DESIGN PROJECT

Write a memo (double spaced - Pitch 12 in MS Word or WordPerfect with disk - write your name, course name, year on the disk) to your boss named Ms. Composite Knowitall about your pressure vessel design for the following problem.

Work this open-ended design problem using the PROMAL program. A cylindrical pressure vessel 2 meters long and 0.5 meters in internal diameter is required to take an internal gauge pressure of 0.75 MPa.

Design (PLY ORIENTATION, STACKING SEQUENCE, NUMBER OF PLIES, PLY MATERIAL, ETC.) the pressure vessel for minimum possible mass and cost. You may be unable to minimize mass and cost simultaneously, that is, the design of the pressure vessel is not the same for the minimum mass and the minimum cost. In that case, give equal weighage to cost and mass, and use this as your function to minimize

$F = A/B + C/D$

where

A = Mass of composite laminate of your design,
B = Mass of composite laminate if design was just based on minimum mass,
C = Cost of composite laminate of your design,
D = Cost of composite laminate if design was just based on minimum cost.

Ms. Composite Knowitall wants to know details about how you chose your **initial designs** and how you came up with your final design. She is interested in knowing how you started the design process and wants to know about **all** the possibilities (**at least 10**) you considered before reaching the final design. She likes you to give her information in a precise and readable manner. She loves information in a tables and graphs, and rewards only those employees who inform her in that manner. However, she does not like the sight of any appendices, incoherent presentation, and computer outputs; those computer outputs gives her a headache (a real one).

Restrictions and Notes

1. Only allowed to use 0, +30, -30, +45, -45, +60, -60, and 90 degree plies.
2. Only Graphite/Epoxy and Glass/Epoxy laminas as given in Table 2.1 are available.
3. You can use hybrid laminates.
4. Cost of a Graphite/Epoxy lamina is 500 units/kgm.
5. Cost of a Glass/Epoxy lamina is 100 units/kgm.
6. Use only SI system of units.
7. Calculate specific gravities of the laminas using Tables 3.1 and 3.2.
8. Thickness of each lamina is 0.125 mm.

Memo is required to be in the following order

- Start the memo by giving her in a table the laminate code, cost and mass for the final design, minimum mass design and minimum cost design.
- Show a sketch of a pressure vessel
- Enumerate all the design requirements and restrictions you had.
- Show how you calculated the loads in the pressure vessel.
- Show how you calculated the density, volume, mass and cost of a Graphite/Epoxy lamina and a Glass/Epoxy lamina.
- Show the lamina hygrothermal properties, thickness, density, volume, mass and cost for the two types of unidirectional lamina in one table.
- Show at least 5 stacking sequences including your final choice for minimum mass design in a table showing the corresponding minimum strength ratio, mass, cost and minimizing function, F.
- Show at least 5 stacking sequences including your final choice for minimum cost design in a table showing the corresponding minimum strength ratio, mass, cost and minimizing function, F.
- Show at least 5 stacking sequences including your final choice for minimizing function, F in a table showing the corresponding minimum strength ratio, mass, cost and minimizing function, F.
- Show calculations for finding mass, cost, function F of your final design.
- Limit your memo to 10 pages or less. No Appendix is allowed. No computer print out directly from PROMAL program is allowed. You can however DUMP any PROMAL output to a text file, and then cut and paste it in the memo.

Example of a Student MEMO. Unedited version

Unknown Student

Final Examination

Ms. Composite Knowitall;

The following report summarizes the design of the pressure vessel as you requested. The final design, design based on minimum mass, and design based on minimum cost appear below in Table 1.

Table 1. Final Design of Plies.

Design	Stacking Sequence	Graphite Plies (#)	Glass Plies(#)	Mass (g)	Cost (units)	Minimizing Function (F)
Final	$[90/0/\overline{90}]_S$	5	0	3578.5	1789.23	2.42
Mass	$[90/0/\overline{90}]_S$	5	0	3578.5	1789.23	2.42

