EML 4230 Introduction to Composite Materials

Chapter 2 Macromechanical Analysis of a Lamina
Tsai-Hill Failure Theory

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Courtesy of the Textbook
Mechanics of Composite Materials by Kaw

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Strength Failure Theories for an Angle Lamina

- The failure theories are generally based on the normal and shear strengths of a unidirectional lamina.
- In the case of a unidirectional lamina, the five strength parameters are:
  - Longitudinal tensile strength $(\sigma_1^T)_{ult}$
  - Longitudinal compressive strength $(\sigma_1^C)_{ult}$
  - Transverse tensile strength $(\sigma_2^T)_{ult}$
  - Transverse compressive strength $(\sigma_2^C)_{ult}$
  - In-plane shear strength $(\tau_{12})_{ult}$

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Tsai-Hill Failure Theory

Based on the distortion energy theory, Tsai and Hill proposed that a lamina has failed if:

$$(G_2+G_3)\sigma_1^2+(G_1+G_3)\sigma_2^2+(G_1+G_2)\sigma_3^2-2G_3\sigma_1\sigma_2-2G_2\sigma_1\sigma_3 -2G_1\sigma_2\sigma_3+2G_4\tau_{23}^2+2G_5\tau_{13}^2+2G_6\tau_{12}^2<1$$

- This theory is based on the interaction failure theory.
- The components G_1 thru G_6 of the strength criteria depend on the strengths of a unidirectional lamina.

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Components of Tsai-Hill Failure Theory

$$(G_2+G_3)\sigma_1^2+(G_1+G_3)\sigma_2^2+(G_1+G_2)\sigma_3^2-2G_3\sigma_1\sigma_2-2G_2\sigma_1\sigma_3 -2G_1\sigma_2\sigma_3+2G_4\tau_{23}^2+2G_5\tau_{13}^2+2G_6\tau_{12}^2<1$$

Apply $\sigma_1=(\sigma_1^T)_{ult}$, to a unidirectional lamina, then the lamina will fail. Hence, Equation reduces to:

$$(G_2+G_3)(\sigma_1^T)_{ult}^2=1$$

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Components of Tsai-Hill Failure Theory

$$(G_2+G_3)\sigma_1^2+(G_1+G_3)\sigma_2^2+(G_1+G_2)\sigma_3^2-2G_3\sigma_1\sigma_2-2G_2\sigma_1\sigma_3 -2G_1\sigma_2\sigma_3+2G_4\tau_{23}^2+2G_5\tau_{13}^2+2G_6\tau_{12}^2<1$$

Apply $\sigma_2=(\sigma_2^T)_{ult}$, to a unidirectional lamina, then the lamina will fail. Hence, Equation reduces to:

$$(G_1+G_3)(\sigma_2^T)_{ult}^2=1$$

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Components of Tsai-Hill Failure Theory

$$(G_2+G_3)\sigma_1^2+(G_1+G_3)\sigma_2^2+(G_1+G_2)\sigma_3^2-2G_3\sigma_1\sigma_2-2G_2\sigma_1\sigma_3 -2G_1\sigma_2\sigma_3+2G_4\tau_{23}^2+2G_5\tau_{13}^2+2G_6\tau_{12}^2<1$$

Apply $\sigma_3=(\sigma_2^T)_{ult}$, to a unidirectional lamina, and assuming that the normal tensile failure strength is the same in direction (2) and (3), then the lamina will fail. Hence, Equation reduces to:

$$(G_1+G_2)(\sigma_2^T)_{ult}^2=1$$

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Components of Tsai-Hill Failure Theory

$$(G_2+G_3)\sigma_1^2+(G_1+G_3)\sigma_2^2+(G_1+G_2)\sigma_3^2-2G_3\sigma_1\sigma_2-2G_2\sigma_1\sigma_3 -2G_1\sigma_2\sigma_3+2G_4\tau_{23}^2+2G_5\tau_{13}^2+2G_6\tau_{12}^2<1$$

Apply $\tau_{12}=(\tau_{12})_{ult}$ to a unidirectional lamina, then the lamina will fail. Hence, Equation reduces to

$$2G_6(\tau_{12})_{ult}^2=1$$

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Components of Tsai-Hill Failure Theory

$$(G_2+G_3)(\sigma_1^T)_{ult}^2=1$$
$$(G_1+G_3)(\sigma_2^T)_{ult}^2=1$$
$$(G_1+G_2)(\sigma_2^T)_{ult}^2=1$$
$$2G_6(\tau_{12})_{ult}^2=1$$

$$G_1=\frac{1}{2}\left(\frac{2}{[(\sigma_2^T)_{ult}]^2}-\frac{1}{[(\sigma_1^T)_{ult}]^2}\right)$$
$$G_2=\frac{1}{2}\left(\frac{1}{[(\sigma_1^T)_{ult}]^2}\right)$$
$$G_3=\frac{1}{2}\left(\frac{1}{[(\sigma_1^T)_{ult}]^2}\right)$$
$$G_6=\frac{1}{2}\left(\frac{1}{[(\tau_{12})_{ult}]^2}\right)$$

