Tsai-Wu Failure Theory

- Tsai-Wu applied the failure theory to a lamina in plane stress. A lamina is considered to be failed if:

\[ H_1 \sigma_1 + H_2 \sigma_2 + H_6 \tau_{12} + H_{11} \sigma_1^2 + H_{22} \sigma_2^2 + H_{66} \tau_{12}^2 + 2H_{12} \sigma_1 \sigma_2 < 1 \]

is violated. This failure theory is more general than the Tsai-Hill failure theory because it distinguishes between the compressive and tensile strengths of a lamina.

- The components \( H_i \) of the failure theory are found using the five strength parameters of a unidirectional lamina.

Components of Tsai-Wu Fail

a) Apply \( \sigma_1 = -\sigma_2, \sigma_2 = 0, \tau_{12} = 0 \) to a unidirectional lamina, the lamina will fail. Equation (2.152) reduces to:

\[ H_1 (\sigma_1)_{ab} + H_2 (\sigma_1)_{ab} = 0 \]

b) Apply \( \sigma_1 = -\sigma_2, \sigma_2 = 0, \tau_{12} = 0 \) to a unidirectional lamina, the lamina will fail. Equation (2.152) reduces to:

\[ -H_1 (\sigma_1)_{ab} + H_2 (\sigma_1)_{ab} = 0 \]

From Equations (2.153) and (2.154),

\[ H_1 = \frac{1}{(\sigma_1)_{ab} - (\sigma_2)_{ab}} \]

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Components of Tsai-Wu Fail

e) Apply \( \sigma_1 = 0, \sigma_2 = 0, \tau_{12} = (\tau_{12})_{\text{lam}} \) to a unidirectional lamina, the lamina will fail. Equation (2.152) reduces to:

\[ H_5 (\tau_{12})_{\text{lam}} + H_6 (\tau_{12})_{\text{lam}} = 1. \]

f) Apply \( \sigma_1 = 0, \sigma_2 = 0, \tau_{12} = - (\tau_{12})_{\text{lam}} \) to a unidirectional lamina, the lamina will fail. Equation (2.152) reduces to:

\[ - H_5 (\tau_{12})_{\text{lam}} + H_6 (\tau_{12})_{\text{lam}} = 1. \]

From Equations (2.157) and (2.158),

\[
H_6 = 0, \\
H_5 = \frac{1}{(\tau_{12})_{\text{lam}}}. 
\]

Determination of \( H_{12} \)

Apply equal tensile loads along the two material axes in a unidirectional composite. If \( \sigma = \sigma_2 = \sigma \sigma_0 = 0 \), is the load at which the lamina fails, then:

\[ (H_1 + H_2) \sigma + (H_{11} + H_{22} + 2H_{66}) \sigma^2 = 1. \]

The solution of the Equation (2.165) gives:

\[
H_1 \sigma_1 + H_2 \sigma_2 + H_3 \tau_{12} + H_4 \sigma_1^2 + H_{22} \sigma_2^2 + H_{66} \tau_{12}^2 + 2H_{11} \sigma_1 \sigma_2 < 1
\]

Empirical Models of \( H_{12} \)

\[
H_{12} = - \frac{1}{2(\sigma_1^2)_{\text{lam}}} \quad \text{as per Tsai-Hill failure theory}^{a} \\
H_{12} = - \frac{1}{2(\sigma_1^2)_{\text{lam}}(\sigma_1^2)_{\text{lam}}} \quad \text{as per Hoffman criterion}^{a} \\
H_{12} = - \frac{1}{2(\sigma_1^2)_{\text{lam}}(\sigma_2^2)_{\text{lam}}(\tau_{12})_{\text{lam}}(\sigma_1^2)_{\text{lam}}} \quad \text{as per Mises-Hencky criterion}^{a}
\]
Example 2.19

Find the maximum value of $S > 0$ if a stress $\sigma_1 = 2S, \sigma_2 = -3S$ and $\tau_{12} = 4S$ are applied to a 60$^\circ$ lamina of Graphite/Epoxy. Use Tsai-Wu failure theory. Use the properties of a unidirectional Graphite/Epoxy lamina from Table 2.1.

Using Equation (2.94), the stresses in the local axes are:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} = \begin{bmatrix}
0.2500 & 0.7500 & 0.8660 \\
0.7500 & 0.2500 & -0.8660 \\
-0.4330 & 0.4330 & -0.5000
\end{bmatrix} \begin{bmatrix} 2S \\ -3S \\ 4S \end{bmatrix}
\]

Substituting these values in Equation (2.152), we obtain:

\[
\begin{bmatrix}
1.0 \times 10^7 \\
-2.714 \times 10^7 \\
-0.4165 \times 10^7
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.1714 \times 10^7 \\
-0.2714 \times 10^7 \\
-0.4165 \times 10^7
\end{bmatrix}
\]

Substituting these values in Equation (2.52), we obtain:

\[
(0)(1.714S) + (2.093 \times 10^4)(2.714S)
\]

\[
+(0)(-4.165S) + (4.444 \times 10^{-19})(1.714S)^2
\]

\[
+ (0.162 \times 10^{-18}) (2.714S)^2 + (2.162 \times 10^{-18})(4.165S)^2
\]

\[
+ 2(-3.30 \times 10^{-18})(1.714S)(2.714S) < 1,
\]

or

\[
S < 22.39 \text{MPa}
\]
Example 2.19

If one uses the other two empirical criteria for $H_{12}$ as per Equation (2.172), one obtains:

\[
S < 22.49 \text{MPa for } H_{12} = \frac{1}{2(\sigma_1 f_{\theta})},
\]

\[
S < 22.49 \text{MPa for } H_{22} = \frac{1}{2(\sigma_2 f_{\theta})}.\]

Summarizing the four failure theories for the same stress-state, the value of $S$ obtained is:

- $S = 16.33$ (Maximum Stress failure theory),
- $S = 16.33$ (Maximum Strain failure theory),
- $S = 10.94$ (Tsai-Hill failure theory),
- $S = 16.06$ (Modified Tsai-Hill failure theory),
- $S = 22.39$ (Tsai-Wu failure theory).