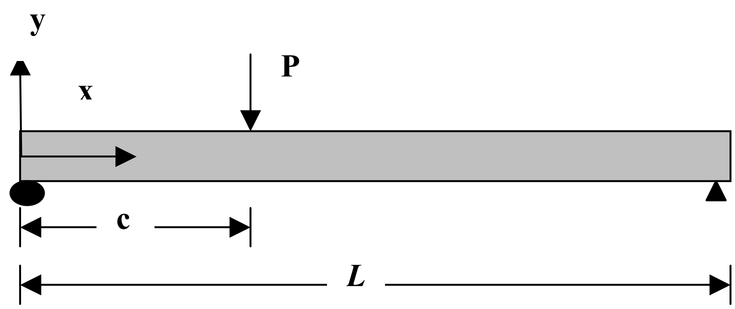
Question.10.25 A simply supported beam carries a load *P* at a distance c away from its left end. Obtain the beam deflection at the point where *P* is applied .Use the Rayleigh-Ritz method. Assume a deflection curve of the form,

v = a.x(L-x),

where *a* is to be determined.



Solution.

In accordance with the Rayleigh-Ritz method,

$$\prod = U - W$$

Where, $\prod$ = Potential energy

U = Strain energy

W = Change in potential

$$U = \int U_0 . dV$$

$$\therefore U = \int \frac{M^2 . y^2}{2E.I^2} . dV$$

$$\therefore U = \iint \frac{M^2 .(y^2 .dA).dx}{2E\, I^2}$$ ……….Since, $dV = dA.dx$

But, $\frac{M}{E.I} = \frac{\partial^2 v}{\partial x^2}$

$$\therefore U = \frac{EI}{2} . \int_0^L (\frac{\partial^2 v}{\partial x^2})^2 .dx$$

$$\because v = a.x(L - x)$$ ……………….assumed deflection curve.

$$\therefore \frac{dv}{dx} = a(L - 2x)$$

$$\therefore \frac{\partial^2 v}{\partial x^2} = -2a$$

$$\therefore U = \frac{EI}{2} . \int_0^L (-2a)^2 .dx$$

$$\therefore U = 2EI.a^2.L$$

And, since

$$W = -P.v_c = -p.a.c.(L - c)$$

Hence, we have

$$\because \prod = U - W$$

$$\therefore \prod = 2EI\, a^2.L + P.a.c.(L - c)$$

As the potential energy must be minimum at equilibrium,

$$\frac{\partial \prod}{\partial a} = 0$$

$$\therefore 2EIL(2a) - P.c.(L - c) = 0$$

$$\therefore a = -\frac{P.c.(L - c)}{4EIL}$$

Now, we substitute this constant back in the assumed deflection curve equation, so that we will be able to calculate the deflection at any section of the beam.

$$v = -\frac{P.c.(L - c)}{4EIL} .x.(L - x)$$

Now, at the desired section, x = c ……(distance from the left end.)

$$\therefore v_c = -\frac{P.c^2 (L - c)^2}{4EIL}$$