simulation study.

4.1. Root-cause identification

There are several diagnostic approaches (Ceglarek and Shi, 1996; Apley and Shi, 1998; Rong et al., 2001) that have achieved considerable success in fixture fault detection. The approach proposed by Apley and Shi (1998) can effectively identify multiple fixture faults. By extending this approach, we use it for sequential root-cause identification:

**Step 1.** Conduct measurements on the features and datum surfaces of the raw workpiece so as to estimate the error sources \( \hat{u}^{(n)} \) for each replicate using Equation (9). The grouped error can be estimated by the average of \( \hat{u}^{(n)} \) over \( N \) measured workpieces, i.e., \( \hat{u} = (1/N) \sum_{n=1}^{N} \hat{u}^{(n)} \), \( n = 1, 2, \ldots, N \). As mentioned in Section 3.2, the fault vector \( u \) contains both error sources and process noise.

**Step 2.** To detect those faults due to process noise, we can use the \( F \)-test statistic introduced by Apley and Shi (1998):

\[
F_i = \frac{\hat{S}_i^2}{[\Gamma_u^T \Gamma_u]^{(n)}_i} \hat{S}_k^2, \quad i = 1, 2, \ldots, 6,
\]

where \( \hat{S}_i^2 = (1/N) \sum_{n=1}^{N} [\hat{u}^{(n)}_i]^2 \), and \( \hat{u}^{(n)}_i \) represents the \( i \)th component of vector \( \hat{u}^{(n)} \). \( (\Gamma_u^T \Gamma_u)^{-1} \) is the \( i \)th diagonal entry of matrix \( (\Gamma_u^T \Gamma_u)^{-1} \). The estimator
for the variance of the noise is:

\[
\hat{\sigma}_n^2 = \frac{1}{N(q - 6)} \sum_{n=1}^{N} \varepsilon(n)^T \varepsilon(n),
\]

and \(\varepsilon(n) = x(n) - \Gamma_n \hat{u}(n)\) is for the noise term. When \(F_i > F_{1-\alpha}(N, Q(q - 6))\), we conclude that the ith fault occurs with a confidence level of 100(1 - \(\alpha\))%. By investigating \(\{u_i(n)\}_{n=1}^{N}\) for mean \(u_i\) (\(H_0: u_i = 0\) vs. \(H_1: u_i \neq 0\)), and variance \(\sigma^2_{u_i}\) (\(H_0: \sigma^2_{u_i} \leq \sigma^2_0\) vs. \(H_1: \sigma^2_{u_i} > \sigma^2_0\)), one can determine whether the error pattern of the faults is mean shift or variance type. \(\sigma^2_0\) is a small value. In the case study, we choose \(\sigma^2_0 = 0.1\) mm². By the normality assumption of EFEs (\(\Delta d, \Delta f, \text{and } \Delta m\)), we can use the \(T\) test statistic.

\[
T = u_i \sqrt{\frac{1}{N(N - 1)} \sum_{n=1}^{N} (u_i(n) - u_i)^2},
\]

and compare it with \(t_{1-\alpha/2}(n - 1)\) to test the mean shift. \(\chi^2 = \sum_{n=1}^{N} (u_i(n) - u_i)^2/\sigma^2_{u_i}\) is used and compared with \(\chi^2_{1-\alpha}(n - 1)\) to test the variance. \(\alpha\) is the significance level. If \(F_i < F_{1-\alpha}(N, Q(q - 6))\), either no faults occur at the \(i\)th locator, or the faults cannot be distinguished from process noise.