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Tsai-Hill Failure Theory – Plane Stress

Because the unidirectional lamina is assumed to be under plane stress - that is, $\sigma_3 = \tau_{31} = \tau_{23} = 0$,

$$(G_2+G_3)\sigma_1^2+(G_1+G_3)\sigma_2^2+(G_1+G_2)\sigma_3^2-2G_3\sigma_1\sigma_2-2G_2\sigma_1\sigma_3 -2G_1\sigma_2\sigma_3+2G_4\tau_{23}^2+2G_5\tau_{13}^2+2G_6\tau_{12}^2<1$$

$$\left[\frac{\sigma_1}{(\sigma_1^T)_{ult}}\right]^2-\left[\frac{\sigma_1\sigma_2}{(\sigma_1^T)_{ult}^2}\right]+\left[\frac{\sigma_2}{(\sigma_2^T)_{ult}}\right]^2+\left[\frac{\tau_{12}}{(\tau_{12})_{ult}}\right]^2<1$$

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Tsai-Hill Failure Theory

- Unlike the Maximum Strain and Maximum Stress Failure Theories, the Tsai-Hill failure theory considers the interaction among the three unidirectional lamina strength parameter.
- The Tsai-Hill Failure Theory does not distinguish between the compressive and tensile strengths in its equation. This can result in underestimation of the maximum loads that can be applied when compared to other failure theories.
- Tsai-Hill Failure Theory underestimates the failure stress because the transverse strength of a unidirectional lamina is generally much less than its transverse compressive strength.

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Example

Find the maximum value of S>0 if a stress of $\sigma_x = 2S, \sigma_y = -3S, \text{and } \tau_{xy} = 4S$ is applied to a 60° lamina of Graphite/Epoxy. Use Tsai-Hill Failure Theory. Use properties of a unidirectional Graphite/Epoxy lamina given in Table 2.1 of the textbook Mechanics of Composite Materials by Autar Kaw.

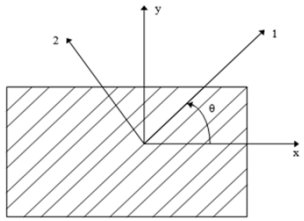

FIGURE 2.33
Off-axis loading in the x-direction

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Solution

The stresses in the local axes are:

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix}=\begin{bmatrix}0.2500 & 0.7500 & 0.8660\\ 0.7500 & 0.2500 & -0.8660\\ -0.4330 & 0.4330 & -0.5000\end{bmatrix}\begin{bmatrix}2S\\ -3S\\ 4S\end{bmatrix}$$

$$=\begin{bmatrix}0.1714\times10^1\\ -0.2714\times10^1\\ -0.4165\times10^1\end{bmatrix}S.$$

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Example

$$\left[\frac{\sigma_1}{(\sigma_1^T)_{ult}}\right]^2-\left[\frac{\sigma_1\sigma_2}{(\sigma_1^T)_{ult}^2}\right]+\left[\frac{\sigma_2}{(\sigma_2^T)_{ult}}\right]^2+\left[\frac{\tau_{12}}{(\tau_{12})_{ult}}\right]^2<1$$

$$\left(\frac{1.714S}{1500\times10^6}\right)^2-\left(\frac{1.714S}{1500\times10^6}\right)\left(\frac{-2.714S}{1500\times10^6}\right)+\left(\frac{-2.714S}{40\times10^6}\right)^2+\left(\frac{-4.165S}{68\times10^6}\right)^2<1$$

$$S<10.94\ MPa$$

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Modified Tsai-Hill Failure Theory

SR = 10.94 (Tsai–Hill failure theory)

SR = 16.33 (maximum stress failure theory)

SR = 16.33 (maximum strain failure theory)

$$\left[\frac{\sigma_1}{X_1}\right]^2-\left[\left(\frac{\sigma_1}{X_2}\right)\left(\frac{\sigma_2}{X_2}\right)\right]+\left[\frac{\sigma_2}{Y}\right]^2+\left[\frac{\tau_{12}}{S}\right]^2<1$$

$$X_1=(\sigma_1^T)_{ult},\ if\ \sigma_1>0$$
$$=(\sigma_1^C)_{ult},\ if\ \sigma_1<0$$

$$X_2=(\sigma_1^T)_{ult},\ if\ \sigma_2>0$$
$$=(\sigma_1^C)_{ult},\ if\ \sigma_2<0$$

$$Y=(\sigma_2^T)_{ult},\ if\ \sigma_2>0$$
$$=(\sigma_2^C)_{ult},\ if\ \sigma_2<0$$

$$S=(\tau_{12})_{ult}$$

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END

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