MECHANICS OF COMPOSITE MATERIALS
FORMULA SHEET FOR CHAPTER 2

$$S_{11} = \frac{1}{E_1},\ S_{12} = -\frac{\nu_{12}}{E_1},\ S_{22} = \frac{1}{E_2},\ S_{66} = \frac{1}{G_{12}}.$$

$$Q_{11} = \frac{E_1}{1 - \nu_{21}\nu_{12}},\ Q_{12} = \frac{\nu_{12} E_2}{1 - \nu_{21}\nu_{12}},\ Q_{22} = \frac{E_2}{1 - \nu_{21}\nu_{12}},\ Q_{66} = G_{12}.$$

$$[T]^{-1} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix}, \quad [T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix},$$

$$\bar{Q}_{11} = Q_{11} c^4 + Q_{22} s^4 + 2(Q_{12} + 2\ Q_{66}) s^2 c^2,$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4\,Q_{66}) s^2 c^2 + Q_{12}(c^4 + s^4),$$

$$\bar{Q}_{22} = Q_{11} s^4 + Q_{22} c^4 + 2(Q_{12} + 2\,Q_{66}) s^2 c^2,$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2\,Q_{66}) c^3 s - (Q_{22} - Q_{12} - 2\,Q_{66}) s^3 c,$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2\,Q_{66}) cs^3 - (Q_{22} - Q_{12} - 2\,Q_{66}) c^3 s,$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2\,Q_{12} - 2\,Q_{66}) s^2 c^2 + Q_{66}(s^4 + c^4).$$

$$\bar{S}_{11} = S_{11} c^4 + (2\,S_{12} + S_{66}) s^2 c^2 + S_{22} s^4,$$

$$\bar{S}_{12} = S_{12}(s^4 + c^4) + (S_{11} + S_{22} - S_{66}) s^2 c^2,$$

$$\bar{S}_{22} = S_{11} s^4 + (2\,S_{12} + S_{66}) s^2 c^2 + S_{22} c^4,$$

$$\bar{S}_{16} = (2\,S_{11} - 2\,S_{12} - S_{66}) sc^3 - (2\,S_{22} - 2\,S_{12} - S_{66}) s^3 c,$$

$$\bar{S}_{26} = (2\,S_{11} - 2\,S_{12} - S_{66}) s^3 c - (2\,S_{22} - 2\,S_{12} - S_{66}) sc^3,$$

$$\bar{S}_{66} = 2(2\,S_{11} + 2\,S_{22} - 4\,S_{12} - S_{66}) s^2 c^2 + S_{66}(s^4 + c^4).$$

$$\overline{Q}_{11} = U_1 + U_2 \operatorname{Cos} 2\theta + U_3 \operatorname{Cos} 4\theta,\ \overline{Q}_{12} = U_4 - U_3 \operatorname{Cos} 4\theta,\ \overline{Q}_{22} = U_1 - U_2 \operatorname{Cos} 2\theta + U_3 \operatorname{Cos} 4\theta,$$

$$\overline{Q}_{16} = \frac{U_2}{2} \operatorname{Sin} 2\theta + U_3 \operatorname{Sin} 4\theta,\ \overline{Q}_{26} = \frac{U_2}{2} \operatorname{Sin} 2\theta - U_3 \operatorname{Sin} 4\theta,\ \overline{Q}_{66} = \frac{1}{2}(U_1 - U_4) - U_3 \operatorname{Cos} 4\theta.$$

$$U_1 = \frac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}),\ U_2 = \frac{1}{2}(Q_{11} - Q_{22}),\ U_3 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}),$$

$$U_4 = \frac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}).$$

$$\overline{S}_{11} = V_1 + V_2 \operatorname{Cos} 2\theta + V_3 \operatorname{Cos} 4\theta,\ \overline{S}_{12} = V_4 - V_3 \operatorname{Cos} 4\theta,\ \overline{S}_{22} = V_1 - V_2 \operatorname{Cos} 2\theta + V_3 \operatorname{Cos} 4\theta,$$

$$\overline{S}_{16} = V_2 \operatorname{Sin} 2\theta + 2V_3 \operatorname{Sin} 4\theta,\ \overline{S}_{26} = V_2 \operatorname{Sin} 2\theta - 2V_3 \operatorname{Sin} 4\theta,\ \overline{S}_{66} = 2(V_1 - V_4) - 4V_3 \operatorname{Cos} 4\theta,$$

$$V_1 = \frac{1}{8}(3S_{11} + 3S_{22} + 2S_{12} + S_{66}),\ V_2 = \frac{1}{2}(S_{11} - S_{22}),\ V_3 = \frac{1}{8}(S_{11} + S_{22} - 2S_{12} - S_{66}),$$

$$V_4 = \frac{1}{8}(S_{11} + S_{22} + 6S_{12} - S_{66}).$$

$$\left[\frac{\sigma_1}{(\sigma_1^T)_{ult}}\right]^2 - \left[\frac{\sigma_1\sigma_2}{(\sigma_1^T)_{ult}^2}\right] + \left[\frac{\sigma_2}{(\sigma_2^T)_{ult}}\right]^2 + \left[\frac{\tau_{12}}{(\tau_{12})_{ult}}\right]^2 < 1.$$

$H_1\sigma_1+H_2\sigma_2+H_6\tau_{12}+H_{11}\sigma_1^2+H_{22}\sigma_2^2+H_{66}\tau_{12}^2+2H_{12}\sigma_1\sigma_2< 1$

$$H_1 = \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}},\ H_{11} = \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}},\ H_2 = \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}},\ H_{22} = \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}.$$

$$H_6 = 0,\ H_{66} = \frac{1}{(\tau_{12})_{ult}^2}.$$

$H_{12} = -\dfrac{1}{2(\sigma_1^T)_{ult}^2}$ as per Tsai-Hill failure theory,

$H_{12} = -\dfrac{1}{2(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}$ as per Hoffman criterion,

$H_{12} = -\dfrac{1}{2}\sqrt{\dfrac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}}$ as per Mises-Hencky criterion.

d