**Step 3.** Use additional measurements to distinguish errors whenever faults are identified. The locator deviation \(\{\Delta f_i^{(n)}\}_{n=1}^{N}\) and the datum surfaces \(\{X_i^{(n)}\}_{i=1}^{N}\) are measured. The EFEs \(\{\Delta d_i^{(n)}\}_{i=1}^{N}\) caused by the datum errors can be calculated using Equation (A.1). If the errors turn out to be a mean shift \((u_i \neq 0\) for a certain \(i\)), the machine tool error in terms of the EFE is \(\Delta m_i = \Delta d_i - \Delta f_i\), where \(\Delta d_i\) and \(\Delta f_i\) are the average EFE over all \(N\) parts. The machine tool error \(\delta m_i\) is then determined by the inverse of Equation (A2):

\[
\delta m_i = K_i^{-1} \Delta m_i.
\]

The variance of grouped error \(\sigma^2_{m_i}\) can then be decomposed as:

\[
\sigma^2_{m_i} = \sigma^2_{d_i} + \sigma^2_{f_i} + \sigma^2_{m_i}.
\]  

If \(\sigma^2_{m_i} > \sigma^2_{0}\), the variances caused by the three types of errors \(\sigma^2_{d_i}, \sigma^2_{f_i}, \text{and } \sigma^2_{m_i}\) can be estimated by the sample variance of \(\{\Delta d_i^{(n)}\}_{n=1}^{N}, \{\Delta f_i^{(n)}\}_{n=1}^{N}, \text{and } \{\Delta m_i^{(n)}\}_{n=1}^{N}\).

The 100(1 - 2\(\alpha\))% Confidence Interval (CI) of \(\Delta m\) is \((\Delta m \pm L)\), where \(z_{1-\alpha}\) follows the cumulative standard normal distribution such that:

\[
\int_{-\infty}^{z_{1-\alpha}} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du = 1 - \alpha, \text{ and } L = \\
\left((z_{1-\alpha}\sqrt{1 \frac{T_{\alpha/2}}{m}})^{-1} \sigma \ldots z_{1-\alpha}\sqrt{1 \frac{T_{\alpha/2}}{m}})^{-1} \sigma \right)^{T}.
\]

The corresponding CI vector for \(\delta m_i\) is \((K_i^{-1} \Delta m \pm K_i^{-1} L)\). The CI for \(\Delta d\) and \(\Delta f\) can be obtained by \((\Delta d_i \pm S_{di}/L_{1-\alpha/2}(n - 1)/\sqrt{m})\) and \((\Delta f_i \pm S_{fi}/L_{1-\alpha/2}(n - 1)/\sqrt{m})\), where \(S_{di}\) and \(S_{fi}\) are the sample variance for \(\{\Delta d_i^{(n)}\}_{n=1}^{N}\) and \(\{\Delta f_i^{(n)}\}_{n=1}^{N}\).

To demonstrate the model and the diagnostic procedure, we intentionally introduced datum and machine tool errors in the milling of five block workpieces. We used the same setup, raw workpiece and fixturing scheme as Wang et al. (2005) (Fig. 5). A xyz coordinate system fixed with a nominal fixture is also introduced to represent the plane. The top plane \(X_1\) and the side plane \(X_2\) are to be milled. Eight vertices are marked as 1–8 and their coordinates in the xyz coordinate system are measured to help to determine \(X_1\) and \(X_2\). In this paper, the units are millimeters for length and radians for angle. For the coordinate system of Fig. 5, the surface specifications are \(X_1 = (0 0 1 5.24)^T\), and \(X_2 = (0 1 0 0 96.5 0)^T\). From Equation (3) and Equation (A8), we get:

\[
x' = \left(\Gamma_1 \Gamma_2\right)(\Delta d' + \Delta f' + \Delta m') + \varepsilon',
\]

![Fig. 5. Nominal part, tolerance, and fixture layout (after Wang et al. (2005)).](image-url)
The number of rows \( q \) in \( \Gamma \) is 12. We set fixture error to be zero (\( \Delta f = 0 \)). The primary datum plane I is pre-machined to be \( X_1 = (0.018 -0.998 0.207 -1.486)^T \) and its corresponding EFE is \( \Delta f = (1.105 0 0 0 0)^T \) mm. The machine tool error is set to be \( \delta q_m = (0.175 -1.44 0.0175 0 0)^T \) by adjusting the orientation and position of the tool path. Based on the measured coordinates of vertices 1–8, the obtained feature deviations are listed in Table 1.

