

***This paper has been accepted for publication  
but has not yet undergone editing or formatting.***

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

# A blind digital signature scheme using elliptic curve digital signature algorithm

**İsmail BÜTÜN\* and Mehmet DEMİRER†**

*\*Department of Electrical Engineering, University of South Florida, Tampa, FL, USA*

*†Department of Electrical Engineering, Hacettepe University, Ankara, Turkey*

*e-mails: ibutun@mail.usf.edu, mehmet@ee.hacettepe.edu.tr*

## **Abstract**

*In this study, we propose a Blind Digital Signature scheme based upon the Elliptic Curve Digital Signature Algorithm that increases the performance significantly. Security of our scheme is based on the difficulty of Elliptic Curve Discrete Logarithm Problem. Therefore, it offers much smaller key lengths for desired security levels, along with much faster cryptographic processes, leading to lesser hardware and software requirements. According to our simulation results, relative performance improvement of our proposed BDS scheme is up to 96% when compared with previously proposed schemes.*

**Key Words:** *Blind digital signature, Elliptic curve digital signature algorithm, Elliptic curve discrete logarithm problem, Digital privacy*

## **1. Introduction and related work**

Nowadays people can accomplish their daily tasks, such as banking transactions, without leaving their homes by using Internet. People also carry out their shopping needs through Internet, which has increased the growing rate of the e-commerce. Now, the challenge is to improve the security and` anonymity of the people in this uncontrollable and dangerous Internet environment. As a solution, we use the concept of Blind Digital Signature (BDS) presented in [1]. Since 1983, several applications of BDS have been developed through the e-commerce and

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

e-voting fields.

Verification of data by using any methods of information technology is called “Electronic Signature”. “Digital Signature” is a special kind of electronic signature, which uses Public Key Cryptography (PKC) algorithms to provide data integrity and authenticity. The function of digital signature is to prevent forged signatures that cannot be distinguished from the original ones and to prevent the modification of the original documents. The aim of digital signature is to form an electronic basis for the replacement of hand written signatures. Today, digital signatures are used for identifying the owners of electronic data. The existence of reliable digital signatures which are being used to secure online transactions, such as web-based financial transactions, motivates individuals and corporations to use digital signatures widespread. Sufficiently reliable digital signature schemes expedite the process of maintaining the obligation of the digital signatures in justice.

Privacy is one of the basic rights for individuals and institutions that need to preserve their confidentiality. BDS is presented as a special application of “Digital Signature” with a distinct property of blindness (unlinkability) and can be used in many applications of cryptography where user privacy is important; such as digital voting systems, digital payment systems, online transactions, electronic government services, etc. [1]-[6].

Digital signature is used for identifying the owners of electronic data, thereby assures the traceability of electronic transactions. On the other hand, BDS disguises the content of a message from its signer, thereby assures the privacy of the users. D. Chaum [1] pioneered the concept of BDS, which he later applied this method to establish customer privacy in electronic payment systems [7]. He attained this by altering traditional digital signature algorithm in an intelligent way.

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

The BDS scheme involves two parties; namely a “signer” and a signature “requester”. The scheme allows the requester to have the message signed by the signer without revealing any information about the message. With a secure BDS scheme, the signer is unable to trace the signed message to the previous signing process, where the requester cannot be traced while using the signed message. For example, a bank can sign an electronic coin without seeing its serial number and later cannot distinguish this particular electronic coin from others. Since, customer’s transactions cannot be traced, the privacy of the customer is ensured.

BDS is used to provide user anonymity and unlinkability of electronic transactions, which prevents the signer from linking a blinded message he signed to the unblinded version that he may be asked to verify. The signed blinded message is unblinded prior to verification in such a way that the signature remains valid for the unblinded version of the message. This is a very important feature to ensure user privacy. By this way, BDS schemes can guarantee anonymity of the customers in secure electronic payment systems [7], [8]; and privacy of the voters in secure electronic voting systems [9]-[11].

Elliptic Curve Cryptography (ECC) is one of the latest methods of PKC, which is becoming more attractive for researchers. This is because; it not only increases the security, but also decreases the resource requirements at the same time. ECC offers the same security level with a shorter key length [12]. It is clear that nobody can regret such a property in nowadays security systems, where increasing the security level is a requirement. This is an appealing development, especially for security applications used in resource<sup>1</sup> limited devices, such as smart cards, cell phones, and other similar palm devices. Therefore, the applications that use ECC on such devices will require less processor loops, less memory size, small key lengths and less power consumption when compared with the applications using other PKC algorithms such as Rivest-

---

<sup>1</sup> Referred “resources” are processor speed, memory size and power supplies (i.e. batteries) of those devices.

