EML 4230 Introduction to Composite Materials

Chapter 2 Macromechanical Analysis of a Lamina
Comparison of Failure Theories

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Courtesy of the Textbook
Mechanics of Composite Materials by Kaw

USF UNIVERSITY OF SOUTH FLORIDA

Strength Failure Theories of an Angle Lamina

- The failure theories are generally based on the normal and shear strengths of a unidirectional lamina.
- An isotropic material generally has two strength parameters:
  normal strength and shear strength.
- In the case of a unidirectional lamina, the five strength parameters are:
  - Longitudinal tensile strength $\left(\sigma_1^T\right)_{ult}$
  - Longitudinal compressive strength $\left(\sigma_1^C\right)_{ult}$
  - Transverse tensile strength $\left(\sigma_2^T\right)_{ult}$
  - Transverse compressive strength $\left(\sigma_2^C\right)_{ult}$
  - In-plane shear strength $\left(\tau_{12}\right)_{ult}$

Experimental Results and Failure Theories

- Tsai and Wu compared the results from various failure theories to some experimental results. He considered an angle lamina subjected to a uniaxial load in the x-direction.

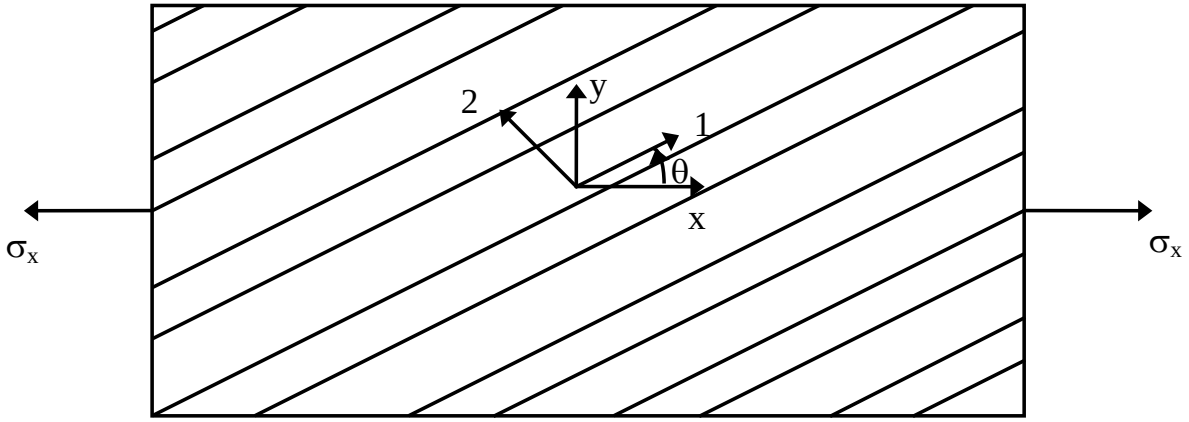


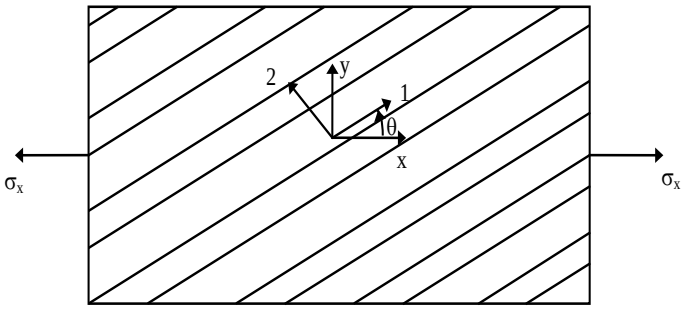
FIGURE 2.33
Off-axis loading in the x-direction