The EFE faults identified after following Steps 1–3 are listed in Tables 2 and 3.

Take \( \alpha \) to be 0.01. Then the threshold value \( F_{0.99}(5,12 - 6) = F_{0.99} (5, 30) = 3.699 \). In Table 3, we can see that \( F_1 = 3.699 \), which indicates that a fault occurs at locator 1.

Table 1. Measured features in units of millimeters

<table>
<thead>
<tr>
<th>( n )</th>
<th>( X_1 )</th>
<th>( X_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>( 0.001 )</td>
<td>( 0.001 )</td>
<td>( 0.001 )</td>
</tr>
<tr>
<td>( 0.001 )</td>
<td>( 0.001 )</td>
<td>( 0.001 )</td>
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<tr>
<td>( 0.001 )</td>
<td>( 0.001 )</td>
<td>( 0.001 )</td>
</tr>
</tbody>
</table>

Table 2. Estimation of \( \mathbf{u} \) for five replicates in units of millimeters

<table>
<thead>
<tr>
<th>( \mathbf{u}^{(1)} )</th>
<th>( \mathbf{u}^{(2)} )</th>
<th>( \mathbf{u}^{(3)} )</th>
<th>( \mathbf{u}^{(4)} )</th>
<th>( \mathbf{u}^{(5)} )</th>
<th>( \mathbf{u} )</th>
<th>( T )</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.937</td>
<td>2.133</td>
<td>1.775</td>
<td>2.697</td>
<td>1.902</td>
<td>2.289</td>
<td>10.119</td>
<td>10.247</td>
</tr>
<tr>
<td>0.050</td>
<td>0.090</td>
<td>-0.064</td>
<td>0.057</td>
<td>0.002</td>
<td>0.027</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>0.002</td>
<td>0.090</td>
<td>-0.0562</td>
<td>0.057</td>
<td>0.020</td>
<td>0.023</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>0.055</td>
<td>-0.031</td>
<td>0.003</td>
<td>0.039</td>
<td>0.015</td>
<td>0.016</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>0.047</td>
<td>-0.031</td>
<td>0.004</td>
<td>0.039</td>
<td>0.018</td>
<td>0.015</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>0.004</td>
<td>0.000</td>
<td>-0.001</td>
<td>0.000</td>
<td>-0.001</td>
<td>0.001</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Error cancellation in matching process control
Table 3. Additional measurement results in units of milimeters

<table>
<thead>
<tr>
<th>Locators</th>
<th>( \hat{u} )</th>
<th>( F_i )</th>
<th>( \Delta f )</th>
<th>( \Delta d )</th>
<th>( \Delta m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.289</td>
<td>19.525</td>
<td>1.105</td>
<td>1.184</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.027</td>
<td>0.051</td>
<td>0</td>
<td>0</td>
<td>0.027</td>
</tr>
<tr>
<td>3</td>
<td>0.023</td>
<td>0.005</td>
<td>0</td>
<td>0</td>
<td>0.023</td>
</tr>
<tr>
<td>4</td>
<td>0.016</td>
<td>0.613</td>
<td>0</td>
<td>0</td>
<td>0.016</td>
</tr>
<tr>
<td>5</td>
<td>0.015</td>
<td>0.073</td>
<td>0</td>
<td>0</td>
<td>0.015</td>
</tr>
<tr>
<td>6</td>
<td>0.001</td>
<td>0.002</td>
<td>0</td>
<td>0</td>
<td>0.003</td>
</tr>
</tbody>
</table>

value of \( \pm 3\sigma_f \). The adjustment for locators \( f_5 \) and \( f_6 \) are all zero and not shown in the figure. One can see that the effect of compensation in the second period is dominant. The compensation for the subsequent periods is relatively small because no significant error sources are introduced in these periods.

