

EML 4230 Introduction to Composite Materials

Chapter 2 Macromechanical Analysis of a Lamina **Invariant Form of Transformed Matrices**

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Courtesy of the Textbook

[Mechanics of Composite Materials by Kaw](#)



Invariant Form of Stiffness Matrix

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix},$$

$$\bar{Q}_{11} = U_1 + U_2 \cos 2\theta + U_3 \cos 4\theta$$

$$\bar{Q}_{12} = U_4 - U_3 \cos 4\theta$$

$$\bar{Q}_{22} = U_1 - U_2 \cos 2\theta + U_3 \cos 4\theta$$

$$\bar{Q}_{16} = \frac{U_2}{2} \sin 2\theta + U_3 \sin 4\theta$$

$$\bar{Q}_{26} = \frac{U_2}{2} \sin 2\theta - U_3 \sin 4\theta$$

$$\bar{Q}_{66} = \frac{1}{2}(U_1 - U_4) - U_3 \cos 4\theta$$

$$U_1 = \frac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66})$$

$$U_2 = \frac{1}{2}(Q_{11} - Q_{22})$$

$$U_3 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})$$

$$U_4 = \frac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66})$$

Invariant Form of Compliance Matrix

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix},$$

$$\bar{S}_{11} = V_1 + V_2 \cos 2\theta + V_3 \cos 4\theta,$$

$$\bar{S}_{12} = V_4 - V_3 \cos 4\theta,$$

$$\bar{S}_{22} = V_1 - V_2 \cos 2\theta + V_3 \cos 4\theta,$$

$$\bar{S}_{16} = V_2 \sin 2\theta + 2V_3 \sin 4\theta,$$

$$\bar{S}_{26} = V_2 \sin 2\theta - 2V_3 \sin 4\theta,$$

$$\bar{S}_{66} = 2(V_1 - V_4) - 4V_3 \cos 4\theta,$$

$$V_1 = \frac{1}{8}(3S_{11} + 3S_{22} + 2S_{12} + S_{66}),$$

$$V_2 = \frac{1}{2}(S_{11} - S_{22}),$$

$$V_3 = \frac{1}{8}(S_{11} + S_{22} - 2S_{12} - S_{66}),$$

$$V_4 = \frac{1}{8}(S_{11} + S_{22} + 6S_{12} - S_{66})$$

END