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

Shamir-Adleman (RSA) algorithm [13], Digital Signature Algorithm (DSA) [14], etc.

With growing potential in e-commerce, ECC systems will be considered to be an important alternative solution to ensure security. Since they increase the security level per bit compared to the other traditional PKC algorithms, the required hardware capacity also decreases. This is a property of prime importance in the systems where the computing resources are limited. Thus, together with ECC systems, we can generate not only high speed hardware implementations, but also robust systems against any known attacks towards PKC systems.

Elliptic Curve Digital Signature Algorithm (ECDSA) [15] helps us in application of this cryptography approach (ECC) to digital signatures. It is developed to match the properties of today's standardized DSA [14]. In this paper, in order to generate effective and satisfactory blind digital signatures, we present an efficient method of ECDSA. This work is based on our previous research published in [16].

In this study, we propose a BDS scheme that offers an increased performance in terms of processing time compared to its counterparts. Relative performance improvement of our scheme is up to 96% compared to [1] and up to 66% compared to [17]. In our proposed scheme, we achieve this by employing ECC (i.e. ECDSA), which is superior to the other cryptography algorithms; such as RSA, DSA, etc.

The rest of the paper is organized as follows: Section 2 provides a brief introduction to the concept of BDS. Section 3 presents our proposed BDS scheme. Section 4 includes analysis and simulation results to provide comparison of our proposed scheme to the schemes presented in [1] and [17]. Finally Section 5 concludes the paper and outlines the future work.

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

## 2. Brief introduction to blind digital signature

BDS is a method of signing in such a way that the signer does not see the content of the document. Moreover, if the signer sees document/sign pair, he cannot determine whether for whom or when the document has been signed (although it is possible to verify the validity of signature). This is equivalent to signing a document blindly. We can verify the signature if we see the document and signature, but it is clear that we would need to remember when or for whom we have signed the document. Initially, this concept may be seen as rather strange – why do we need to sign a document without reading it? It has been shown that this concept can efficiently be applied to the systems where user privacy is in primary significance. Online voting and electronic payment are very good examples of these systems: When we vote online, we would like to keep secret for whom we voted, just same as in the case of voting with the ballots. When shopping with cash money, a customer does not show an ID to a vendor (in most of the cases, assuming age restrictions are fulfilled). Besides, the vendor does not recognize the customer, but can tell whether customer's money is forged or not. In the same manner, through an online transaction, we do not want anybody to learn when or what we have paid for, but a vendor would be able to verify legitimacy of our payment.

The first BDS scheme that appears in the cryptography literature is based on factoring algorithm problem proposed by Chaum [1]. According to Chaum's BDS scheme there are five phases: initialization, blinding, signing, unblinding, and verifying. And a BDS scheme must satisfy the following properties:

- **Correctness:** the correctness of the signature of a message signed through the proposed BDS scheme can be checked by anyone using the signer's public key.
- **Blindness:** the content of the message should be blind to the signer.

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

- **Unforgeability:** the signature is the proof of the signer, and no one else can derive any forged signature and pass through the verification.
- **Unlinkability:** the signer of the BDS is unable to link the message/signature pair even when the signature has been revealed to the public.

In [1], BDS transaction has two parties, namely the requester and signer. When a requester requires a BDS of the signer in response to a message, the system blinds the message by multiplying with a blinding factor. Next requester sends the blinded message to the signer. Then, the signer signs the blind message by using his own private key and then sends the resultant BDS to the requester. Afterwards, the requester unblinds (extracts) the signers' digital signature from the message by deducting the blinding factor. At the end of transaction, requester obtains the signer's signature on the original message without revealing the original message to the signer. The signers' public key can be used for authentication purposes (to authenticate the signature when needed).

### **3. Proposed blind digital signature scheme**

In [17], the proposed BDS scheme was derived from a variation of the DSA. Whereas our BDS scheme is derived from a variation of the ECDSA and again it has five phases:

- Initialization
- Blinding
- Signing
- Unblinding
- Verifying

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

In our proposed scheme we used the elliptic curves over the  $F_p$  prime field, which has been suggested by National Institute of Science and Technology (NIST) and called as Federal Information Processing Standard (FIPS) 186-2 [18], [19].

