

# TABLE OF CONTENTS

## INTRODUCTION, APPROXIMATION & ERRORS

1

### **Chapter 01.01 Introduction to numerical methods 1**

Multiple-choice test 7

Problem set 9

### **Chapter 01.02 Measuring errors 11**

True error 11

Relative true error 12

Approximate error 13

Relative approximate error 14

Significant digits 16

Multiple-choice test 18

Problem set 20

### **Chapter 01.03 Sources of error 22**

What is round off error? 22

What problems can be created by round off errors? 22

What is truncation error? 23

Can you give me other examples of truncation error? 24

Multiple-choice test 28

Problem set 30

### **Chapter 01.04 Binary representation of numbers 34**

Multiple-choice test 41

Problem set 43

### **Chapter 01.05 Floating point representation 44**

Multiple-choice test 52

Problem set 54

### ~~Chapter 01.06 Propagation of errors 55~~

~~Multiple-choice test 58~~

### **Chapter 01.07 Taylor theorem revisited 60**

Multiple-choice test 68

## DIFFERENTIATION

70

### Physical problems

- Chapter 02.00A Physical problem - general engineering 70
- Chapter 02.00B Physical problem - chemical engineering 72
- ~~Chapter 02.00D Physical problem - computer engineering 74~~
- ~~Chapter 02.00E Physical problem - electrical engineering 78~~
- ~~Chapter 02.00F Physical problem - industrial engineering 82~~
- Chapter 02.00G Physical problem - mechanical engineering 86

### Chapter 02.01 Primer on differential calculus (View it on the web)

Go to <http://numericalmethods.eng.usf.edu>

>Keyword

> Primer on differential calculus

Multiple-choice test 90

Problem set 92

### Chapter 02.02 Differentiation of continuous functions 94

- Forward difference approximation of the first derivative 94
- Backward difference approximation of the first derivative 97
- Forward difference approximation from the Taylor series 98
- Finite difference approximation of higher derivatives 101
- Multiple-choice test 106
- Problem set 108

### Chapter 02.03 Differentiation of discrete functions 110

- Forward difference approximation of the first derivative 110
- Direct fit polynomials 112
- ~~Lagrange polynomial 114~~
- Multiple-choice test 116
- Problem set 119

## NONLINEAR EQUATIONS

121

### Physical problems

- Chapter 03.00A Physical problem - general engineering 121
- Chapter 03.00B Physical problem - chemical engineering 125
- ~~Chapter 03.00C Physical problem - civil engineering 128~~

Chapter 0.3.00D Physical problem - computer engineering 134  
~~Chapter 03.00E Physical problem - electrical engineering 137~~  
~~Chapter 03.00F Physical problem - industrial engineering 140~~  
Chapter 03.00G Physical problem - mechanical engineering 146

**Chapter 03.01 Solution of quadratic equations 150**

Multiple-choice test 153  
Problem set 155

**Chapter 03.03 Bisection method of solving a nonlinear equation 157**

Bisection method 157  
Algorithm for the bisection method 160  
Advantages of bisection method 163  
Drawbacks of bisection method 163  
Multiple-choice test 166  
Problem set 168

**Chapter 03.04 Newton-Raphson method of solving a nonlinear equation 170**

Introduction 170  
Derivation 170  
Algorithm 171  
Drawbacks of the Newton-Raphson method 174  
What is an inflection point? 175  
Derivation of Newton Raphson method from Taylor series 178  
Multiple-choice test 179  
Problem set 181

~~Chapter 03.05 Secant method of solving nonlinear equations 183~~

~~What is the secant method and why would I want to use it instead of the  
Newton-Raphson method? 183  
Multiple-choice test 188  
Problem set 190~~

**SIMULTANEOUS LINEAR EQUATIONS**

**192**

**Physical problems**

Chapter 04.00A Physical problem - general engineering 192  
Chapter 04.00B Physical problem - chemical engineering 195  
Chapter 04.00C Physical problem - civil engineering 197  
~~Chapter 04.00D Physical problem - computer engineering 202~~  
~~Chapter 04.00E Physical problem - electrical engineering 207~~

~~Chapter 04.00F Physical problem - industrial engineering 213~~

Chapter 04.00G Physical problem - mechanical engineering 216

### **Chapter 4.1 Introduction to matrix algebra 222**

What is a matrix? 222

What are the special types of matrices? 223

Square matrix 224

Upper triangular matrix 224

Lower triangular matrix 224

Diagonal matrix 225

Identity matrix 225

Zero matrix 225

Tridiagonal matrices 226

When are two matrices considered to be equal? 226

How do you add two matrices? 227

How do you subtract two matrices? 228

How do I multiply two matrices? 229

What is a scalar product of a constant and a matrix? 231

what is a linear combination of matrices? 232

What are some of the rules of binary matrix operations? 232

Transpose of a matrix 235

Symmetric matrix 235

Matrix algebra is used for solving system of equations. Can you illustrate this concept? 236

