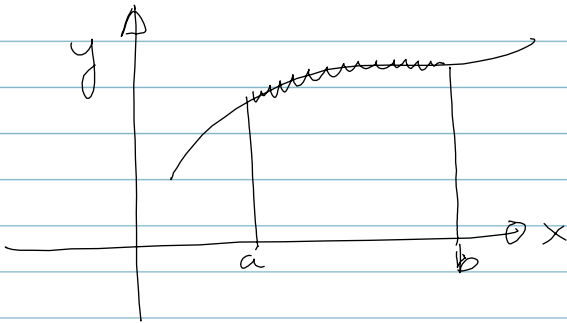


An interpolant passes thru (2, 4),
(3, 9), (4, 16) and is given by $y = x^2$.
Find the length of the path from
 $x = 2$ to $x = 4$.

Method 1:

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$



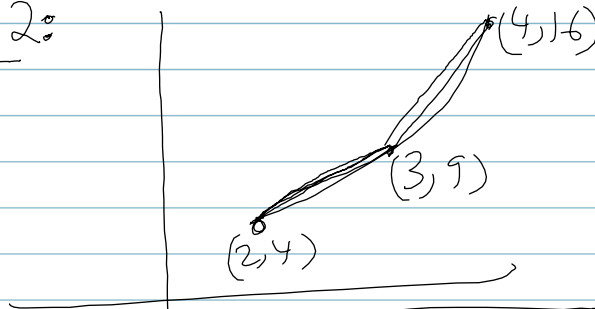
$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

$$S = \int_2^4 \sqrt{1 + (2x)^2} dx = \int_2^4 \sqrt{1 + 4x^2} dx$$

$$\begin{aligned} &= \left[\frac{1}{2} x \sqrt{1+4x^2} + \frac{1}{4} \sinh^{-1}(2x) \right]_2^4 \\ &= \left[\frac{1}{2} (4) \sqrt{1+4(4)^2} + \frac{1}{4} \sinh^{-1}(2 \times 4) \right] \\ &\quad - \left[\frac{1}{2} (2) \sqrt{1+4(2)^2} + \frac{1}{4} \sinh^{-1}(2 \times 2) \right] \\ &= \underline{\underline{12.172}} \end{aligned}$$

Method 2:



$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\begin{aligned} S &\approx \sqrt{(9 - 4)^2 + (3 - 2)^2} \\ &\quad + \sqrt{(16 - 9)^2 + (4 - 3)^2} \end{aligned}$$

$$= |2.171$$

$$y = x^2$$

$(2, 4), (2.5, 6.25), (3, 9),$
 $(3.5, 12.25), (4, 16)$

Homework.

Method 3

$$S = \int_2^4 \sqrt{1 + 4x^2} dx$$

$$\int_a^b f(x) dx \approx \frac{\underline{b-a}}{\underline{6}} \left[\underline{f(a)} + \underline{4 f\left(\frac{a+b}{2}\right)} + \underline{f(b)} \right]$$

Simpson's $\frac{1}{3}$ rule.

$$= \frac{4-2}{6} \left[f(2) + 4 f(\underline{3}) + f(4) \right]$$

$$= \frac{4-2}{6} \left[\sqrt{1+4(2)^2} + 4\sqrt{1+4(3)^2} \right]$$

$$+ \sqrt{1 + 4(4)^2}$$

$$= \frac{4-2}{6} [\sqrt{17} + 4\sqrt{37} + \sqrt{5}]$$

$$= 12.172$$

END