<u>COMPOUND PRESSURE VESSEL</u>

A cylinder for a high-pressure compressor having an internal diameter of 25 mm is to be made from a steel grade (E=207 GPa) for which the maximum allowable hoop stress (tensile or compressive) is 400 MPa. **There is no limit on the maximum allowable radial stress.**

a) If the internal pressure is to be 600 MPa, show that a simple cylinder will not be satisfactory.

b) Investigate the use of two compounded cylinders to take an internal pressure of 600 MPa, each having a ratio of external to internal diameter of 2 (the inner diameter of the compounded cylinder assembly still is 25 mm), and suggest a suitable radial interference (with tolerances) between the cylinders before compounding.

(Hint and Answer: The minimum interface pressure to avoid excessive tensile hoop stress at the bore is 105 MPa. Maximum interface pressure to avoid excessive tensile hoop stress in the outer surface at the interface is 120 MPa, which is lower than the maximum interface pressure to avoid excessive hoop stress at the bore due to compounding alone, namely, 150 MPa. Hence, $0.0423 \le \delta \le 0.0483$*, or* $\delta = 0.0453 \pm 0.003$ *mm)*

a) Single Cylinder

$$\sigma_\theta|_{\max} = p_i \frac{b^2 + a^2}{b^2 - a^2}$$

Since $b^2 + a^2 > b^2 - a^2$ for $b>a$, $a>0$

$$\frac{b^2 + a^2}{b^2 - a^2} > 1$$

$$\sigma_\theta|_{\max} > p_i > 600\,MPa$$

Hence a single cylinder is not a viable option.

B) Compound Cylinder:

Eqn. 8.23 gives interface pressure due to interference.

$$p = \frac{E\delta}{b} \frac{(b^2 - a^2)(c^2 - b^2)}{2b^2(c^2 - a^2)}$$

where

a = inner nominal radius of cylinder 1 (0.0125 mm)

b = outer nominal radius of cylinder 1 (0.025 mm)

b = inner nominal radius of cylinder 1 (0.025 mm)

c = outer nominal radius of cylinder 2 (0.050 mm)

E = 207 GPa.

$$ä = \frac{pb}{E}\frac{2b^2(c^2-a^2)}{(b^2-a^2)(c^2-b^2)}$$

$$ä = \frac{p.b}{E}\frac{2(4a^2)(16a^2-a^2)}{(4a^2-a^2)(16a^2-4a^2)} \, as \begin{pmatrix} c = 2b \\ b = 2a \end{pmatrix}$$

$$ä = \frac{pb}{E}\frac{120}{36}$$

$$ä = \frac{10\,pb}{3\,E}$$

$$ä = \frac{10\,p(25)(10^{-3})}{3(207)(10^9)}$$

ä = $4.026\ (10^{-13})\ p$

1) Hoop stress due to internal pressure of 600 MPa.

$$s_q^1 = \frac{600(10^6)0.0125^2}{0.05^2-0.0125^2}\left(1+\frac{0.05^2}{r^2}\right)$$

$$= 40(10^6)\left(1+\frac{0.0025}{r^2}\right)$$

r (mm)	óè (MPa)
12.5 25 50	680 200 80

2) Hoop stress due to interface pressure p

Inner cylinder (Eqn. 8.16)

$$s_q = \frac{p\,(0.025^2)}{0.025^2 - 0.0125^2}\left(1 + \frac{0.0125^2}{r^2}\right)$$

$$= -\frac{4}{3}\,p\left(1 + \frac{1.05625(10^{-4})}{r^2}\right)$$

Outer cylinder (Eqn. 8.13)

$$s_q = \frac{p\,(0.025^2)}{0.05^2 - 0.025^2}\left(1 + \frac{0.05^2}{r^2}\right)$$

$$= \frac{1}{3}\,p\left(1 + \frac{2.5\,(10^{-3})}{r^2}\right)$$

r (mm)	óè (MPa)
12.5 25- 25+ 50	-8/3 p -5/3 p 5/3 p 2/3 p

Total hoop stresses due to internal pressure and interference pressure

r (mm)	óè (MPa)
12.5	$680 - \frac{8}{3}p$
25-	$200 - \frac{5}{3}p$
25+	$200 + \frac{5}{3}p$
50	$80 + \frac{2}{3}p$

Since the ultimate tensile and compressive hoop stress is limited to 400 MPa,

$$-400 < 680 - \frac{8}{3}p < 400$$

$$-400 < 200 - \frac{5}{3}p < 400$$

$$-400 < 200 + \frac{5}{3}p < 400$$

$$-400 < 200 + \frac{5}{3}p < 400$$

Hence

$$105 < p < 405$$
$$-240 < p < 360$$
$$-360 < p < 120$$
$$-720 < p < 480$$

Therefore $105 < p < 120$ as max (105, -240, -360, -720) = 105 and min (405, 360, 120, 480) = 120.

Lower limit - Hoop stress at inside radius of cylinder 1

Upper limit - Hoop stress in cylinder 2 at interface

Since

ä = 4.026 (10^{-13})p

$$105\ (10^{6}) < \frac{\boldsymbol{d}}{4.026(10^{-13})} < 120\ (10^{6})$$

4.2273 (10^{-5})m < ä < 4.8312 (10^{-5})m

0.042273 mm < ä < 0.048312 mm.

ä = 0.0452925±0.0030195 mm.