Experimental Results and Maximum Stress Failure Theory

$$-\left(\sigma_1^C\right)_{ult} \le \sigma_1 \le \left(\sigma_1^T\right)_{ult}$$

$$-\left(\sigma_2^C\right)_{ult} \le \sigma_2 \le \left(\sigma_2^T\right)_{ult}$$

$$-\left(\tau_{12}\right)_{ult} \le \tau_{12} \le \left(\tau_{12}\right)_{ult}$$



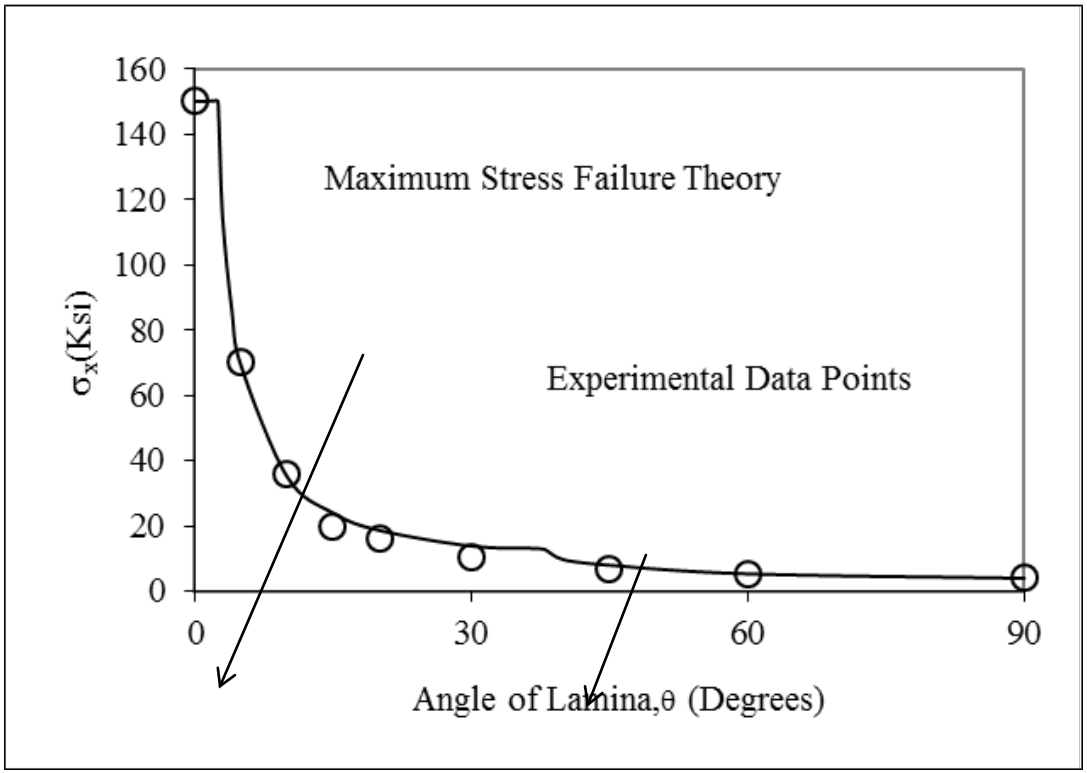


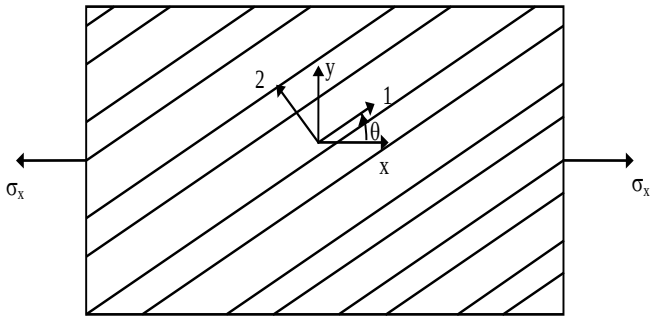
FIGURE 2.34
Maximum normal tensile stress in x-direction as function of angle of lamina using maximum stress failure theory

Experimental Results and Maximum Strain Failure Theory

$$-\left(\varepsilon_1^C\right)_{ult} < \varepsilon_1 < \left(\varepsilon_1^T\right)_{ult}$$

$$-\left(\varepsilon_2^C\right)_{ult} < \varepsilon_2 < \left(\varepsilon_2^T\right)_{ult}$$

$$-\left(\gamma_{12}\right)_{ult} < \gamma_{12} < \left(\gamma_{12}\right)_{ult}$$