According to Standards for Efficient Cryptography Group [20], elliptic curve domain parameters over  $F_p$  are defined as a sextuple:

$$T = (p, F_p, a, b, G, n, h) \quad (1)$$

where  $p$  is an integer specifying the  $F_p$  finite field;  $a, b \in F_p$  are integers specifying the elliptic curve  $E(F_p)$  defined by (2):

$$E(F_p): y^2 \equiv x^3 + ax + b \pmod{p} \quad (2)$$

where  $G = (x_G, y_G)$  is a base point on  $E(F_p)$ ,  $n$  is a prime number defining the order of  $G$ , and  $h$  is an integer defining the cofactor:  $h = \# \frac{E(F_p)}{n}$

### 3.1 Initialization and key pair generation for ECDSA

The signer defines the elliptic curve domain parameters  $T$ , defined as in (1). Then, for each request, an integer  $k$  is randomly selected by the user and the elliptic curve point  $\hat{R}$  is calculated accordingly:<sup>2</sup>

$$\hat{R} = kG = (x'_1, y'_1) \quad (3)$$

$$\hat{r} = x'_1 \pmod{n} \quad (4)$$

In addition, the signer checks whether (5) holds.

$$\hat{r} \neq 0 \quad (5)$$

If the result is true, the signer sends the elliptic curve point  $\hat{R}$  to the requester. If the result is

---

<sup>2</sup> Refer to Table 1 for interpretation of the abbreviations used in this section.

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

**Table 1** Interpretation of abbreviations used throughout the Section 3

Abbreviation	Interpretation
$T$	elliptic curve domain parameters
$p$	order of the finite field $F_p$ , integer
$F_p$	finite field
$a, b$	coefficients defining the elliptic curve
$G$	generator point
$n$	order of $G$ , a prime number
$h$	cofactor, integer
$ECC$	elliptic curve cryptography
$ECDSA$	elliptic curve digital signature algorithm
$H(\cdot)$	hash value
$d$	private key of the signer
$Q$	public key of the signer, a point on elliptic curve
$m$	message
$\hat{m}$	blinded message
$s$	signature
$\hat{s}$	blind signature
$r$	$x$ coordinate of $R$
$\hat{r}$	$x$ coordinate of $\hat{R}$
$R, \hat{R}$	points on elliptic curve
$A, B, k$	random integer numbers
$(x, y)$	coordinates for the Cartesian System

false, then the signer selects another  $k$  randomly and repeats (3) and (4) till he finds an  $\hat{r}$  fulfilling (5).

To generate private and public key of the signer, the following steps are followed:

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

1. Integer  $d$  is chosen randomly in the range  $(1, n-1)$ .
2. The elliptic curve point of  $Q$  is calculated as in (6):

$$Q = dG = (x_Q, y_Q) \quad (6)$$

With these calculations, the public key of the signer is assigned as the point  $Q$  and private key of the signer is assigned as the integer  $d$ .

### 3.2 Blinding phase

In order to blind the message  $m$ , the owner of the message  $m$  needs the elliptic curve domain parameters  $T$  of the signer, refer to (1). Blinding is achieved through the following steps which are shown in Figure 1.

1. Signer sends the elliptic point  $\hat{R}$ , refer to (3), to the requester, which will be used as blinding coefficient.
2. Requester calculates  $\hat{r}$  from the elliptic point  $\hat{R}$  as shown in (4).
3. Requester randomly chooses integers  $A$  and  $B$ , which are in the range of  $(1, n-1)$ .
4. Requester calculates the elliptic point  $R$ :

$$R = A\hat{R} + BG = (x_1, y_1) \quad (7)$$

5. Requester calculates  $r$  from the elliptic point  $R$ , which was given in (7):

$$r = x_1 \pmod{n} \quad (8)$$

6. Requester generates the blinded message  $\hat{m}$  and sends it back to the signer for signing operation:

$$\hat{m} = A H(m) \hat{r} r^{-1} \pmod{n} \quad (9)$$

where,  $H$  is the “Hash” function and in our scheme we use SHA-1 [21] algorithm as the hash function.

