Cartesian Co-ordinate System Equations

Equilibrium Equations

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}+X=0$$

$$\frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yz}}{\partial z}+Y=0$$

$$\frac{\partial \sigma_z}{\partial z}+\frac{\partial \tau_{zx}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+Z=0$$

Strain–Displacement Equations

$$\in_x=\frac{\partial u}{\partial x},\in_y=\frac{\partial v}{\partial y},\in_z=\frac{\partial w}{\partial z}$$

$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x},\gamma_{zx}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x},\gamma_{yz}=\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}$$

Hooke's law

$$\in_x=\frac{\sigma_x}{E}-\frac{\nu\sigma_y}{E}-\frac{\nu\sigma_z}{E}$$

$$\in_y=-\frac{\nu\sigma_x}{E}+\frac{\sigma_y}{E}-\frac{\nu\sigma_z}{E}$$

$$\in_z=-\frac{\nu\sigma_x}{E}-\frac{\nu\sigma_y}{E}+\frac{\sigma_z}{E}$$

$$\gamma_{xy}=\frac{\tau_{xy}}{G},\gamma_{yz}=\frac{\tau_{yz}}{G},\gamma_{zx}=\frac{\tau_{zx}}{G}$$

Boundary Conditions

$$\overline{X}=\sigma_x l+\tau_{xy}m+\tau_{zx}n,\overline{Y}=\sigma_y m+\tau_{yz}n+\tau_{xy}l,$$

$$\overline{Z}=\sigma_z n+\tau_{zx}l+\tau_{yz}m$$

Hooke's law – Plane Stress

$$\in_x=\frac{1}{E}\left(\sigma_x-\nu\sigma_y\right),\in_y=\frac{1}{E}\left(\sigma_y-\nu\sigma_x\right),\gamma_{xy}=\frac{\tau_{xy}}{G}$$

Hooke's law – Plane Strain

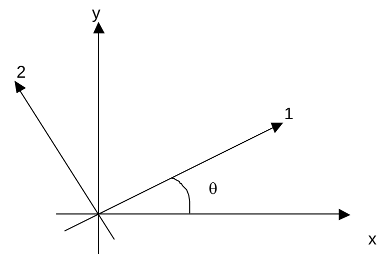
$$\in_x=\frac{1+\nu}{E}\left[(1-\nu)\sigma_x-\nu\sigma_y\right],\in_y=\frac{1+\nu}{E}\left[(1-\nu)\sigma_y-\nu\sigma_x\right]$$

$$\gamma_{xy}=\frac{\tau_{xy}}{G}.$$

Transformation of strains and stresses

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix}=[T]\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix},\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}/2\end{bmatrix}=[T]\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}/2\end{bmatrix},$$

$$[T]=\begin{bmatrix}c^2 & s^2 & 2sc\\ s^2 & c^2 & -2sc\\ -sc & sc & c^2-s^2\end{bmatrix}, s=\sin(\theta),\ c=\cos(\theta)$$



Equations of Compatibility

$$\frac{\partial^2\in_x}{\partial y^2}+\frac{\partial^2\in_y}{\partial x^2}=\frac{\partial^2\gamma_{xy}}{\partial x\,\partial y}$$

$$2\frac{\partial^2\in_x}{\partial y\,\partial z}=\frac{\partial}{\partial x}\left(-\frac{\partial\gamma_{yz}}{\partial x}+\frac{\partial\gamma_{xz}}{\partial y}+\frac{\partial\gamma_{xy}}{\partial z}\right)$$

$$\frac{\partial^2\in_y}{\partial z^2}+\frac{\partial^2\in_z}{\partial y^2}=\frac{\partial^2\gamma_{yz}}{\partial y\,\partial z}$$

$$2\frac{\partial^2\in_y}{\partial x\,\partial z}=\frac{\partial}{\partial y}\left(\frac{\partial\gamma_{yz}}{\partial x}-\frac{\partial\gamma_{xz}}{\partial y}+\frac{\partial\gamma_{xy}}{\partial z}\right)$$