Can you divide two matrices? 238

Can I use the concept of the inverse of a matrix to find the solution of a set of equations  $[A][X] = [C]$ ? 239

How do I find the inverse of a matrix? 239

If the inverse of a square matrix  $[A]$  exists, is it unique? 242

Multiple-choice test 243

Problem set 246

### **Chapter 04.06 Gaussian elimination 250**

How are a set of equations solved numerically? 250

Forward elimination of unknowns 251

Back substitution 252

Are there any pitfalls of Naïve Gauss elimination method? 253

Round-off error 257

What are the techniques for improving Naïve Gauss elimination method? 259

How does Gaussian elimination with partial pivoting differ from Naïve Gauss elimination? 259

Can we use Naïve Gauss elimination methods to find the determinant of a square matrix? 262

What if I cannot find the determinant of the matrix using Naive Gauss elimination method, for example, if I get division by zero problems during Naïve Gauss elimination method? 263

Multiple-choice test 265

Problem set 268

#### **Chapter 04.07 LU decomposition 270**

I hear about LU decomposition used as a method to solve a set of simultaneous linear equations? What is it and why do we need to learn different methods of solving a set of simultaneous linear equations? 270

How do I decompose a non-singular matrix  $[A]$ , that is, how do I find  $[A] = [L][U]$ ? 272

How do I find the inverse of a square matrix using LU decomposition? 276

Multiple-choice test 280

Problem set 284

#### ~~Chapter 04.08 Gauss-Seidel method 286~~

~~Why do we need another method to solve a set of simultaneous linear equations? 286~~

~~The above system of equations does not seem to converge. Why? 290~~

~~Multiple-choice test 296~~

~~Problem set 300~~

## **INTERPOLATION**

**301**

### **Physical problems**

Chapter 05.00A Physical problem - general engineering 301

Chapter 05.00B Physical problem - chemical engineering 303

Chapter 05.00C Physical problem - civil engineering 307

Chapter 05.00D Physical problem - computer engineering 310

~~Chapter 05.00E Physical problem - electrical engineering 313~~

~~Chapter 05.00F Physical problem - industrial engineering 316~~

Chapter 05.00G Physical problem - mechanical engineering 318

### **Chapter 05.01 Background of interpolation**

Multiple-choice test 322

### **Chapter 05.02 Direct method of interpolation 324**

What is interpolation? 324  
Direct method 325  
Multiple-choice test 332  
Problem set 334

~~Chapter 05.03 Newton's divided difference interpolation 336~~

~~What is interpolation? 336~~  
~~Newton's divided difference polynomial method 336~~  
~~Linear interpolation 337~~  
~~Quadratic interpolation 339~~  
~~General form of Newton's divided difference polynomial 342~~  
~~Multiple-choice test 347~~  
~~Problem set 349~~

**Chapter 05.05 Spline method of interpolation 351**

What is interpolation? 351  
Linear spline interpolation 354  
Quadratic splines 356  
Multiple-choice test 361  
Problem set 364

**Chapter 05.06 Extrapolation is a bad idea 366**

**Chapter 05.07 Higher order interpolation is a bad idea 370**

**Chapter 05.08 Why do we need splines? 373**

**Chapter 05.10 Shortest path of a robot 376**

**REGRESSION**

**381**

**Physical problems**

Chapter 06.00A Physical problem - general engineering 381  
~~Chapter 06.00B Physical problem - chemical engineering 385~~  
Chapter 06.00C Physical problem - civil engineering 388  
~~Chapter 06.00D Physical problem - computer engineering 391~~  
~~Chapter 06.00E Physical problem - electrical engineering 394~~  
Chapter 06.00F Physical problem - industrial engineering 398  
Chapter 06.00G Physical problem - mechanical engineering 400

**Chapter 06.01 Statistics background of regression analysis 405**

Review of statistical terminologies 405  
Elementary statistics 405  
A brief history of regression 409

**Chapter 06.02 Introduction of regression analysis 411**

What is regression analysis? 411  
Comparison of regression and correlation 412  
Uses of regression analysis 412  
Abuses of regression analysis 412  
Extrapolation 412  
Least squares methods 415  
Why minimize the sum of the square of the residuals? 415  
Multiple-choice test 417  
Problem set 419

