EML 4230 Introduction to Composite Materials

Chapter 2 Macromechanical Analysis of a Lamina
Tsai-Wu Failure Theory

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Courtesy of the Textbook
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$H_1\sigma_1$+$H_2\sigma_2$+$H_6\tau_{12}$+H_{11}

+H_{22}
+H_{66}
+$2H_{12}\sigma_1\sigma_2$< 1

Tsai-Wu Failure Theory

- Tsai-Wu applied the failure theory to a lamina in plane stress. A lamina is considered to be failed if:

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1$$

is violated. This failure theory is more general than the Tsai-Hill failure theory because it distinguishes between the compressive and tensile strengths of a lamina.

- The components $H_1 - H_{66}$ of the failure theory are found using the five strength parameters of a unidirectional lamina.

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Components of Tsai-Wu Fail

a) Apply $\sigma_1 = (\sigma_1^T)_{ult}, \sigma_2 = 0, \tau_{12} = 0$ to a unidirectional lamina, the lamina will fail. Equation (2.152) reduces to:

$$H_1(\sigma_1^T)_{ult} + H_{11}(\sigma_1^T)_{ult}^2 = 1.$$

b) Apply $\sigma_1 = -(\sigma_1^C)_{ult}, \sigma_2 = 0, \tau_{12} = 0$ to a unidirectional lamina, the lamina will fail. Equation (2.152) reduces to:

$$-H_1(\sigma_1^C)_{ult} + H_{11}(\sigma_1^C)_{ult}^2 = 1.$$

From Equations (2.153) and (2.154),

$$H_1 = \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}},$$

$$H_{11} = \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}},$$

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Components of Tsai-Wu Fail

c) Apply $\sigma_1 = 0, \sigma_2 = (\sigma_2^T)_{ult}, \tau_{12} = 0$ to a unidirectional lamina, the lamina will fail. Equation (2.152) reduces to

$$H_2(\sigma_2^T)_{ult} + H_{22}(\sigma_2^T)_{ult}^2 = 1.$$

d) Apply $\sigma_1 = 0, \sigma_2 = -(\sigma_2^C)_{ult}, \tau_{12} = 0$ to a unidirectional lamina, the lamina will fail. Equation (2.152) reduces to:

$$-H_2(\sigma_2^C)_{ult} + H_{22}(\sigma_2^C)_{ult}^2 = 1.$$

From Equations (2.157) and (2.158):

$$H_2 = \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}},$$

$$H_{22} = \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}.$$

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Components of Tsai-Wu Fail

e) Apply $\sigma_1 = 0, \sigma_2 = 0, \tau_{12} = (\tau_{12})_{ult}$ to a unidirectional lamina, the lamina will fail. Equation (2.152) reduces to:

$$H_6 (\tau_{12})_{ult} + H_{66} (\tau_{12})^2_{ult} = 1.$$

f) Apply $\sigma_1 = 0, \sigma_2 = 0, \tau_{12} = -(\tau_{12})_{ult}$ to a unidirectional lamina, the lamina will fail. Equation (2.152) reduces to:

$$-H_6 (\tau_{12})_{ult} + H_{66} (\tau_{12})^2_{ult} = 1.$$

From Equations (2.157) and (2.158),

$$H_6 = 0,$$

$$H_{66} = \frac{1}{(\tau_{12})^2_{ult}}.$$

Determination of H_{12}

Apply equal tensile loads along the two material axes in a unidirectional composite. If $\sigma_x = \sigma_y = \sigma$ $\tau_{xy} = 0$, is the load at which the lamina fails, then:

$$(H_1 + H_2)\sigma + (H_{11} + H_{22} + 2H_{12})\sigma^2 = 1.$$

The solution of the Equation (2.165) gives:

$$H_{12} = \frac{1}{2\sigma^2}\left[1 - (H_1 + H_2)\sigma - (H_{11} + H_{22})\sigma^2\right]$$

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1$$

Determination of H_{12}

Take a 45° lamina under uniaxial tension σ_x. The stress σ_x at failure is noted.