### 3.3 Signing phase

After the signer receives the blinded message  $m'$  from the requester, he generates the blind signature  $s'$  by following steps which are also shown in Figure 1:

1. Signer calculates  $r'$  from the elliptic point  $R'$  as shown in (4).
2. The private key of the signer,  $d$  was generated in the initialization phase.
3.  $k$ , a random integer which was generated in the initialization phase.
4.  $s'$  is calculated as shown in (10):

$$s' = dr' + km' \pmod{n} \quad (10)$$

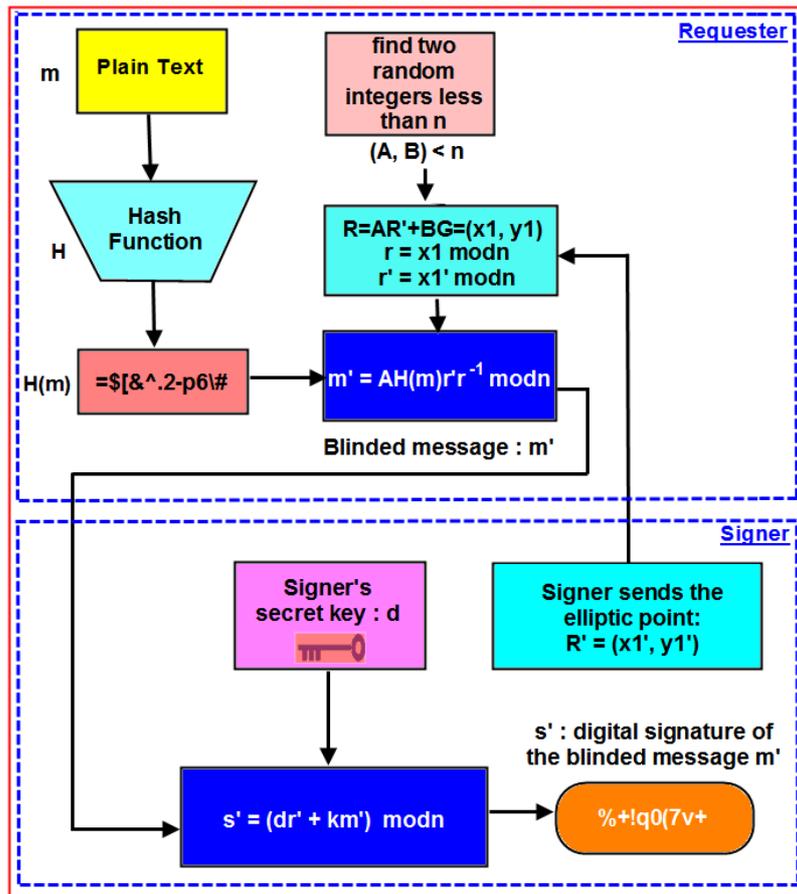


Figure 1. “Blinding” and “Signing” phases of the proposed BDS scheme

### 3.4 Unblinding phase

When the requester receives the blind digital signature  $\hat{s}$  from the signer, the unblinding operation is needed to obtain the digital signature  $(s, R)$  on message  $m$  as shown in Figure 2.

1. Requester calculates  $\hat{r}$  from the elliptic point  $\hat{R}$  as shown in (4).
2. Requester verifies whether  $\hat{r}$  and  $\hat{s}$  are in the range of  $(1, n-1)$ . If it is so, requester generates the digital signature  $(s, R)$  of the signer on the message  $m$  as shown in (11):