The effect of error compensation can be illustrated with the quality improvement of two features, the plane distance along the \( z \) axis (\( l_z \)) and the \( y \) axis (\( l_y \)) as shown in Fig. 5. \( l_z \) can be estimated by the mean and standard deviation of the lengths of edges \( l_{15}, l_{26}, l_{37}, \) and \( l_{48} \) and \( l_y \) can be estimated by \( l_{14}, l_{23}, l_{67}, \) and \( l_{58} \) for each machining period, where \( l_{mn} \) is the distance between the vertices \( m \) and \( n \) and is estimated by the edge length of five parts in each period. The milling of planes \( X_1 \) and \( X_2 \) impacts on the plane distance along the \( z \) and \( y \) axes. The nominal part should have the same edge lengths along the \( z \) and \( y \) directions (15.24 and 96.5 mm, see the dashed line in Fig. 7), respectively. However, in the first adjustment period (\( i = 1 \)) without error compensation, the standard error of the edge lengths are beyond the specified tolerance. In periods 2–5 when the compensation algorithm is applied the deviations of \( l_z \) and \( l_y \) are significantly reduced.

Fig. 7. Mean and standard deviation of the two features.

5. Conclusions

This paper has investigated error cancellation among datum, fixture, and machine tool errors as a method to improve quality control in machining processes. Based on the EFE concept error cancellation was modeled as a linear combination of EFEs. A process fault model was then derived in terms of grouped EFEs to allow us to conduct a fault diagnosis and error compensation of a machining process. The EFE methodology helped to reveal the structure of the matrix of the fault model. We mathematically proved that a machining process with datum, fixture, and machine tool errors cannot be diagnosed by simply measuring the part features. To solve this problem, we developed the procedure of sequential root-cause identification. First, datum error and machine tool error are grouped with the fixture.
error and the existence and locations of EFEs is detected. Additional measurements on the process variable (locator deviation) only need to be implemented if faults are detected. This procedure can detect the mean shift and variance of process faults from the process noise. A case study of a milling process of block parts has shown that the proposed approach can effectively identify the error sources. An error cancellation study has also suggested that the machine tool and datum errors can be compensated by adjusting the length of the fixture locators. An integral control algorithm for the compensation of static errors has been presented. The procedure was demonstrated by means of a simulation study.

A future area of study involves applying the EFE concept to processes with dynamic disturbances so as to determine the disturbance model and find out the optimal control rule to minimize the MSE.

References


Appendix

Appendix A: Derivation of the EFE

Assume that all of the three datum surfaces are planar. By linearizing Equation (5), we can derive the EFE \((\Delta d_1, \Delta d_2, \Delta d_3)\) caused by the datum error as:

\[
\Delta d_i = -f_{ix} \Delta v_{hx} - f_{iy} \Delta v_{hy} - \Delta p_{iI}, \quad i = 1, 2, 3,
\]

\[
\Delta d_i = -f_{ix} \Delta v_{hx} - f_{ix} \Delta v_{hy} - \Delta p_{iII}, \quad i = 4, 5,
\]

or \(\Delta d = K_1 \begin{pmatrix} x_I \\ x_{II} \\ x_{III} \end{pmatrix}\).

\[
\Delta d_{i6} = -f_{ix} \Delta v_{I11}, -f_{ix} \Delta v_{I11} - \Delta p_{I11}, \quad i = 6. \quad (A1)
\]

The mapping matrix that relates the datum error to \(\Delta d\) is:

\[
K_1 = \begin{pmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{pmatrix},
\]

where

\[
G_1 = \begin{pmatrix} f_{1x} & f_{1y} & 0 & 0 & 0 & 1 \\ f_{2x} & f_{2y} & 0 & 0 & 0 & 1 \\ f_{3x} & f_{3y} & 0 & 0 & 0 & 1 \end{pmatrix},
\]

\[
G_2 = \begin{pmatrix} f_{4x} & 0 & f_{4z} & 0 & 1 & 0 \\ f_{5x} & 0 & f_{5z} & 0 & 1 & 0 \end{pmatrix},
\]

and \(G_3 = -(0, f_{6y}, f_{6z}, 1, 0, 0)\).
When deriving $\Delta \mathbf{m}$, we use the relationship between $\mathbf{X}_j$ and the machine tool error $\delta \mathbf{q}_m$. Linearization of Equation (5) then yields:

$$\Delta \mathbf{m} = K_2 \delta \mathbf{q}_m,$$  \hspace{1cm} (A2)

where

$$K_2 = \begin{pmatrix} 0 & 0 & -1 & -f_{1y} & f_{1x} & 0 \\ 0 & 0 & -1 & -f_{2y} & f_{2x} & 0 \\ 0 & 0 & -1 & -f_{3y} & f_{3x} & 0 \\ 0 & -1 & 0 & f_{4z} & 0 & -f_{4x} \\ 0 & -1 & 0 & f_{5z} & 0 & -f_{5x} \\ -1 & 0 & 0 & 0 & -f_{6z} & f_{6y} \end{pmatrix}.$$

Note that if the datum surfaces are not planes then the datum surfaces $\mathbf{X}_j$ become tangential planes to each locating point and there are then six datum surfaces.

**Appendix B: Derivation of $\mathbf{G}_j$**

Wang and Huang (2003) have modeled the setup and cutting operation using a homogeneous transformation matrix. Feature deviation can then be expressed (Wang et al., 2005):

$$\mathbf{x}_j = (\mathbf{A}_{jd} \mid \mathbf{A}_{df} \mid \mathbf{A}_{jm}) \delta \mathbf{q} + \varepsilon_j,$$  \hspace{1cm} (A3)

where

$$\mathbf{A}_{jd}(k) = \mathbf{A}_{df}(k) = -\mathbf{A}_{jd}(k) = \begin{pmatrix} 0 & 0 & 0 & -2\psi_{1y} & 2\psi_{1z} & 0 \\ 0 & 0 & 0 & 2\psi_{2y} & 0 & -2\psi_{2z} \\ 0 & 0 & 0 & -2\psi_{3y} & 2\psi_{3z} & 0 \\ -1 & 0 & 0 & 0 & -2\phi_{1y} & 2\phi_{1z} \\ 0 & -1 & 0 & 2\phi_{2y} & 0 & -2\phi_{2z} \\ 0 & 0 & -1 & 0 & 2\phi_{3y} & 0 \end{pmatrix}, \text{ and rank} (\mathbf{A}_{jd}(k)) \leq 5.$$

$$\delta \mathbf{q} = (x_d \ y_d \ z_d \ \delta e_{1x} \ \delta e_{2x} \ \delta e_{3x} \ \delta e_{1y} \ \delta e_{2y} \ \delta e_{3y} \ \delta e_{1z} \ \delta e_{2z} \ \delta e_{3z} \ \delta e_{im} \ \delta e_{2m} \ \delta e_{3m})^T, (\delta e_{1x} \ \delta e_{2x} \ \delta e_{3x})^T, (\delta e_{1y} \ \delta e_{2y} \ \delta e_{3y})^T \text{ and } (\delta e_{1z} \ \delta e_{2z} \ \delta e_{3z})^T \text{ are the Euler parameters of the rotation caused by the three types of errors respectively. Under a small deviation they are one-half of the Euler angles, i.e., } \delta e_1 = 0.5\alpha, \delta e_2 = 0.5\beta, \text{ and } \delta e_3 = 0.5\gamma.$$

Parameters $(x_d/y_d/z_d \ \delta e_{1y} \ \delta e_{2y} \ \delta e_{3y})$ represent the transformation of surface due to the faulty setup with datum error, and $(x_t/y_t/z_t \ \delta e_{1t} \ \delta e_{2t} \ \delta e_{3t})$ represent the transformation due to the fixture error. $\varepsilon_j$ is the noise term that corresponds to the $j$th feature. Using the variational approach proposed by Cai et al. (1997), we can find the relationship between parameters in $\delta \mathbf{q}$ and the error sources. This approach can be directly applied for the fixture error, i.e.,

$$\begin{pmatrix} x_t & y_t & z_t & \delta e_{1t} & \delta e_{2t} & \delta e_{3t} \end{pmatrix}^T = -J^{-1}\Phi \mathbf{E} \Delta \mathbf{f},$$  \hspace{1cm} (A4)