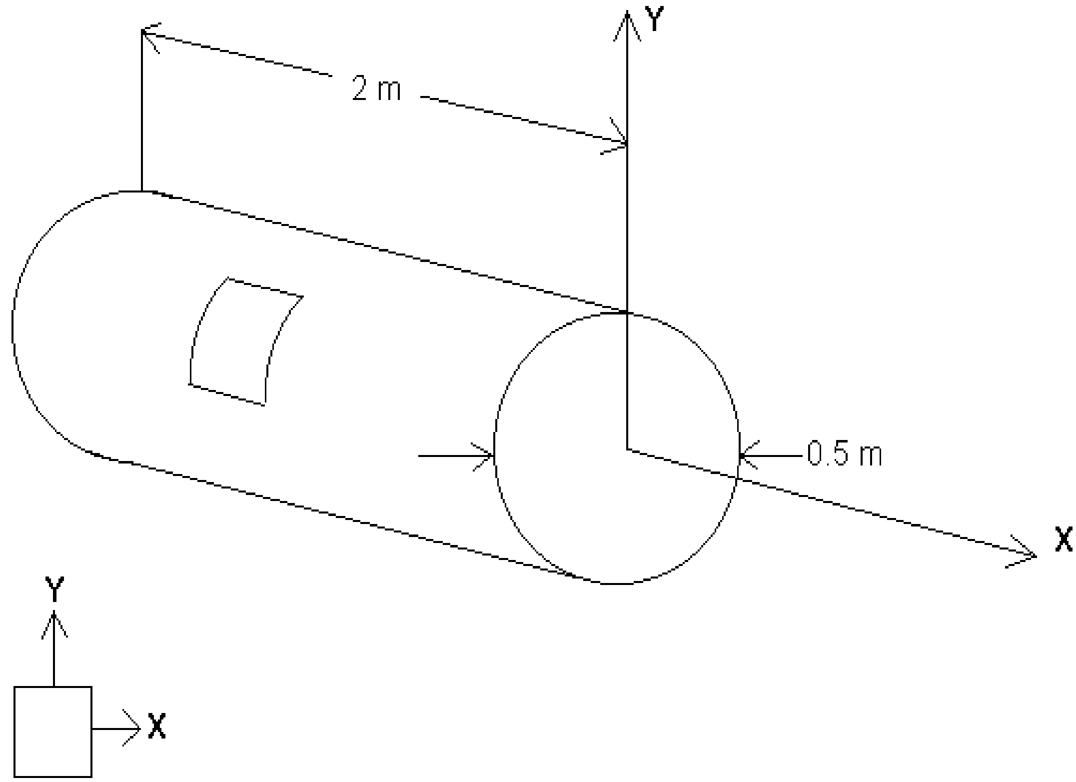

Figure 1. Pressure Vessel and Representative Element Showing Global X and Y Directions.

Cost	$[90/90/0/90/90/90/0/90]_s$	0	16	12617.4	1261.74

From the above table, it can be seen that the final design as well as the design based on minimum mass are one and the same. Due to the severe increase in mass accompanied with the use of only glass/epoxy plies, resulting from the lower strength, the minimizing function occurs such that the mass factor weighs more in the equation than the cost factor, as can be seen by the similar cost between the minimum mass and the minimum cost design. Specific details of the parameters of each follow.

The pressure vessel as you requested design information on appears below in Figure 1. The Global X direction was taken in the direction of the vessel (longitudinal) and the Global Y direction was taken around the circumference of the pressure vessel (hoop direction).

In the design of the pressure vessel there were several limitations on the parameters utilized. Ply constituents were limited to Graphite/Epoxy and Glass/Epoxy with properties listed below in Table 2. In addition to a fixed ply material (i.e. fiber volume fraction and the corresponding ultimate strength of the ply), the ply angles were limited to 0^o, $\pm30^o$, $\pm45^o$,

±60°, and 90°. The use of hybrid laminates, or combinations of the plies, was allowed. In addition to these ply properties, the thickness of each ply was limited to 0.125mm.

As far as cost considerations are concerned, the Graphite/Epoxy laminas were taken to cost 500 units/kg, and the Glass/Epoxy laminas were taken to cost 100 units/kg. Thus, the cost of the lamina is directly proportional to the mass, with the Graphite/Epoxy cost five times that of the Glass/Epoxy lamina, per unit weight.

Table 2. Properties of Laminas Used in Design.