$$\frac{\partial^2\in_z}{\partial x^2}+\frac{\partial^2\in_x}{\partial z^2}=\frac{\partial^2\gamma_{xz}}{\partial x\,\partial z}$$

$$2\frac{\partial^2\in_z}{\partial x\,\partial y}=\frac{\partial}{\partial z}\left(\frac{\partial\gamma_{yz}}{\partial x}+\frac{\partial\gamma_{xz}}{\partial y}-\frac{\partial\gamma_{xy}}{\partial z}\right)$$

$$(1+\nu)\nabla^2\sigma_x+\frac{\partial^2\theta}{\partial x^2}=0,(1+\nu)\nabla^2\tau_{yz}+\frac{\partial^2\theta}{\partial y\,\partial z}=0$$

$$(1+\nu)\nabla^2\sigma_y+\frac{\partial^2\theta}{\partial y^2}=0,(1+\nu)\nabla^2\tau_{xz}+\frac{\partial^2\theta}{\partial x\,\partial z}=0$$

$$(1+\nu)\nabla^2\sigma_z+\frac{\partial^2\theta}{\partial z^2}=0,(1+\nu)\nabla^2\tau_{xy}+\frac{\partial^2\theta}{\partial x\,\partial y}=0$$

$$\theta=\sigma_x+\sigma_y+\sigma_z$$

Polar Coordinates Equations

$$x=r\,Cos\,\theta\quad;\quad y=r\,Sin\,\theta$$

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$

Equilibrium Equations

$$\frac{\partial\sigma_r}{\partial r}+\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta}+\frac{\sigma_r-\sigma_\theta}{r}+F_r=0$$

$$\frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta}+\frac{\partial\tau_{r\theta}}{\partial r}+\frac{2\tau_{r\theta}}{r}+F_\theta=0$$

Stress – Airy Stresses Equations

$$\sigma_r=\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2},\ \sigma_\theta=\frac{\partial^2\phi}{\partial r^2},\tau_r=-\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right)$$

Strain – Displacement Equations

$$\in_r = \frac{\partial u_r}{\partial r}, \in_\theta = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}, \gamma_{r\theta} = \frac{\partial u_\theta}{\partial r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r}$$

Hooke's law – Plane Stress

$$\in_r = \frac{1}{E}(\sigma_r - \nu\sigma_\theta), \in_\theta = \frac{1}{E}(\sigma_\theta - \nu\sigma_r), \gamma_{r\theta} = \frac{\tau_{r\theta}}{G}$$

Hooke's law – Plane Strain

$$\in_r = \frac{1+\nu}{E}[(1-\nu)\sigma_r - \nu\sigma_\theta], \in_\theta = \frac{1+\nu}{E}[(1-\nu)\sigma_\theta - \nu\sigma_r]$$

$$\gamma_{r\theta} = \frac{\tau_{r\theta}}{G}$$

Transformation equations.

$$\begin{bmatrix}\sigma_r\\ \sigma_\theta\\ \tau_{r\theta}\end{bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc\\ -s^2 & c^2 & -2sc\\ -sc & sc & c^2 - s^2\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}$$

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix} c^2 & s^2 & -2sc\\ s^2 & c^2 & 2sc\\ sc & -sc & c^2 - s^2\end{bmatrix}\begin{bmatrix}\sigma_r\\ \sigma_\theta\\ \tau_{r\theta}\end{bmatrix}$$

Internal Pressure Only

$$\sigma_r = \frac{a^2 p_i}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right), \ \sigma_\theta = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

$$u = \frac{a^2 p_i r}{E(b^2 - a^2)}\left[(1-\nu) + (1+\nu)\frac{b^2}{r^2}\right]$$

External Pressure Only

$$\sigma_r = -\frac{p_0 b^2}{b^2 - a^2}\left(1 - \frac{a^2}{r^2}\right), \sigma_\theta = -\frac{p_0 b^2}{b^2 - a^2}\left(1 + \frac{a^2}{r^2}\right)$$

$$u = -\frac{b^2 p_0 r}{E(b^2 - a^2)}\left[(1-\nu) + (1+\nu)\frac{a^2}{r^2}\right]$$

