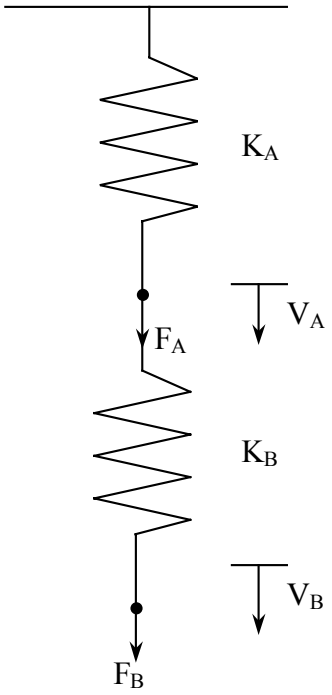
Principle of Stationery Potential Energy

Among all admissible displaced configurations of a conservative system those that satisfy equations of equilibrium make the potential energy stationery with respect to small admissible variations of displacement. If the stationery condition is a relative minimum, the equilibrium state is stable.

Example:

Consider the system of two springs as shown. Find the displacements, V_A and V_B of the two springs from the state when no forces are applied.



Solution

$$\Pi = U - W$$

$$U = \frac{1}{2}K_A V_A^2 + \frac{1}{2}K_B\left(V_B - V_A\right)^2$$

$$W = F_A V_A - F_B V_B$$

$$\Pi = \frac{1}{2}K_A V_A^2 + \frac{1}{2}K_B\left(V_B - V_A\right)^2 - F_A V_A - F_B V_B$$

$$\frac{d\Pi}{dV_A} = 0$$

gives

$$K_A V_A - K_B\left(V_B - V_A\right) = F_A \text{.....(1)}$$

$$\frac{d\Pi}{dV_B} = 0$$

gives

$$K_B\left(V_B - V_A\right) = F_B \text{.....(2)}$$

From (1) and (2), we get the displacements

$$V_A = \frac{F_A + F_B}{K_B}$$

$$V_B = \frac{F_B}{K_B} + \frac{F_A F_B}{K_A}$$