$$s = \hat{s} \hat{r}^{-1} + BH(m) \pmod{n} \quad (11)$$

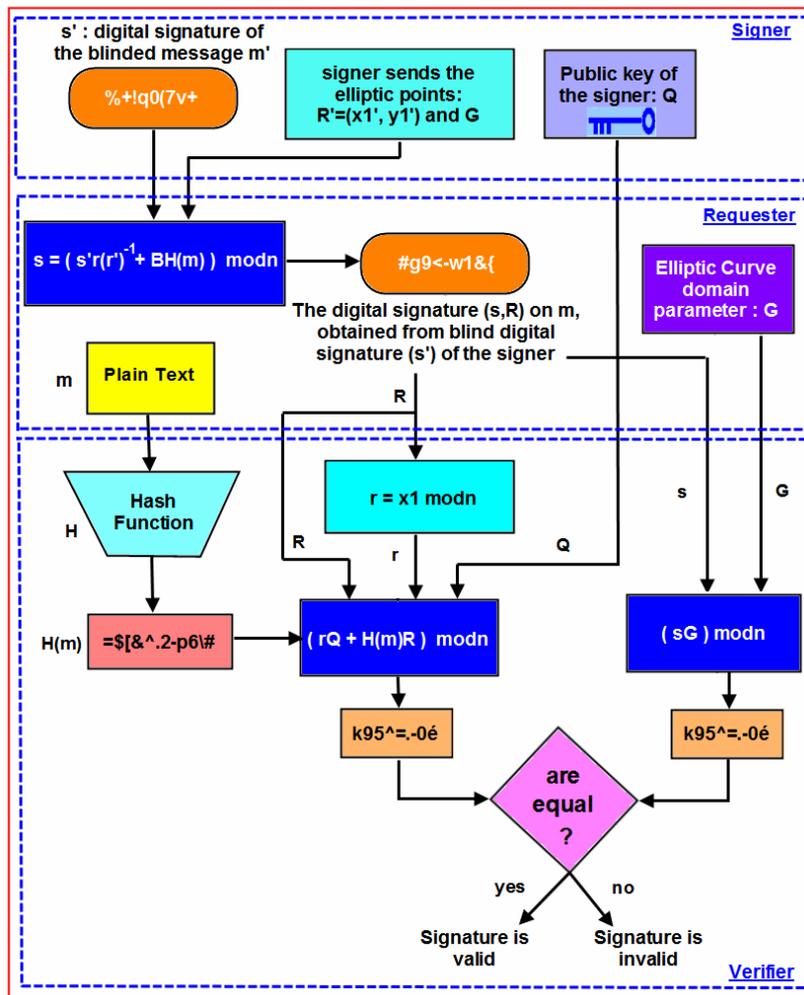


Figure 2. “Unblinding” and “Verifying” phases of the proposed BDS scheme

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

### 3.5 Verifying phase

Any party who has the elliptic domain parameters  $T$  of the signer, refer to (1), can verify the digital signature of  $(s, R)$  on the message of  $m$  by following these steps which are also shown in Figure 2:

$$u_1 = sG \pmod{n} \quad (12)$$

$u_2$  is calculated by using public key of the signer,  $Q$ :

$$u_2 = rQ + H(m)R \pmod{n} \quad (13)$$

If the statement of  $u_1 = u_2$  is met, then the signature is verified as valid, otherwise it is considered as invalid.

### 3.6 Correctness proof of the proposed scheme

We begin with expanding  $u_2$  defined in (13), by substituting  $Q$  with  $dG$  according to (6):

$$u_2 = rdG + H(m)R \pmod{n} \quad (14)$$

Since from (7) we know that  $R = A\acute{R} + BG$ , then we can expand (14) as follows:

$$u_2 = rdG + H(m)A\acute{R} + H(m)BG \pmod{n} \quad (15)$$

By using (3), we substitute  $\acute{R}$  with  $kG$  and we get,

$$u_2 = rdG + H(m)AkG + H(m)BG \pmod{n} \quad (16)$$

Now, by expanding  $u_1$  defined in (12), we need to achieve the same expression shown in (16). Since from (11) we know that  $s = \acute{s} r \acute{r}^{-1} + BH(m) \pmod{n}$ , then equation (12) becomes:

$$u_1 = \acute{s} r \acute{r}^{-1}G + BH(m)G \pmod{n} \quad (17)$$

By substituting  $\acute{s}$  with  $d\acute{r} + k\acute{m} \pmod{n}$  from (10), (17) results:

$$u_1 = d\acute{r} r \acute{r}^{-1}G + k\acute{m} r \acute{r}^{-1}G + BH(m)G \pmod{n} \quad (18)$$

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

By rearranging (18) we get:

$$u_1 = rdG\acute{r}\acute{r}^{-1} + k\acute{m} r \acute{r}^{-1}G + H(m)BG \pmod{n} \quad (19)$$

From (9), by substituting  $\acute{m}$  with  $A H(m) \acute{r} r^{-1} \pmod{n}$  in (19) results:

$$u_1 = rdG\acute{r}\acute{r}^{-1} + k A H(m) \acute{r} r^{-1}r \acute{r}^{-1}G + H(m)BG \pmod{n} \quad (20)$$

From modular arithmetic, we know that  $\acute{r}\acute{r}^{-1} = 1 \pmod{n}$  and  $rr^{-1} = 1 \pmod{n}$ . By substituting these into (20) we get:

$$u_1 = rdG + k A H(m)G + H(m)BG \pmod{n} \quad (21)$$

Equation (21) is the same expression shown in (16). Therefore we have proved that  $u_1 = u_2$  by showing that (16) and (21) are equal to the same expression.

#### 4. Simulation results and discussions

In the applications, key length of the algorithm is determined according to the desired security level. Today, it is most practical to use 160-192 bits key length for ECC systems. In case of RSA, the key length is 1024 bits for commercial applications and 2048 bits for more critical applications (where more security is needed). These key lengths correspond to the 192 bits and 224 bits ECC key lengths, respectively [22].