Property	Symbol	Units	Glass/ Epoxy	Graphite/ Epoxy
Fiber Volume Fraction	V_f	--	0.45	0.70
Longitudinal Elastic Modulus	E_1	GPa	38.6	181
Transverse Elastic Modulus	E_2	GPa	8.27	10.30
Major Poisson's Ratio	ν_{12}	—	0.26	0.28
Shear Modulus	G_{12}	GPa	4.14	7.17
Ultimate Longitudinal Tensile Strength	$(\sigma_1^T)_{ult}$	MPa	1062	1500
Ultimate Longitudinal Compressive Strength	$(\sigma_1^C)_{ult}$	MPa	610	1500
Ultimate Transverse Tensile Strength	$(\sigma_2^T)_{ult}$	MPa	31	40
Ultimate Transverse Compressive Strength	$(\sigma_2^C)_{ult}$	MPa	118	246
Ultimate In-Plane Shear Strength	$(\tau_{12})_{ult}$	MPa	72	68

Once the design requirements in terms of material properties were known, the loading of the pressure vessel was determined. Plane-stress analysis was utilized, and the known relationship between internal gauge pressure and the radius and thickness of the pressure vessel was utilized to calculate the stresses. The formulas for the stress in the Global X and Y direction, as shown in Figure 1 appear below in equations 1 and 2, respectively.

$$\sigma_x = \frac{Pr}{2t} \qquad (1)$$

$$\sigma_y = \frac{Pr}{t} \qquad (2)$$

Where: P = Internal Gauge Pressure, MPa
r = Internal Radium, m
t = Thickness, m

As the design of composites utilizes force per unit width, the stresses were then multiplied by the thickness to correspond to the correct format for load input. The resulting formulas and values used in the design appear below in equations 3 and 4.

$$N_x = \sigma_x * t = \frac{0.75(10^6) * 0.25}{2 * t} * t = 0.094(10^6)\frac{N}{m} \qquad (3)$$

$$N_y = \sigma_y * t = \frac{0.75(10^6) * 0.25}{t} * t = 0.188(10^6)\frac{N}{m} \qquad (4)$$

From the above calculations, it is seen that the force in the Global Y, or hoop direction of $0.188(10^6)$ N/m is twice that of the Global X, or longitudinal direction of $0.094(10^6)$ N/m.

In order to determine the mass and cost (which was directly related to the mass as previously stated), the density and volume for each type of ply was calculated. The values for the specific gravity for each type of fiber as well as the matrix appear below in Table 3.

Table 3. Densities of Lamina Constituents

Property	Epoxy	Glass Fibers	Graphite Fibers
Specific Gravity	1.2	2.5	1.8
Density (g/cm^3)	1.2	2.5	1.8

With the specific gravity of each material known, the corresponding density was calculated knowing that density is specific gravity times the density of water, taken as 1 g/cm^3. The density of each material appears above in Table 3.

Once the density for each constituent of the lamina was known, the density of the composite lamina could be calculated. Using the known fiber volume fractions (V_f) from Table 2, the lamina densities were calculated using equation 5, below.

$$\rho_c = V_f * \rho_f + (1 - V_f) * \rho_m \qquad (5)$$

Where: ρ_c = Density of composite lamina, g/cm^3
ρ_f = Density of fiber, g/cm^3
ρ_m = Density of matrix, g/cm^3

In order to calculate the mass of each lamina, the volume of the lamina was needed. The surface area was first calculated for the pressure vessel and then multiplied by the ply thickness, as shown in equation 6. As more plies are introduced to the composite laminate, the volume of each new lamina will increase due to the enlarged radius; however, as the radius increase was minimal (0.125 mm) compared to the radius (0.25 m), this effect was neglected.

$$V_l = \left(2 * \frac{\pi * D^2}{4} + \pi * D * L\right) * t \qquad (6)$$

Where: V_l = Volume of Lamina, m^3
D = Internal Diameter, m
L = Length, m
t = thickness, m

Once the density for each type of lamina, as well as the volume, was calculated, the resulting mass of each lamina was found by simply multiplying the density by the volume and solving for the mass (g). The cost of the lamina was simply the cost times the mass of the lamina. The resulting thickness, density, volume, mass, cost, and hygrothermal properties for the two types of lamina plies are shown below in Table 4.

Table 4. Properties of Laminas.

Property	Units	Glass/ Epoxy	Graphite/ Epoxy
Thickness	mm	0.125	0.125
Density	g/cm^3	1.785	1.62
Volume	cm^3	441.79	441.79
Mass	g	788.59	715.69
Cost	units	78.86	357.85
Longitudinal Coefficient of Thermal Expansion	$\mu m/m/^oC$	8.6	0.02
Transverse Coefficient of Thermal Expansion	$\mu m/m/^oC$	22.1	22.5
Longitudinal Coefficient of Moisture Expansion	m/m/kg/kg	0.00	0.00
Transverse Coefficient of Moisture Expansion	m/m/kg/kg	0.60	0.60

Once all the properties had been determined for each lamina type, a trial design was begun. The design for minimum mass was done first, and since the mass for both lamina types was similar, yet the strength of the Graphite/Epoxy was higher than that of the Glass/Epoxy, as seen in Table 2, a design based on Graphite/Epoxy was chosen. Five different trial stacking sequences appear below in Table 5, along with the final design chosen for minimum mass.