**Chapter 06.03 Linear regression 420**

Why minimize the sum of the square of the residuals? 420  
Multiple-choice test 433  
Problem set 435

**Chapter 06.04 Nonlinear models for regression 437**

Nonlinear models using least squares 437  
Exponential model 437  
Growth model 441  
Polynomial models 443  
Linearization of data 447  
Exponential model 447  
Logarithmic functions 450  
Power functions 453  
Multiple-choice test 458  
Problem set 460

**Chapter 06.05 Adequacy of models for regression 464**

Quality of fitted model 464  
Caution in the use of  $r^2$  468  
What else should I check for the adequacy of the model in example 1?  
468  
Adequacy of coefficient of regression 470  
Problem set 471

**Physical problems**

- Chapter 07.00A Physical problem - general engineering 474
- ~~Chapter 07.00B Physical problem - chemical engineering 477~~
- ~~Chapter 07.00C Physical problem - civil engineering 480~~
- ~~Chapter 07.00D Physical problem - computer engineering 486~~
- ~~Chapter 07.00E Physical problem - electrical engineering 497~~
- Chapter 07.00F Physical problem – industrial engineering 502
- Chapter 07.00G Physical problem - mechanical engineering 506

**Chapter 07.01 Primer on integration (View it on the web)**

Go to <http://numericalmethods.eng.usf.edu>

- >Keyword
- > Primer on integral calculus
- Multiple-choice test 510
- Problem set 512

**Chapter 07.02 Trapezoidal rule of integration 515**

- What is integration? 515
- What is the trapezoidal rule? 515
- Derivation of the trapezoidal rule 516
- Multiple-segment trapezoidal rule 522
- Error in multiple-segment trapezoidal rule 528
- Multiple-choice test 531
- Problem set 533

~~**Chapter 07.03 Simpson's 1/3 rule of integration 536**~~

- ~~What is integration? 536~~
- ~~Simpson's 1/3 rule 536~~
- ~~Multiple-segment Simpson's 1/3 rule 543~~
- ~~Error in multiple-segment Simpson's 1/3 rule 546~~
- ~~Multiple-choice test 548~~
- ~~Problem set 550~~

**Chapter 07.05 Gauss quadrature 552**

- What is integration? 552
- Gauss quadrature rule 553
- Derivation of two-point Gaussian quadrature rule 554
- Higher point Gaussian quadrature formulas 556
- Arguments and weighing factors for n-point Gauss quadrature rules 557
- Multiple-choice test 566
- Problem set 569



**Chapter 07.06 Integrating discrete functions 571**

What is integration? 571

Integrating discrete functions 572

Trapezoidal rule for discrete functions with unequal segments 576

Problem set 579

**Chapter 07.07 Integrating improper functions 582**

What is integration? 582

What is an improper integral? 583

Problem set 593

**ORDINARY DIFFERENTIAL EQUATIONS**

**594**

**Physical problems**

Chapter 08.00A Physical problem - general engineering 594

~~Chapter 08.00B Physical problem - chemical engineering 598~~

Chapter 08.00C Physical problem - civil engineering 600

~~Chapter 08.00D Physical problem - computer engineering 602~~

~~Chapter 08.00E Physical problem - electrical engineering 606~~

~~Chapter 08.00F Physical problem - industrial engineering 611~~

Chapter 08.00G Physical problem - mechanical engineering 617

**Chapter 08.01 Primer for ordinary differential equations (View it on web)**

Go to <http://numericalmethods.eng.usf.edu>

>Keyword

> Primer on ordinary differential equations

Multiple-choice test 623

Problem set 625

**Chapter 08.02 Euler's method for ordinary differential equations 627**

What is Euler's method? 627

Derivation of Euler's method 628

Multiple-choice test 636

Problem set 639

**Chapter 08.03 Runge-Kutta 2nd order method 643**

What is the Runge-Kutta 2nd order method? 644

Heun's method 646

Midpoint method 646

Ralston's method 647

How do these three methods compare with results obtained if we found  $f'(x, y)$  directly? 650

How do we get the 2nd order Runge-Kutta method equations? 650

Multiple-choice test 654

Problem set 657

**Chapter 08.04 Runge-Kutta 4th order method 661**

What is the Runge-Kutta 4th order method? 661

How does one write a first order differential equation in the above form?  
661

Multiple-choice test 668

Problem set 672

**Chapter 08.05 On Solving higher order equations 676**

Problem set 685

**Chapter 08.07 Finite difference method 687**

What is the finite difference method? 687

Multiple-choice test 695

Problem set 700