While the security of the Chaum's [1] BDS scheme is based on the difficulty of the factorization problem [23], security of the Chamenish et al.'s [17] BDS scheme is based on the difficulty of the discrete logarithm problem [24]. On the other hand, security of our BDS scheme relies on elliptic curve discrete logarithm problem, which is considered much more difficult than both problems [12].

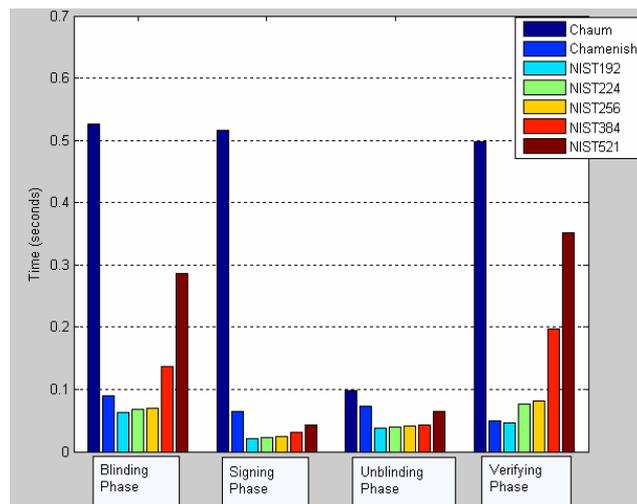
In our work, to provide comparisons to the reader, implementation and simulation of both BDS schemes of [1] and [17] has been accomplished. 1024 bits RSA key length is chosen for the

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

implementation of [1] and 1024 bits DSA key length is chosen for the implementation of [17]. To provide further comparison to the reader, we have issued our scheme with a variety of NIST suggested elliptic curves (NIST192, NIST224, NIST256, NIST384 and NIST521). This means that the key length of our scheme changes depending on the curve (192 bits, 224 bits, 256 bits, 384 bits and 521 bits, respectively). For example, if the NIST192 elliptic curve is chosen for our scheme, then the key length is apparently 192 bits.

The test-bed system consists of 1733 MHz processor with 512 MB of DDR-2 533 MHz RAM. Implementation is based upon C programming language. For elliptic curve arithmetic operations, Miracle Library is used [25].

In order to compare the time consumptions of the algorithms, the clock command of the C programming language has been used. It gives the time that is spent on the processor between two events. For the same plain text message (m consists of 431 bytes), the time (in seconds) spent on the processor for the relevant algorithms are given in the Figure 3.

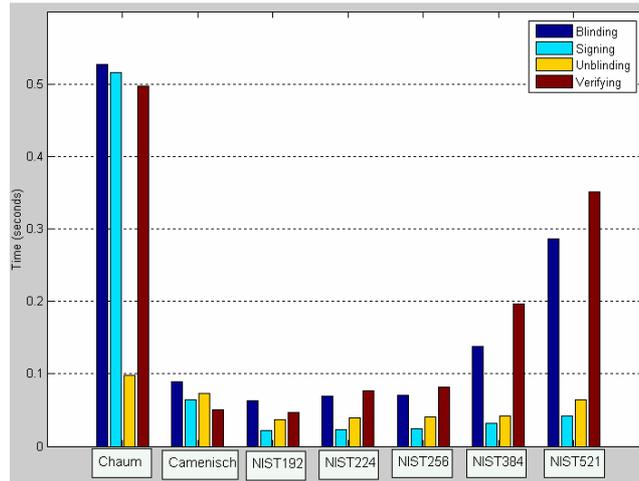


**Figure 3.** Comparisons of processing time for various BDS schemes classified according to phases.

Figure 3 is sorted according to the phases (blinding, signing, unblinding, verifying) of the

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

BDS schemes, while Figure 4 is sorted according to the types of the BDS schemes (Chaum’s [1], Chamenisch et al.’s [17], and our scheme with following elliptic curves: NIST192, NIST224, NIST256, NIST384 and NIST521).



**Figure 4.** Comparisons of processing time for various BDS schemes classified according to schemes.

Table 2 gives the processing time (sec) of our scheme compared to other schemes, when NIST192 elliptic curve is used for our scheme. Table 3 gives the performance improvement (%) of our scheme compared to other schemes, when NIST192 elliptic curve is used for our scheme. In this case, it is clear that in terms of processing time, our scheme outperforms Chaum’s scheme [1] by about 96% and Chamenisch et al.’s scheme [17] by about 66%.

