# EML 4230 Introduction to Composite Materials

Chapter 2 Macromechanical Analysis of a Lamina
**Hygrothermal Stresses and Strains**

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Courtesy of the Textbook
**Mechanics of Composite Materials by Kaw**

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# Hygrothermal Stress-Strain Relationship

- For a unidirectional lamina

$$\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix} = \begin{bmatrix}S_{11} & S_{12} & 0\\ S_{12} & S_{22} & 0\\ 0 & 0 & S_{66}\end{bmatrix}\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} + \begin{bmatrix}\varepsilon_1^T\\ \varepsilon_2^T\\ 0\end{bmatrix} + \begin{bmatrix}\varepsilon_1^C\\ \varepsilon_2^C\\ 0\end{bmatrix} \quad (2.174)$$

- Thermally induced strains:

$$\begin{bmatrix}\varepsilon_1^T\\ \varepsilon_2^T\\ 0\end{bmatrix} = \Delta T\begin{bmatrix}\alpha_1\\ \alpha_2\\ 0\end{bmatrix} \quad (2.175)$$

- Moisture induced strains:

$$\begin{bmatrix}\varepsilon_1^C\\ \varepsilon_2^C\\ 0\end{bmatrix} = \Delta C\begin{bmatrix}\beta_1\\ \beta_2\\ 0\end{bmatrix} \quad (2.176)$$

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# Hygrothermal Stress-Strain Relationship

- For a unidirectional lamina

$$\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix} = \begin{bmatrix}S_{11} & S_{12} & 0\\ S_{12} & S_{22} & 0\\ 0 & 0 & S_{66}\end{bmatrix}\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} + \begin{bmatrix}\varepsilon_1^T\\ \varepsilon_2^T\\ 0\end{bmatrix} + \begin{bmatrix}\varepsilon_1^C\\ \varepsilon_2^C\\ 0\end{bmatrix} \quad (2.174)$$

$$\Downarrow$$

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1 - \varepsilon_1^T - \varepsilon_1^C\\ \varepsilon_2 - \varepsilon_2^T - \varepsilon_2^C\\ \gamma_{12}\end{bmatrix} \quad (2.177)$$

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# Hygrothermal Stress-Strain Relationship

- For an angular lamina

$$\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} = \begin{bmatrix}\bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16}\\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26}\\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66}\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} + \begin{bmatrix}\varepsilon_x^T\\ \varepsilon_y^T\\ \gamma_{xy}^T\end{bmatrix} + \begin{bmatrix}\varepsilon_x^C\\ \varepsilon_y^C\\ \gamma_{xy}^C\end{bmatrix} \quad (2.178)$$

- Thermally induced strains:

$$\begin{bmatrix}\varepsilon_x^T\\ \varepsilon_y^T\\ \gamma_{xy}^T\end{bmatrix} = \Delta T\begin{bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{bmatrix} \quad (2.179)$$

- Moisture induced strains:

$$\begin{bmatrix}\varepsilon_x^C\\ \varepsilon_y^C\\ \gamma_{xy}^C\end{bmatrix} = \Delta C\begin{bmatrix}\beta_x\\ \beta_y\\ \beta_{xy}\end{bmatrix} \quad (2.180)$$

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## Transformation of CTE

- For an angular lamina

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy}/2 \end{bmatrix} = [T]^{-1} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{bmatrix} \quad (2.181)$$

$$[T]^{-1} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2-s^2 \end{bmatrix} \quad (2.95) \qquad [T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix} \quad (2.96)$$

$$c = \text{Cos}\,(\theta)$$
$$s = \text{Sin}\,(\theta) \quad (2.97a,b)$$

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## Transformation of Coefficients of Moisture Expansion

- For an angular lamina

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy}/2 \end{bmatrix} = [T]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{bmatrix} \quad (2.182)$$

$$[T]^{-1} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2-s^2 \end{bmatrix} \quad (2.95) \qquad [T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix} \quad (2.96)$$

$$c = \text{Cos}\,(\theta)$$
$$s = \text{Sin}\,(\theta) \quad (2.97a,b)$$

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## Example 2.20

Find the following for a $60^0$ angle lamina of Glass/Epoxy

a) coefficients of thermal expansion,

b) coefficients of moisture expansion,

c) strains under a temperature change of $-100^0$C and a moisture absorption of 0.02 kg/kg.

Use properties of unidirectional Glass/Epoxy lamina from Table 2.1.

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## Example 2.20

a) From Table 2.1,

$$\alpha_1 = 8.6\times 10^{-6}\ m/m/^0C,$$
$$\alpha_2 = 22.1\times 10^{-6}\ m/m\,/^0C.$$

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy}/2 \end{bmatrix} = [T]^{-1} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{bmatrix}. \quad (2.181)$$

Using Equation (2.181), gives

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy}/2 \end{bmatrix} = \begin{bmatrix} 0.2500 & 0.7500 & -0.8660 \\ 0.7500 & 0.2500 & 0.8660 \\ 0.4330 & -0.4330 & -0.5000 \end{bmatrix} \begin{bmatrix} 8.6\times 10^{-6} \\ 22.1\times 10^{-6} \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} = \begin{bmatrix} 18.73\times 10^{-6} \\ 11.98\times 10^{-6} \\ -11.69\times 10^{-6} \end{bmatrix} m/m\,/^0C.$$

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## Example 2.20

b) From Table 2.1

$\beta_1 = 0$ m/m/kg/kg,

$\beta_2 = 0.6$ m/m/kg/kg.

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy}/2 \end{bmatrix} = [T]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{bmatrix} \quad (2.182)$$

Using Equation (2.182) gives

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy}/2 \end{bmatrix} = \begin{bmatrix} 0.2500 & 0.7500 & -0.8660 \\ 0.7500 & 0.2500 & 0.8660 \\ 0.4330 & -0.4330 & -0.5000 \end{bmatrix} \begin{bmatrix} 0.0 \\ 0.6 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} = \begin{bmatrix} 0.4500 \\ 0.1500 \\ -0.5196 \end{bmatrix} m/m/kg/kg$$

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## Example 2.20

c) Now using Equations (2.179) and (2.180) to calculate the strains as.

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 18.73 \times 10^{-6} \\ 11.98 \times 10^{-6} \\ -11.69 \times 10^{-6} \end{bmatrix} (-100) + \begin{bmatrix} 0.4500 \\ 0.1500 \\ -0.5196 \end{bmatrix} (0.02)$$

$$= \begin{bmatrix} 0.7127 \times 10^{2} \\ 0.1802 \times 10^{2} \\ -0.9223 \times 10^{2} \end{bmatrix} m/m$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} + \begin{bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \end{bmatrix} + \begin{bmatrix} \varepsilon_x^C \\ \varepsilon_y^C \\ \gamma_{xy}^C \end{bmatrix} \quad (2.178)$$

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## END

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