EML 4230 Introduction to Composite Materials

Chapter 2 Macromechanical Analysis of a Lamina

Engineering Constants

Part 6

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Courtesy of the Textbook

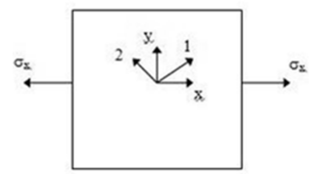
Mechanics of Composite Materials by Kaw

UNIVERSITY OF SOUTH FLORIDA

1

Pure Axial Load in Direction x

$\sigma_x \neq 0, \sigma_y = 0, \tau_{xy} = 0$



(a)

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \cancelto{0}{\sigma_y} \\ \cancelto{0}{\tau_{xy}} \end{bmatrix}$$

FIGURE 2.23
Application of stresses to find engineering constants of an angle lamina

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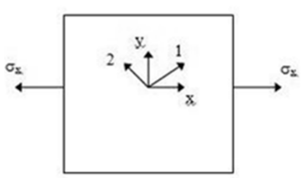
Engineering Constants

$\varepsilon_x = \bar{S}_{11}\sigma_x$ $\quad$ $\varepsilon_y = \bar{S}_{12}\sigma_x$ $\quad$ $\gamma_{xy} = \bar{S}_{16}\sigma_x$

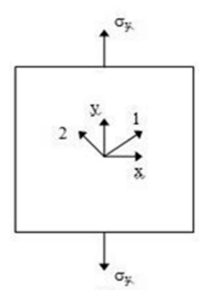
$$E_x \equiv \frac{\sigma_x}{\varepsilon_x} = \frac{1}{\bar{S}_{11}} \qquad \nu_{xy} \equiv -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\bar{S}_{12}}{\bar{S}_{11}} \qquad \frac{1}{m_x} \equiv -\frac{\sigma_x}{\gamma_{xy} E_1} = -\frac{1}{\bar{S}_{16} E_1}$$



3

Pure Axial Load in Direction y

$\sigma_x = 0, \sigma_y \neq 0, \tau_{xy} = 0$



(b)

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{bmatrix} \cancelto{0}{\sigma_x} \\ \sigma_y \\ \cancelto{0}{\tau_{xy}} \end{bmatrix}$$

FIGURE 2.23
Application of stresses to find engineering constants of an angle lamina

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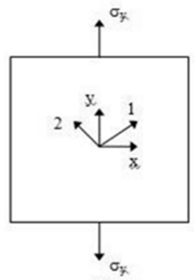
Engineering Constants

$$\varepsilon_x = \bar{S}_{12}\sigma_y \qquad \varepsilon_y = \bar{S}_{22}\sigma_y \qquad \gamma_{xy} = \bar{S}_{26}\sigma_y$$

$$\nu_{yx} \equiv -\frac{\varepsilon_x}{\varepsilon_y} = -\frac{\bar{S}_{12}}{\bar{S}_{22}} \qquad E_y \equiv \frac{\sigma_y}{\varepsilon_y} = \frac{1}{\bar{S}_{22}} \qquad \frac{1}{m_y} \equiv -\frac{\sigma_y}{\gamma_{xy}E_1} = -\frac{1}{\bar{S}_{26}E_1}$$

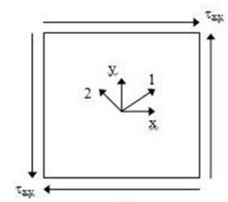
$$\frac{\nu_{yx}}{E_y} = \frac{\nu_{xy}}{E_x}$$



5

Pure Shear Load in x-y Plane

$$\sigma_x = 0, \sigma_y = 0, \tau_{xy} \neq 0$$



(c)

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{bmatrix} \cancelto{0}{\sigma_x} \\ \cancelto{0}{\sigma_y} \\ \tau_{xy} \end{bmatrix}$$

FIGURE 2.23
Application of stresses to find engineering constants of an angle lamina

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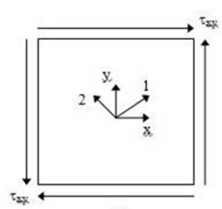
Engineering Constants

$$\varepsilon_x = \bar{S}_{16}\tau_{xy} \qquad \varepsilon_y = \bar{S}_{26}\tau_{xy} \qquad \gamma_{xy} = \bar{S}_{66}\tau_{xy}$$

$$\frac{1}{m_x} = -\frac{\tau_{xy}}{\varepsilon_x E_1} = -\frac{1}{\bar{S}_{16}E_1} \qquad \frac{1}{m_y} = -\frac{\tau_{xy}}{\varepsilon_y E_1} = -\frac{1}{\bar{S}_{26}E_1} \qquad G_{xy} \equiv \frac{\tau_{xy}}{\gamma_{xy}} = \frac{1}{\bar{S}_{66}}$$



7

Stress-Strain Relationships for Angle Lamina

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & -\frac{m_x}{E_1} \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{m_y}{E_1} \\ -\frac{m_x}{E_1} & -\frac{m_y}{E_1} & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

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Engineering Constant E_x of an Angle Lamina

$$\frac{1}{E_x} = \bar{S}_{11}$$

$$= S_{11}c^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}s^4$$

$$= \frac{1}{E_1}c^4 + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)s^2c^2 + \frac{1}{E_2}s^4$$

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Engineering Constant E_y of an Angle Lamina

$$\frac{1}{E_y} = \bar{S}_{22}$$

$$= S_{11}s^4 + (2S_{12} + S_{66})c^2s^2 + S_{22}c^4$$

$$= \frac{1}{E_1}s^4 + \left(-\frac{2\nu_{12}}{E_1} + \frac{1}{G_{12}}\right)c^2s^2 + \frac{1}{E_2}c^4$$

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Engineering Constant G_{xy} of an Angle Lamina

$$\frac{1}{G_{xy}} = \bar{S}_{66}$$

$$= 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})s^2c^2 + S_{66}(s^4 + c^4)$$

$$= 2\left(\frac{2}{E_1} + \frac{2}{E_2} + \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)s^2c^2 + \frac{1}{G_{12}}(s^4 + c^4)$$

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Engineering Constant m_x of an Angle Lamina

$$m_x = -\bar{S}_{16}E_1$$

$$= -E_1\left[(2S_{11} - 2S_{12} - S_{66})sc^3 - (2S_{22} - 2S_{12} - S_{66})s^3c\right]$$

$$= E_1\left[\left(-\frac{2}{E_1} - \frac{2\nu_{12}}{E_1} + \frac{1}{G_{12}}\right)sc^3 + \left(\frac{2}{E_2} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)s^3c\right]$$

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Engineering Constant m_y of an Angle Lamina

$$m_y = -\bar{S}_{26} E_1$$

$$= -E_1\left[(2S_{11} - 2S_{12} - S_{66})s^3 c - (2S_{22} - 2S_{12} - S_{66})s c^3\right]$$

$$= E_1\left[\left(-\frac{2}{E_1} - \frac{2\nu_{12}}{E_1} + \frac{1}{G_{12}}\right)s^3 c + \left(\frac{2}{E_2} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)s c^3\right]$$

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END

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