If this stress is $\sigma_x = 0$ then using Equation (2.94), the local stresses at failure are:

$$\sigma_1 = \frac{\sigma}{2},$$

$$\sigma_2 = \frac{\sigma}{2},$$

$$\tau_{12} = -\frac{\sigma}{2}.$$

Substituting the above local stresses in Equation (2.152):

$$(H_1 + H_2)\frac{\sigma}{2} + \frac{\sigma^2}{4}(H_{11} + H_{22} + H_{66} + 2H_{12}) = 1,$$

$$H_{12} = \frac{2}{\sigma^2} - \frac{(H_1 + H_2)}{\sigma} - \frac{1}{2}(H_{11} + H_{22} + H_{66}).$$

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1$$

Empirical Models of H_{12}

$$H_{12} = -\frac{1}{2(\sigma_1^T)^2_{ult}}$$ as per Tsai-Hill failure theory[8]

$$H_{12} = -\frac{1}{2(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}$$ as per Hoffman criterion[10]

$$H_{12} = -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}}$$ as per Mises-Hencky criterion[11]

Example 2.19

Find the maximum value of $S > 0$ if a stress $\sigma_x = 2S, \sigma_y = -3S$ and $\tau_{xy} = 4S$ are applied to a 60° lamina of Graphite/Epoxy. Use Tsai-Wu failure theory. Use the properties of a unidirectional Graphite/Epoxy lamina from Table 2.1.

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Example 2.19

- Using Equation (2.94), the stresses in the local axes are:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} 0.2500 & 0.7500 & 0.8660 \\ 0.7500 & 0.2500 & -0.8660 \\ -0.4330 & 0.4330 & -0.5000 \end{bmatrix} \begin{bmatrix} 2S \\ -3S \\ 4S \end{bmatrix}$$

$$= \begin{bmatrix} 0.1714 \times 10^1 \\ -0.2714 \times 10^1 \\ -0.4165 \times 10^1 \end{bmatrix} S.$$

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H_{12}=

Example 2.19

$$H_1 = \frac{1}{1500 \times 10^6} - \frac{1}{1500 \times 10^6} = 0 \ Pa^{-1},$$

$$H_2 = \frac{1}{40 \times 10^6} - \frac{1}{246 \times 10^6} = 2.093 \times 10^{-8} \ Pa^{-1},$$

$$H_6 = 0 \ Pa^{-1},$$

$$H_{11} = \frac{1}{(1500 \times 10^6)(1500 \times 10^6)} = 4.4444 \times 10^{-19} Pa^{-2},$$

$$H_{22} = \frac{1}{(40 \times 10^6)(246 \times 10^6)} = 1.0162 \times 10^{-16} Pa^{-2},$$

$$H_{66} = \frac{1}{(68 \times 10^6)^2} = 2.1626 \times 10^{-16} Pa^{-2},$$

$$H_{12} = -0.5\left[(4.4444 \times 10^{-19})(1.0162 \times 10^{-16})\right]^{\frac{1}{2}} = -3.360 \times 10^{-18} Pa^{-2}.$$

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Example 2.19

Substituting these values in Equation (2.152), we obtain:

$$(0)(1.714S) + (2.093 \times 10^{-8})(-2.714S)$$
$$+(0)(-4.165S) + (4.4444 \times 10^{-19})(1.714S)^2$$
$$+(1.0162 \times 10^{-16})(-2.714S)^2 + (2.1626 \times 10^{-16})(4.165S)^2$$
$$+2(-3.360 \times 10^{-18})(1.714S)(-2.714S) < 1,$$

or

$$S < 22.39 MPa$$

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Example 2.19

If one uses the other two empirical criteria for H_{12} as per Equation (2.171), one obtains:

$$S < 22.49 MPa \ for \ H_{12} = -\frac{1}{2(\sigma_1^T)_{ult}^2},$$

$$S < 22.49 MPa \ for \ H_{12} = -\frac{1}{2}\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}.$$

Summarizing the four failure theories for the same stress-state, the value of *S* obtained is:

S = 16.33 (Maximum Stress failure theory),
= 16.33 (Maximum Strain failure theory),
= 10.94 (Tsai-Hill failure theory),
= 16.06 (Modified Tsai-Hill failure theory),
= 22.39 (Tsai-Wu failure theory).

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