where for generic workpiece, the Jacobian matrix $\mathbf{J}$ is:

$$\mathbf{J} = \begin{pmatrix} -\nu_x & -\nu_y & -\nu_z & 2(-f_{1x}v_x + f_{1y}v_y) & 2(f_{2x}v_x - f_{2y}v_y) & 2(-f_{3x}v_x + f_{3y}v_y) \\ -\nu_x & -\nu_y & -\nu_z & 2(-f_{1x}v_x + f_{1y}v_y) & 2(f_{2x}v_x - f_{2y}v_y) & 2(-f_{3x}v_x + f_{3y}v_y) \\ -\nu_x & -\nu_y & -\nu_z & 2(-f_{1x}v_x + f_{1y}v_y) & 2(f_{2x}v_x - f_{2y}v_y) & 2(-f_{3x}v_x + f_{3y}v_y) \\ -\nu_x & -\nu_y & -\nu_z & 2(-f_{1x}v_x + f_{1y}v_y) & 2(f_{2x}v_x - f_{2y}v_y) & 2(-f_{3x}v_x + f_{3y}v_y) \\ -\nu_x & -\nu_y & -\nu_z & 2(-f_{1x}v_x + f_{1y}v_y) & 2(f_{2x}v_x - f_{2y}v_y) & 2(-f_{3x}v_x + f_{3y}v_y) \\ -\nu_x & -\nu_y & -\nu_z & 2(-f_{1x}v_x + f_{1y}v_y) & 2(f_{2x}v_x - f_{2y}v_y) & 2(-f_{3x}v_x + f_{3y}v_y) \end{pmatrix}.$$

$$\nu = (\nu_1, \nu_2, \nu_3)^T$$ is the orientation vector of the datum planes $j = 1, 2,$ and $3$. The Jacobian matrix is definitely full rank because the workpiece is deterministically located. The inverse of the Jacobian matrix therefore exists. Matrix $\Phi$ is diag$(\mathbf{v}_1^T \mathbf{v}_2^T \mathbf{v}_3^T \mathbf{v}_4^T \mathbf{v}_5^T \mathbf{v}_6^T)$. $\mathbf{E}$ is an $18 \times 6$ matrix, that is, diag $(\mathbf{E}_1 \mathbf{E}_2 \mathbf{E}_3 \mathbf{E}_4 \mathbf{E}_5 \mathbf{E}_6)$, where $\mathbf{E}_1 = (0 \ 0 \ 1)^T, \mathbf{E}_2 = (0 \ 1 \ 0)^T, \text{ and } \mathbf{E}_3 = (1 \ 0 \ 0)^T$. We can also extend the variational approach for the EFE due to datum and machine tool errors, $\Delta \mathbf{d}$ and $\Delta \mathbf{m}$, respectively:

$$\begin{pmatrix} x_d & y_d & z_d & \alpha_d & \beta_d & \gamma_d \end{pmatrix}^T = -J^{-1}\Phi \mathbf{E} \Delta \mathbf{d},$$  \hspace{1cm} (A6)

$$\begin{pmatrix} x_m & y_m & z_m & \alpha_m & \beta_m & \gamma_m \end{pmatrix}^T = (-J^{-1}\Phi \mathbf{E} \Delta \mathbf{m}) = J^{-1}\Phi \mathbf{E} \Delta \mathbf{m}.$$  \hspace{1cm} (A7)

Equation (A7) has an additional minus sign due to the inverse transformation caused by the machine tool error transforming the workpiece from a nominal position to its real position. Combining Equations (A5), (A6), and (A9–A10), we get input matrix $\mathbf{G}_j$ corresponding to the machined surface $j$:

$$\mathbf{G}_j = -\mathbf{A}_{jd}^{-1}\Phi \mathbf{E}.$$  \hspace{1cm} (A8)

We can see that the matrices $\mathbf{G}_j$ corresponding to three EFEs are the same.

**Biographies**

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*Contributed by the Department*