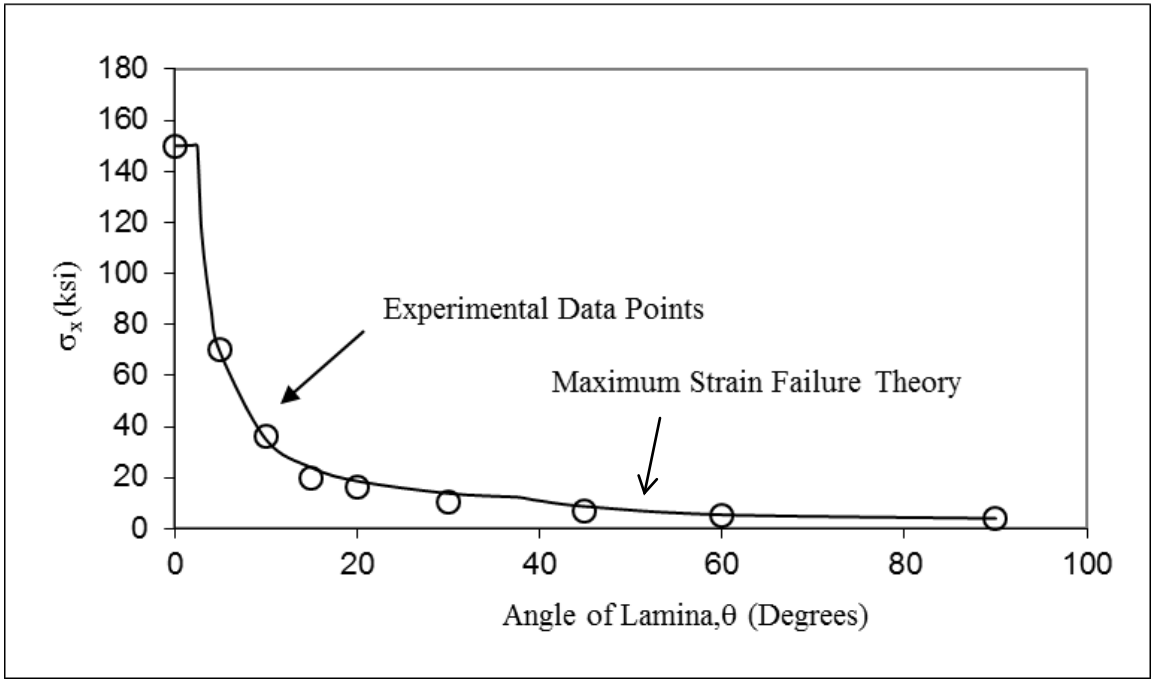
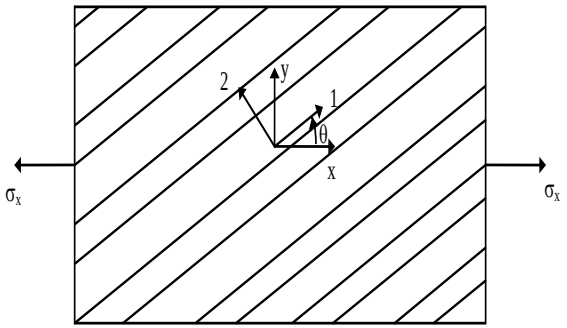


FIGURE 2.35
Maximum normal tensile stress in x-direction as function of angle of lamina using maximum Strain failure theory

Experimental Results and Tsai-Hill Failure Theory

$$\left[\frac{\sigma_1}{(\sigma_1^T)_{ult}}\right]^2 - \left[\frac{\sigma_1\sigma_2}{(\sigma_1^T)^2_{ult}}\right] + \left[\frac{\sigma_2}{(\sigma_2^T)_{ult}}\right]^2 + \left[\frac{\tau_{12}}{(\tau_{12})_{ult}}\right]^2 < 1$$



