EML 4230 Introduction to Composite Materials

Chapter 2 Macromechanical Analysis of a Lamina

2D Stiffness and Compliance Matrix for Unidirectional Lamina

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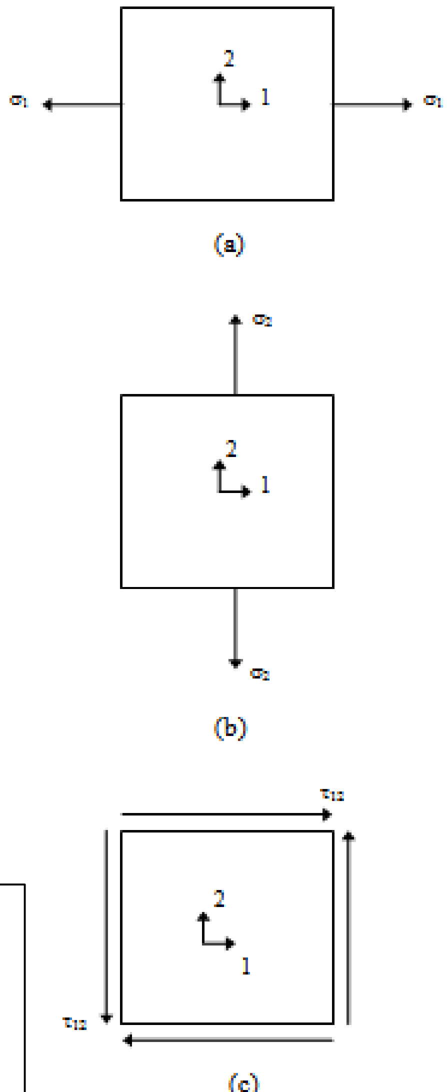
Courtesy of the Textbook

Mechanics of Composite Materials by Kaw

USF UNIVERSITY OF SOUTH FLORIDA

Compliance and Stiffness Matrix Elements in Terms of Elastic Constants



(a) Tensile load in direction 1

(b) Tensile load in direction in 2

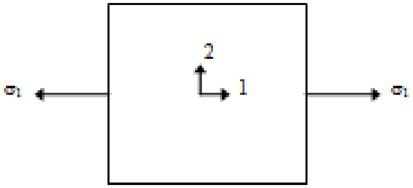
(c) Pure shear stress in plane of 1-2

FIGURE 2.18
Application of stresses to find engineering constants of a unidirectional lamina

Pure Axial Load in Direction 1

Apply a pure axial load in direction 1

$\sigma_1 \neq 0, \ \sigma_2 = 0, \ \tau_{12} = 0$



$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \longrightarrow \begin{matrix} \varepsilon_1 = S_{11}\sigma_1 \\ \varepsilon_2 = S_{12}\sigma_1 \\ \gamma_{12} = 0 \end{matrix}$$

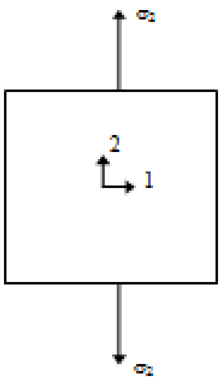
$$E_1 \equiv \frac{\sigma_1}{\varepsilon_1} = \frac{1}{S_{11}} \longrightarrow S_{11} = \frac{1}{E_1}$$

$$\nu_{12} \equiv -\frac{\varepsilon_2}{\varepsilon_1} = -\frac{S_{12}}{S_{11}}. \longrightarrow S_{12} = -\frac{\nu_{12}}{E_1}$$

Pure Axial Load in Direction 2

Apply a pure axial load in direction 2

$\sigma_1 = 0, \ \sigma_2 \neq 0, \ \tau_{12} = 0$



$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \longrightarrow \begin{matrix} \varepsilon_1 = S_{12}\sigma_2 \\ \varepsilon_2 = S_{22}\sigma_2 \\ \gamma_{12} = 0 \end{matrix}$$

$$E_2 \equiv \frac{\sigma_2}{\varepsilon_2} = \frac{1}{S_{22}} \longrightarrow S_{22} = \frac{1}{E_2}$$

$$\nu_{21} \equiv -\frac{\varepsilon_1}{\varepsilon_2} = -\frac{S_{12}}{S_{22}}. \longrightarrow S_{12} = -\frac{\nu_{21}}{E_2}$$

Reciprocal Relationship

$$S_{12} = -\frac{v_{12}}{E_1}$$

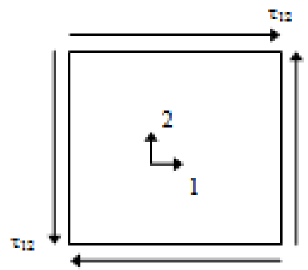
$$S_{12} = -\frac{v_{21}}{E_2}$$

$$\frac{v_{12}}{E_1} = \frac{v_{21}}{E_2}$$

Pure Shear Load in Plane 12

Apply a pure shear load in Plane 12

$$\sigma_1 = 0,\ \sigma_2 = 0,\ \tau_{12} \neq 0$$



$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \longrightarrow \begin{matrix} \varepsilon_1 = 0 \\ \varepsilon_2 = 0 \\ \gamma_{12} = S_{66}\tau_{12} \end{matrix}$$

$$G_{12} \equiv \frac{\tau_{12}}{\gamma_{12}} = \frac{1}{S_{66}} \longrightarrow S_{66} = \frac{1}{G_{12}}$$

Compliance Matrix

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

Coefficients of Stiffness Matrix

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

$$Q_{11} = \frac{S_{22}}{S_{11} S_{22} - S_{12}^2}$$

$$Q_{12} = -\frac{S_{12}}{S_{11} S_{22} - S_{12}^2}$$

$$Q_{22} = \frac{S_{11}}{S_{11} S_{22} - S_{12}^2}$$

$$Q_{66} = \frac{1}{S_{66}}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \frac{E_1}{1 - \nu_{21}\nu_{12}} & \frac{\nu_{12} E_2}{1 - \nu_{21}\nu_{12}} & 0 \\ \frac{\nu_{12} E_2}{1 - \nu_{21}\nu_{12}} & \frac{E_2}{1 - \nu_{21}\nu_{12}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

END