**Table 2.** Processing time (sec) of BDS schemes

	<b>Our Scheme</b>	<b>Chaum’s Scheme</b>	<b>Chamenish et al.’s Scheme</b>
<b>Blinding Phase</b>	0.0624	0.5267	0.0892
<b>Signing Phase</b>	0.0218	0.5156	0.0641
<b>Unblinding Phase</b>	0.0374	0.0984	0.0732
<b>Verifying Phase</b>	0.0470	0.4971	0.0499

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

**Table 3.** Relative performance improvement (%) of our scheme regarding to other schemes

	<b>Chaum's Scheme</b>	<b>Chamenish et al.'s Scheme</b>
<b>Blinding Phase</b>	88.15	30.04
<b>Signing Phase</b>	95.77	65.99
<b>Unblinding Phase</b>	61.99	48.90
<b>Verifying Phase</b>	90.55	5.81

For all the phases (blinding, signing, unblinding and verifying), the fastest scheme is the one proposed by this study which is using the NIST192 elliptic curve (in other words, the one which has a key length of 192 bits), and the slowest of all is the Chaum's [1] scheme which is using 1024 bits RSA key length. It is important to mention that the key lengths for the considered schemes are selected to provide equal security levels. For example, it has been proved that, the security levels of the 1024 bits key length RSA algorithm, 1024 bits key length DSA algorithm and 160 bits key length ECC algorithm are the same [26], [27]. The computational effort needed to factor a 1024 bits size integer using the General Number Field Sieve method is  $3 \times 10^{11}$  MIPS (Million Instructions Per Second) years, whereas the same effort is needed to compute elliptic curve logarithms of the 160 bits size elliptic point with the Pollard  $\rho$ -method [12]. In [12], it is suggested that 192 bits size NIST elliptic curve is comparable to 1024 bits size RSA and DSA key lengths in terms of intended cryptanalysis strength. Hence we issued 192 bits key length ECC algorithm, in this case our scheme is not only faster but also more secure. Table 4 gives the comparable key sizes of the ECDSA and RSA/DSA algorithms in terms of the computational effort for cryptanalysis [28].

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

**Table 4.** Comparable Key Sizes in Terms of Computational Effort for Cryptanalysis [28]

<b>ECDSA (size of the prime field in bits)</b>	<b>RSA/DSA (modulus size in bits)</b>
112	512
160	1024
224	2048
256	3072
384	7680
512	15360

#### **4. Conclusions and future remarks**

In this study, we have briefly introduced the concept of BDS and later on our contribution to the field is presented. Our proposed BDS scheme has lower complexity (i.e. in terms of computational load) and provides better security compared to [1] and [17].

Our proposed scheme uses ECC (ECDSA), providing all of its advantages over the other PKC algorithms. It offers smaller key lengths for desired security levels, along with high speed cryptographic processes, leading to low complexity hardware and software requirements [12]. These advantages are indispensable for the applications where resource shortage is of prime importance especially in mobile platforms. Our proposed scheme can be used in the applications where not only user anonymity but also processing time is critical under certain hardware constraints.

According to the results, our proposed scheme outperforms [1] by 96% and [17] by 66% in terms of processing time. Thus our proposed scheme leads to an apparent improvement in BDS systems. Eventually, this enhancement will drastically reduce the total cost of the commercial systems that are using BDS.

Application of our scheme to smart cards, e-commerce and e-voting is left as a future work for us to consider.

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

## **Acknowledgement**

We would like to give special thanks to our colleagues Dr. Murad Khalid, Dr. Hasari Çelebi and Prof. Ravi Sankar for their reviews and valuable comments regarding publication of this work.

## **References**

- [1] D. Chaum, "Blind signatures for untraceable payments" in Proc. CRYPTO 82, New York, Plenum Press, pp.199-203, 1983.
- [2] D. Chaum, "Blind signature system" in Proc. CRYPTO 83, New York, Plenum Press, pp. 153-153, 1984.
- [3] D. Chaum, A. Fiat, M. Naor, "Untraceable electronic cash" in Proc. CRYPTO 88, Lecture Notes in Computer Science No. 403, Springer-Verlag, pp. 319-327, 1988.
- [4] A. Juels, M. Luby, R. Ostrovsky, "Security of blind digital signatures" in Proc. CRYPTO 97, Lecture Notes in Computer Science No. 1294, Springer-Verlag, pp 150-164, 1997.
- [5] D. Pointcheval, J. Stern, "Provably secure blind signature schemes" in Advances in Cryptology- ASIACRYPT'96, Lecture Notes in Computer Science No. 1163, Springer-Verlag, pp 252-265, 1996.
- [6] Y.C. Lai, M.S. Hwang, "A study on digital blind signature and its applications to electronic voting and electronic cash", Master thesis, Chaoyang University of Technology, Taiwan, June 2002.
- [7] D. Chaum, "Privacy protected payment, SMART CARD 2000", Proceedings, North-Holland, Amsterdam, pp. 69-93, 1989.