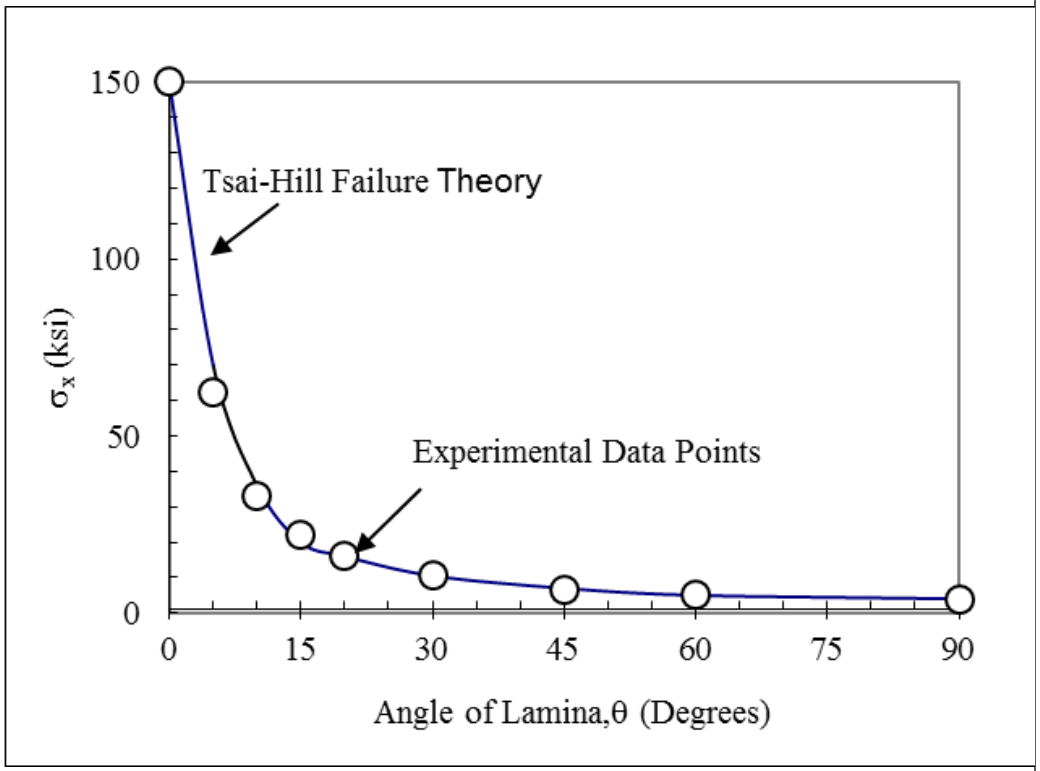
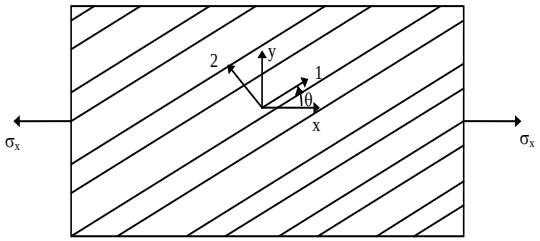


FIGURE 2.36
Maximum normal tensile stress in x-direction as function of angle of lamina using Tsai-Hill failure theory