Shrink Fit

$$p = \frac{E\delta}{b}\frac{(b^2 - a^2)(c^2 - b^2)}{2b^2(c^2 - a^2)}$$

Rotating Disks

$$u = -\frac{\rho\omega^2 r^3(1-\nu^2)}{8E} + C_1 r + \frac{C_2}{r}$$

$$\sigma_r = \frac{E}{1-\nu^2}\left[\begin{array}{c}\dfrac{-(3+\nu)(1-\nu^2)\rho\omega^2 r^2}{8E}\\ + (1+\nu)C_1 - (1-\nu)\dfrac{C_2}{r^2}\end{array}\right]$$

$$\sigma_\theta = \frac{E}{1-\nu^2}\left[\begin{array}{c}\dfrac{-(1+3\nu)(1-\nu^2)\rho\omega^2 r^2}{8E}\\ + (1+\nu)C_1 + (1-\nu)\dfrac{C_2}{r^2}\end{array}\right]$$

Solid disk

$$u = \frac{1-\nu}{8E}[(3+\nu)b^2 - (1+\nu)r^2]\rho\omega^2 r$$

$$\sigma_r = \frac{3+\nu}{8}\left[b^2 - r^2\right]\rho\omega^2$$

$$\sigma_\theta = \frac{3+\nu}{8}\left[b^2 - \frac{1+3\nu}{3+\nu}r^2\right]\rho\omega^2$$

Annular disk

$$u = \frac{(3+\nu)(1-\nu)}{8E}\left[\begin{array}{l} a^2 + b^2 - \dfrac{1+\nu}{3+\nu}r^2\\ + \dfrac{1+\nu}{1-\nu}\dfrac{a^2 b^2}{r^2}\end{array}\right]\rho\omega^2 r$$

$$\sigma_r = \frac{3+\nu}{8}\left[a^2 + b^2 - r^2 - \frac{a^2 b^2}{r^2}\right]\rho\omega^2$$

$$\sigma_\theta = \frac{3+\nu}{8}\left[a^2 + b^2 - \frac{1+3\nu}{3+\nu}r^2 + \frac{a^2 b^2}{r^2}\right]\rho\omega^2$$

Hole in Plate

$$\sigma_r = \frac{1}{2}\sigma_o\left[\left(1 - \frac{a^2}{r^2}\right) + \left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)\cos 2\theta\right]$$

$$\sigma_\theta = \frac{1}{2}\sigma_o\left[\left(1 + \frac{a^2}{r^2}\right) - \left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$\tau_{r\theta} = -\frac{1}{2}\sigma_o\left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2}\right)\sin 2\theta$$

Three Dimensional Stresses/Strains

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0, \ I_1 = \sigma_x + \sigma_y + \sigma_z$$

$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}{}^2 - \tau_{yz}{}^2 - \tau_{zx}{}^2$$

$$I_3 = \begin{vmatrix}\sigma_x & \tau_{xy} & \tau_{zx}\\ \tau_{xy} & \sigma_y & \tau_{yz}\\ \tau_{zx} & \tau_{yz} & \sigma_z\end{vmatrix}, K = \frac{E}{3(1-2\nu)}$$

$$U_0 = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - \frac{\nu}{E}(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x) + \frac{1}{2G}(\tau_{xy}{}^2 + \tau_{yz}{}^2 + \tau_{zx}{}^2)$$

$$U_{od} = \frac{3}{4G}\tau_{oct}{}^2,$$

$K_{1C} = 1.12\sigma\sqrt{\pi a}$ (edge cracks); $= \sigma\sqrt{\pi a}$ (a = half crack length)

$$\tau_{oct} = \frac{1}{3}\left[\begin{array}{l}(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2\\ + 6(\tau_{xy}{}^2 + \tau_{yz}{}^2 + \tau_{zx}{}^2)\end{array}\right]^{\frac{1}{2}}$$