In terms of a final design parameter, a minimizing function was calculated based on minimum mass and minimum cost design. The minimizing function weighs each factor equally, and thus was chosen to represent the best design. The formula for the minimizing function appears below, in equation 7.

To illustrate the differences in the use of the two types of laminas, the mass and cost of each lamina are shown as a function of the number of plies below in Figure 2.

$$F = \frac{A}{B} + \frac{C}{D} \qquad (7)$$

Where: F = Minimizing Value

A = Mass of Composite In Design

B = Mass of Composite Based on Minimum Mass Criteria

C = Cost of Composite In Design

D = Cost of Composite Based on Minimum Cost Criteria

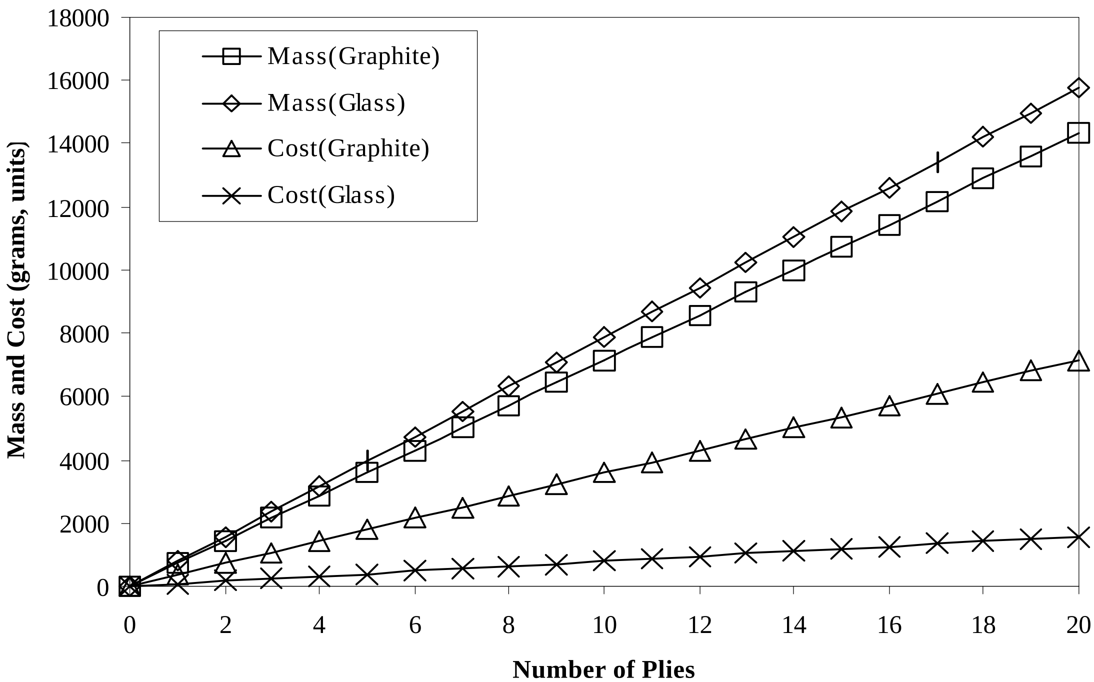

Figure 2. Mass and Cost of Plies.

Table 5. Trial Designs Based on Minimum Mass.

Stacking Sequence	Glass Plies (#)	Graphite Plies (#)	Strength Ratio	Mass (g)	Cost (units)	Minimizing Function
$[90/\overline{0}]_S$	0	3	0.7446	2147.08	1073.54	1.45
$[90/0]_S$	0	4	0.8840	2862.77	1431.39	1.93
$[0/90/\overline{0}]_S$	0	5	0.9323	3578.47	1789.23	2.42
$[90/60/\overline{0}]_S$	0	5	0.7269	3578.47	1789.23	2.42
$[90/0/\overline{90}]_S$	0	5	1.199	3578.47	1789.23	2.42

As it can be seen from Table 5, the stacking sequence for the minimum mass design involved five graphite plies arranged in a stacking sequence of $[90/0/\overline{90}]_S$. Trial designs were performed using both Glass/Epoxy laminas as well as hybrid lamina, yet the design above resulted in the lowest overall mass and thus was chosen.