Experimental Results and Tsai-Wu Failure Theory

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1$$



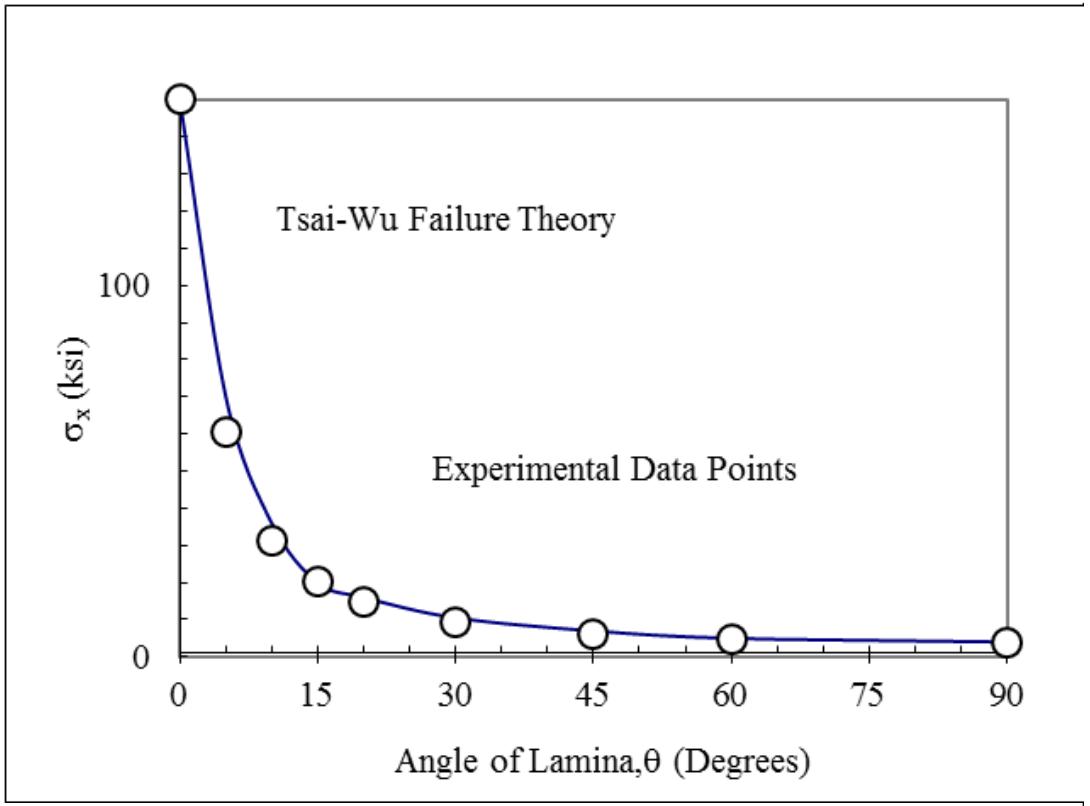
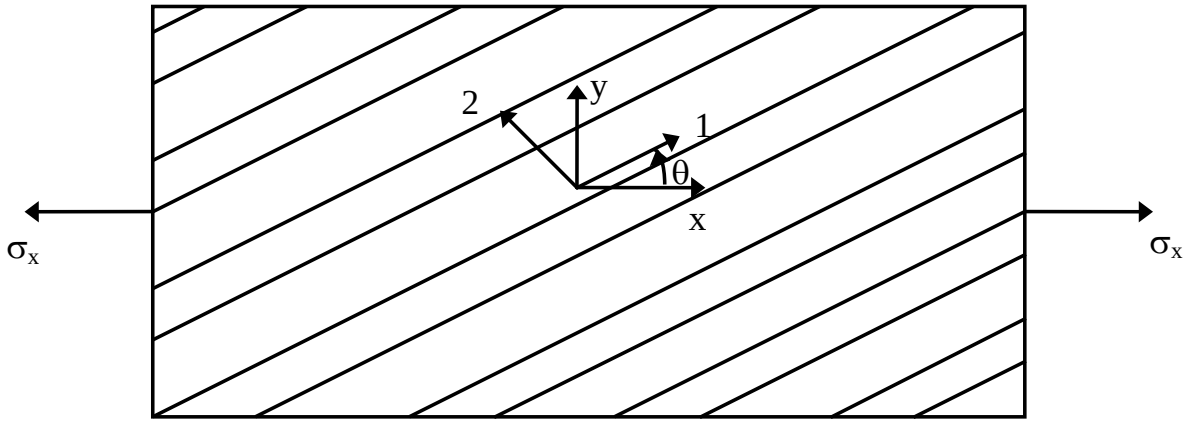


FIGURE 2.37
Maximum normal tensile stress in x-direction as function of angle of lamina using Tsai-Wu failure theory

Comparison of Strength Ratios

S = 16.33 (Maximum Stress failure theory),
= 16.33 (Maximum Strain failure theory),
= 10.94 (Tsai-Hill failure theory),
= 16.06 (Modified Tsai-Hill failure theory),
= 22.39 (Tsai-Wu failure theory)



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