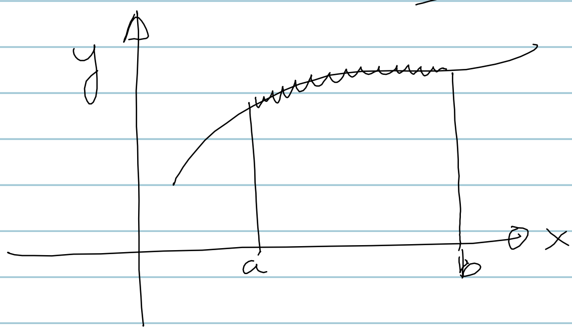
An interpolant passes thru (2, 4), (3, 9), (4, 16) and is given by $y = x^2$. Find the length of the path from $x = 2$ to $x = 4$.

Method 1:

$$S = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$



$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

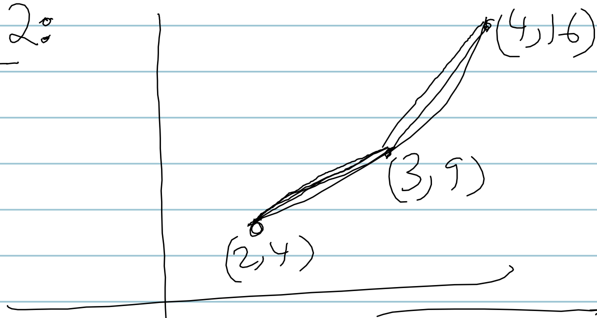
$$S = \int_2^4 \sqrt{1+(2x)^2}\, dx = \int_2^4 \sqrt{1+4x^2}\, dx$$

$$= \left[\frac{1}{2}x\sqrt{1+4x^2} + \frac{1}{4}\sinh^{-1}(2x)\right]_2^4$$

$$= \left[\frac{1}{2}(4)\sqrt{1+4(4)^2} + \frac{1}{4}\sinh^{-1}(2*4)\right]$$
$$- \left[\frac{1}{2}(2)\sqrt{1+4(2)^2} + \frac{1}{4}\sinh^{-1}(2*2)\right]$$

$$= \underline{\underline{12.172}}$$

Method 2:



$$L = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$S \approx \sqrt{(9-4)^2 + (3-2)^2}$$
$$+ \sqrt{(16-9)^2 + (4-3)^2}$$

$= 12.171$

$y = x^2$

$(2, 4), (2.5, 6.25), (3, 9),$
$(3.5, 12.25), (4, 16)$

Homework.

Method 3

$$S = \int_2^4 \sqrt{1+4x^2}\, dx$$

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Simpson's $1/3$ rule.

$$= \frac{4-2}{6}\left[f(2) + 4f(3) + f(4)\right]$$

$$= \frac{4-2}{6}\left[\sqrt{1+4(2)^2} + 4\sqrt{1+4(3)^2}\right.$$

$$+\sqrt{1+4(4)^2}\,)$$

$$=\frac{4-2}{6}\left[\sqrt{17}+4\sqrt{37}+\sqrt{5}\right]$$

$$\approx 12.172$$

$$\boxed{\text{END}}$$