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

- [8] N. Ferguson, "Single term off-line coins", Advances in Cryptology-EUROCRYPT'93, pp. 318-328, 1994.
- [9] C.I. Fan, C.L. Lei, "A multi-recastable ticket scheme for electronic elections", Advances in Cryptology ASIACRYPT'96, pp. 116-124, 1996.
- [10] W.S. Juang, C.L. Lei, "A collision-free secret ballot protocol for computerized general elections", Computers and Security, Elsevier, pp. 339-348, 1996.
- [11] W.S. Juang, C.L. Lei, "A secure and practical electronic voting scheme for real world environments", IEICE Transactions on Fundamentals of Electronics Communications and Computer Sciences, pp. 64-71, 1997.
- [12] A. Lenstra, E. Verhuel, "Selecting cryptographic key sizes", Journal of Cryptography, Volume 14, Number 4, Springer, pp. 255-293, 2001.
- [13] R.L. Rivest, A. Shamir, L.M. Adleman, "Cryptographic communications system and method." in US Patent 4,405,829, September 1983.
- [14] D.W. Kravitz, "Digital Signature Algorithm (DSA)", US Patent 5,231,668, July 1993.
- [15] D. Johnson, A. Menezes, S. Vanstone, "The elliptic curve digital signature algorithm (ECDSA)", International Journal of Information Security, Vol. 1, Springer Berlin, pp. 36-63, 2001.
- [16] I. Butun, "Blind digital signature system development and implementation", Master thesis, Hacettepe University, Ankara, Turkey, 2006. Available at:  
[http://www.eng.usf.edu/~ibutun/masters/ismail\\_butun\\_master\\_thesis\\_published.pdf](http://www.eng.usf.edu/~ibutun/masters/ismail_butun_master_thesis_published.pdf)
- [17] J.L. Camenisch, J.M. Piveteau, M.A. Stadler, "Blind signatures based on the discrete logarithm problem", in Advances in Cryptology - EUROCRYPT, Springer, pp. 428-432, 1995.

REPLACE THIS LINE WITH YOUR PAPER IDENTIFICATION NUMBER

- [18] M. Brown, D. Hankerson, J. Lopez, A. Menezes, “Software Implementation of the NIST Elliptic Curves over Primes Fields” in Proc. of CT-RSA 2001, LNCS, Vol. 2020, Springer-Verlag, pp. 250-265, 2001.
- [19] “Digital Signature Standard”, NIST, FIPS Publication 186-2, February 2000.
- [20] “SEC 1: Elliptic Curve Cryptography”, Standards for Efficient Cryptography Group, available at: <http://www.secg.org/> , cited in October, 2011.
- [21] D. Eastlake, P. Jones, “US Secure Hash Algorithm 1 (SHA1)”, in RFC 3174, September 2001.
- [22] “PKI and Contents Protection”, available at: [www.softforum.co.kr](http://www.softforum.co.kr) , cited in May 2010.
- [23] N. Koblitz, A. Menezes, S. Vanstone, “The State of Elliptic Curve Cryptography”, Journal of Designs, Codes and Cryptography, Vol. 19, Springer, pp. 173-193, 2000.
- [24] S. Pohlig, M. Hellman, “An improved algorithm for computing logarithms over GF(P) and its cryptographic significance”, IEEE Transactions on Information Theory, pp. 106-110, 1978.
- [25] “MIRACL Elliptic Curve Library”, Shamus Software, available at: <http://www.shamus.ie/index.php?page=elliptic-curves> , cited in May 2010.
- [26] M.J. Wiener, “Performance comparison of public-key cryptosystems”, RSA CryptoBytes, Vol. 4, Summer 1998.
- [27] A. Menezes, “Elliptic curve cryptosystems”, CryptoBytes, Vol. 1, Summer 1995.
- [28] W. Stallings, Cryptography and Network Security: Principles and Practice, Prentice Hall, pp. 312-313, 2006.