Programming Concepts in Mechanical Engineering

How low can you go?

***Title*:** Aren't you glad you live in Florida?

***Background*:** In laying water main lines "up north", water utilities are concerned with the possibility of water lines freezing in winter.

Although soil and weather conditions are complicated, reasonable approximations can be made based on the assumption that soil is uniform in all directions. In that case, the temperature of the soil, *T(x,t)* will be a function of time *t* and depth *x*.

At a distance of *x* meters below the surface and *t* seconds after beginning of a cold snap, the temperature of the soil, *T=T(x, t)* is approximated by

$$\frac{T - T_s}{T_i - T_s} = erf\left(\frac{x}{2\sqrt{\alpha\, t}}\right)$$

where

T_s is the constant surface temperature during the cold period,

T_i is the initial soil temperature before the cold snap,

α is the thermal diffusivity (m^2/s), and

Thermal diffusivity of a material depends on thermal conductivity, density and specific heat of the material.

erf is the error function

Specifications:

A. Write a program that **inputs** the data

T_i, initial soil temperature before the cold snap (^oC),

T_s, constant surface temperature during the cold period (^oC),

. x, depth of water main line (m).

α, thermal diffusivity (m^2/s),

n, number of days of exposure at constant surface temperature, T_S (^oC),

and **outputs** the data

the temperature of the soil at the depth of the water main line.

***Example for Testing*:** Find if the water main line freezes for the following conditions. The soil is initially at a uniform temperature of 20^oC. Assume that under worst conditions, the soil will be subjected to a sudden temperature of -15^oC for 60 days. The thermal diffusivity of the soil is 0.138×10^{-6} m^2/s. The water main line is 0.5m below the soil surface.

Solution: T_s = -15^oC, T_i = 20^oC, $\alpha=0.138\times10^{-6}$ m^2/s, n=60 days (t=60*24*3600=), x=

0.5m

$$\frac{T-(-15)}{20-(-15)} = erf\left(\frac{0.5}{2\sqrt{0.138\,(10^{-6})\,(60)\,(24)\,(3600)}}\right)$$

$$\frac{T+15}{35} = erf\ (0.2956)$$

erf(0.2956) = 0.3241

$$T = 35 \times erf\ (0.2956) - 15$$

T = -3.6565° C
showing the water main line is not deep enough to prevent freezing.

B. Write a program now that **inputs** the data

T_i, the initial soil temperature before the cold snap (^oC),

T_s, the constant surface temperature during the cold period (^oC),

. α, the thermal diffusivity (m^2/s),

n, number of days of exposure at constant surface temperature, T_s (^oC),

and **outputs** the data

x, depth of water main line (m) so that the waterline does not freeze

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