In contrast to the lowest mass design requirement, where both laminas were similar in mass, the cost design criteria were quite different, as the Graphite/Epoxy laminas cost five times that of the Glass/Epoxy laminas, per unit weight. Even though the Graphite/Epoxy laminas weighed slightly less, the increase in strength over the Glass/Epoxy laminas did not justify their use to minimize the cost. Four trial designs as well as the final design based on minimum cost appear below in Table 6.

Table 6. Trial Designs Based on Minimum Cost.

Stacking Sequence	Glass Plies (#)	Graphite Plies (#)	Strength Ratio	Mass (g)	Cost (units)	Minimizing Function
$[90/0/\overline{90}]_S$	5	0	0.2999	3942.34	394.29	1.41
$[90/60/0/-60/90]_S$	10	0	0.6384	7885.88	788.59	2.83

$[90/0/90/60/-60/90]_S$	14	0	0.8581	11040.2	1104.02	3.96
$[90/0/90/0/60/-60/90/\overline{90}]_S$	15	0	0.9380	11828.8	1182.88	4.24
$[90/90/0/90/90/90/0/90]_S$	16	0	1.060	12617.4	1261.74	4.53

From Table 6 it is seen that to design a pressure vessel based solely on cost requires over three times the material of a design based on mass. The minimizing function suffers as well, with a value above 4.5. It should be noted that although the cost is in fact lower for this design, the difference in cost as compared to mass is minimal, and thus the minimizing function is higher.

Hybrid laminas were once again tried in design, however with the high cost of Graphite/Epoxy laminas, it was necessary to replace almost five Glass/Epoxy plies to equal one Graphite/Epoxy ply. Thus, hybrid laminas were ineffective in minimizing the cost.

The final design, based on the lowest minimizing function, is shown below in Table 7. It can be seen that the minimum mass and final design are one and the same, due to the low value of the minimizing function. Many hybrid designs, which gave a lower minimizing function value as shown in Table 7, were analyzed, yet none met the required strength requirements.

Table 6. Trial Designs Based on Minimizing Function.

Stacking Sequence	Glass Plies (#)	Graphite Plies (#)	Strength Ratio	Mass (g)	Cost (units)	Minimizing Function
$[90^{Gr}/0/\overline{90}]_S$	2	3	0.5247	3724.2	1231.2	2.02
$[90/0/\overline{90}^{Gl}]_S$	1	4	0.9484	3651.3	1510.2	2.22
$[0/0^{Gl}/90]_S$	2	4	0.9048	4439.9	1589.1	2.50
$[90^{Gr}/0/0]_S$	4	2	0.7077	4585.7	1031.1	2.10
$[90/0/\overline{90}]_S$	0	5	1.199	3578.4	1789.2	2.42

Where the superscript "Gr" stands for the graphite plies and the superscript "Gl" stands for the glass plies in the hybrid laminates.

From the report, it is seen that in order to satisfy the design requirements, the laminate must have a strength ratio greater than one. In all cases, a minimum value was obtained to result in a strength ratio greater than one, and the values in Table 1 represent these design laminates. It should be noted, however, that no hygrothermal analysis was done, as this may cause for variances in results, should the pressure vessel be utilized to store a liquid under temperature excursions from ambient. However, for the sake of design as you specified, the above data is deemed to be valid and accurate for the intended purpose.

In terms of the failure criteria, the first ply failure theory was utilized. That is, the composite was deemed to be adequate only if all the values of the strength ratio were greater than one. If the load were to increase by the value of the strength ratio, the ply which gave the lowest value of strength ratio would fail. If this were to happen, the remaining plies would be forced to carry more load. It was not analyzed as to what effect the failure of the first ply would have on the remaining plies, and thus it must be assumed that if the first ply fails, the entire pressure vessel would fail.

To ensure that no sudden failure would occur, a factor of safety is often imposed on the design criteria, whereby the load is increased by some factor, in effect making the vessel that factor times stronger than necessary. Again, this was not utilized in the design of the pressure vessel; however, the strength ratio of the final design has a minimum value of 1.199, effectively making the pressure vessel 1.199 times stronger (in terms of ultimate strength) than the applied load.

Again, this report summarizes the steps taken to design the pressure vessel as you requested. All work was done using the PROMAL™ computer software for analysis, as well as an Excel® spreadsheet to analyze the cost and mass for each trial stacking sequence.