

Chapter 4 & 5: Macromechanics of a Laminate

$$A_{ij} = \sum_{k=1}^n [(\bar{Q}_{ij})]_k (h_k - h_{k-1}) \quad i = 1,2,3; j = 1,2,3,$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^n [(\bar{Q}_{ij})]_k (h_k^2 - h_{k-1}^2), \quad i = 1,2,3; j = 1,2,3,$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^n [(\bar{Q}_{ij})]_k (h_k^3 - h_{k-1}^3), \quad i = 1,2,3; j = 1,2,3,$$

$$[A]_{\text{quasi-isotropic}} = \begin{bmatrix} U_1 & U_4 & 0 \\ U_4 & U_1 & 0 \\ 0 & 0 & (U_1 - U_4)/2 \end{bmatrix} h.$$

$$\sum_{k=1}^N \cos kx \equiv \frac{\sin\left(N + \frac{1}{2}\right)x}{2\sin\left(\frac{x}{2}\right)} - \frac{1}{2}, \quad \sum_{k=1}^N \sin kx \equiv \frac{\cos\left(\frac{x}{2}\right) - \cos\left(N + \frac{1}{2}\right)x}{2\sin\left(\frac{x}{2}\right)},$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

$$[N^T] = \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \Delta T \sum_{k=1}^n \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k (h_k - h_{k-1})$$

$$[M^T] = \begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} = \frac{1}{2} \Delta T \sum_{k=1}^n \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k (h_k^2 - h_{k-1}^2)$$

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$$\begin{aligned}
 [N^C] &= \begin{bmatrix} N_x^C \\ N_y^C \\ N_{xy}^C \end{bmatrix} = \Delta C \sum_{k=1}^n \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix}_k (h_k - h_{k-1}) \\
 [M^C] &= \begin{bmatrix} M_x^C \\ M_y^C \\ M_{xy}^C \end{bmatrix} = \frac{1}{2} \Delta C \sum_{k=1}^n \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix}_k (h_k^2 - h_{k-1}^2)
 \end{aligned}$$

$$T_c = (2\pi r_m^2 t)(0.272)(E_x E_y^3)^{1/4} \left(\frac{t}{r_m} \right)^{3/2}$$

$$f_n = \frac{\pi}{2} \sqrt{\frac{E_x I}{mL^4}}$$

$\sigma_x = pr/(2t)$; $\sigma_y = pr/t$ for cylindrical pressure vessel

$\sigma_x = pr/(2t)$; $\sigma_y = pr/(2t)$ for spherical pressure vessel