EML 4230 Introduction to Composite Materials

Chapter 2 Macromechanical Analysis of a Lamina
Maximum Stress Failure Theory

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Courtesy of the Textbook
Mechanics of Composite Materials by Kaw

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Strength Failure Theories for an Angle Lamina

- The failure theories are generally based on the normal and shear strengths of a unidirectional lamina.
- In the case of a unidirectional lamina, the five strength parameters are:
  - Longitudinal tensile strength $\left(\sigma_1^T\right)_{ult}$
  - Longitudinal compressive strength $\left(\sigma_1^C\right)_{ult}$
  - Transverse tensile strength $\left(\sigma_2^T\right)_{ult}$
  - Transverse compressive strength $\left(\sigma_2^C\right)_{ult}$
  - In-plane shear strength $\left(\tau_{12}\right)_{ult}$

Maximum Stress Failure Theory

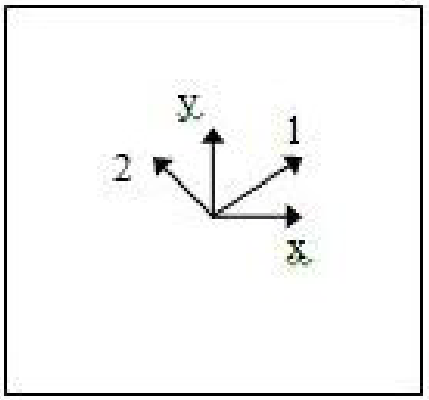
- The lamina is considered to be failed if:

$$-\left(\sigma_1^C\right)_{ult} \le \sigma_1 \le \left(\sigma_1^T\right)_{ult}, \quad or$$

$$-\left(\sigma_2^C\right)_{ult} \le \sigma_2 \le \left(\sigma_2^T\right)_{ult}, \quad or$$

$$-\left(\tau_{12}\right)_{ult} \le \tau_{12} \le \left(\tau_{12}\right)_{ult}$$

is violated.



- Note that all five strength parameters are positive numbers.
- Each component of stress does not interact with each other.

Example

Find the maximum value of S>o if a stress of $\sigma_x = 2S, \sigma_y = -3S, \text{and } \tau_{xy} = 4S$ is applied to a 60° lamina of Graphite/Epoxy. Use Maximum Stress failure theory. Use properties of a unidirectional Graphite/Epoxy lamina given in Table 2.1 of the textbook Mechanics of Composite Materials by Autar Kaw.

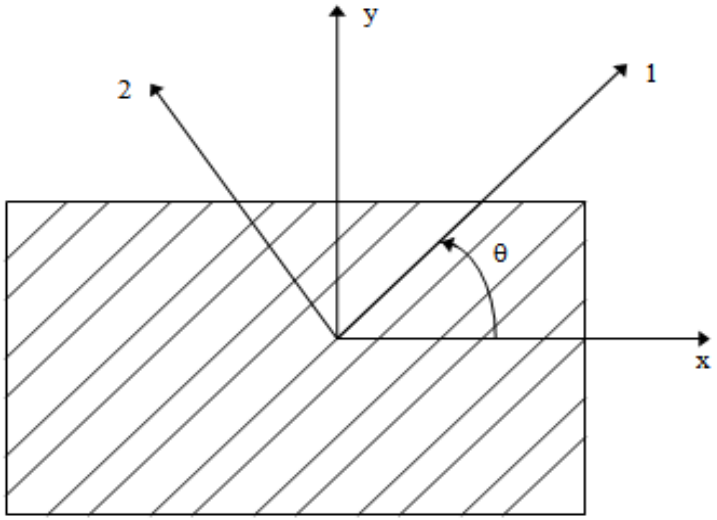


FIGURE 2.33
Off-axis loading in the x-direction

Solution

The stresses in the local axes are

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} 0.2500 & 0.7500 & 0.8660 \\ 0.7500 & 0.2500 & -0.8660 \\ -0.4330 & 0.4330 & -0.5000 \end{bmatrix} \begin{bmatrix} 2S \\ -3S \\ 4S \end{bmatrix}$$

$$= \begin{bmatrix} 0.1714 \times 10^1 \\ -0.2714 \times 10^1 \\ -0.4165 \times 10^1 \end{bmatrix} S.$$

Solution

The ultimate strengths of a unidirectional Graphite/Epoxy lamina are:

$$\left(\sigma_1^T\right)_{ult} = 1500 \text{ MPa},$$

$$\left(\sigma_1^C\right)_{ult} = 1500 \text{ MPa},$$

$$\left(\sigma_2^T\right)_{ult} = 40 \text{ MPa},$$

$$\left(\sigma_2^C\right)_{ult} = 246 \text{ MPa},$$

$$\left(\tau_{12}\right)_{ult} = 68 \text{ MPa}.$$

Solution

Then using the inequalities of the Maximum Stress Failure Theory:

$$-1500(10^6) < 0.1714(10^1)S < 1500(10^6),$$
$$-246(10^6) < -0.2714(10^1)S < 40(10^6),$$
$$-68(10^6) < -0.4165(10^1)S < 68(10^6),$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} 0.1714 \times 10^1 \\ -0.2714 \times 10^1 \\ -0.4165 \times 10^1 \end{bmatrix} S.$$

or,

$$-875.1(10^6) < S < 875.1(10^6),$$

$$-14.73(10^6) < S < 90.64(10^6)$$

$$-16.33(10^6) < S < 16.33(10^6).$$

All the inequality conditions (and $S > 0$)are satisfied if $0 < S < 16.33\,\text{MPa}$.
The above inequalities also show that the angle lamina will fail in shear. The maximum stress that can be applied before failure is:

$$\sigma_x = 32.66\ MPa, \sigma_x = -48.99\ MPa, \tau_{xy} = 65.32\ MPa.$$

END