# EML 4230 Introduction to Composite Materials

Chapter 3 Micromechanical Analysis of a Lamina
**Volume Fractions, Weight Fractions, Density, and Void Content**

Dr. Autar Kaw
Department of Mechanical Engineering
University of South Florida, Tampa, FL 33620

Courtesy of the Textbook
**Mechanics of Composite Materials by Kaw**

USF UNIVERSITY OF SOUTH FLORIDA

1

# Volume Fractions

$$V_f = \frac{v_f}{v_c}, \text{ and } \quad V_m = \frac{v_m}{v_c}$$

$$V_f + V_m = 1$$

$$v_f + v_m = v_c$$

2

# Weight Fractions

$$W_f = \frac{w_f}{w_c}, \text{ and } \quad W_m = \frac{w_m}{w_c}$$

$$W_f + W_m = 1$$

$$w_f + w_m = w_c$$

3

# Volume and Weight Fractions

$$w_c = \rho_c \, v_c \,,$$

$$w_f = \rho_f \, v_f \,,$$

$$w_m = \rho_m \, v_m \,, \text{ and}$$

$$w_c = w_f + w_m$$

4

## Volume and Weight Fractions

$$W_f = \frac{\rho_f}{\rho_c} V_f \text{, and}$$

$$W_m = \frac{\rho_m}{\rho_c} V_m$$

5

## Volume and Weight Fractions

$$W_f = \frac{\frac{\rho_f}{\rho_m}}{\frac{\rho_f}{\rho_m} V_f + V_m} V_f$$

$$W_m = \frac{1}{\frac{\rho_f}{\rho_m}(1 - V_m) + V_m} V_m$$

6

## Density

$$\rho_c \mathrm{v_c} = \rho_f \mathrm{v_f} + \rho_m \mathrm{v_m}$$

$$\rho_c = \rho_f \frac{\mathrm{v_f}}{\mathrm{v_c}} + \rho_m \frac{\mathrm{v_m}}{\mathrm{v_c}}$$

$$\rho_c = \rho_f V_f + \rho_m V_m$$

$$\frac{1}{\rho_c} = \frac{W_f}{\rho_f} + \frac{W_m}{\rho_m}$$

7

## Example

***Example 3.1***

A Glass/Epoxy lamina consists of a 70% fiber volume fraction. Use properties of glass and epoxy from Tables 3.1 and 3.2, respectively to determine the

a) density of lamina

b) mass fractions of the glass and epoxy

c) volume of composite lamina if the mass of the lamina is 4 kg.

d) volume and mass of glass and epoxy in part (c).

8

# Example

Table 3.1 Typical Properties of Fibers (SI system of units)

| Property | Units | Graphite | Glass | Aramid |
|---|---|---|---|---|
| Axial modulus | GPa | 230 | 85 | 124 |
| Transverse modulus | GPa | 22 | 85 | 8 |
| Axial Poisson's ratio | - - - | 0.30 | 0.20 | 0.36 |
| Transverse Poisson's ratio | - - - | 0.35 | 0.20 | 0.37 |
| Axial shear modulus | GPa | 22 | 35.42 | 3 |
| Axial coefficient of thermal expansion | μm/m/°C | -1.3 | 5 | -5.0 |
| Transverse coefficient of thermal expansion | μm/m/°C | 7.0 | 5 | 4.1 |
| Axial tensile strength | MPa | 2067 | 1550 | 1379 |
| Axial compressive strength | MPa | 1999 | 1550 | 276 |
| Transverse tensile strength | MPa | 77 | 1550 | 7 |
| Transverse compressive strength | MPa | 42 | 1550 | 7 |
| Shear strength | MPa | 36 | 35 | 21 |
| Specific gravity | - - - | 1.8 | 2.5 | 1.4 |

9

# Example

Table 3.2 Typical Properties of Matrices (SI system of units)

| Property | Units | Epoxy | Aluminum | Polyamide |
|---|---|---|---|---|
| Axial modulus | GPa | 3.4 | 71 | 3.5 |
| Transverse modulus | GPa | 3.4 | 71 | 3.5 |
| Axial Poisson's ratio | - - - | 0.3 | 0.30 | 0.35 |
| Transverse Poisson's ratio | - - - | 0.3 | 0.30 | 0.35 |
| Axial shear modulus | GPa | 1.308 | 27 | 1.3 |
| Coefficient of thermal expansion | μm/m/°C | 63 | 23 | 90 |
| Coefficient of moisture expansion | m/m/kg/kg | 0.33 | 0.00 | 0.33 |
| Axial tensile strength | MPa | 72 | 276 | 54 |
| Axial compressive strength | MPa | 102 | 276 | 108 |
| Transverse tensile strength | MPa | 72 | 276 | 54 |
| Transverse compressive strength | MPa | 102 | 276 | 108 |
| Shear strength | MPa | 34 | 138 | 54 |
| Specific gravity | - - - | 1.2 | 2.7 | 1.2 |

10

# Example

Table 3.3 Typical Properties of Fibers (USCS system of units)

| Property | Units | Graphite | Glass | Aramid |
|---|---|---|---|---|
| Axial Modulus | Msi | 33.35 | 12.33 | 17.98 |
| Transverse modulus | Msi | 3.19 | 12.33 | 1.16 |
| Axial Poisson's ratio | - - - | 0.30 | 0.20 | 0.36 |
| Transverse Poisson's ratio | - - - | 0.35 | 0.20 | 0.37 |
| Axial shear modulus | Msi | 3.19 | 5.136 | 0.435 |
| Axial coefficient of thermal expansion | μin/in/°F | -0.7222 | 2.778 | -2.778 |
| Transverse coefficient of thermal expansion | μin/in/°F | 3.889 | 2.778 | 2.278 |
| Axial tensile strength | ksi | 299.7 | 224.8 | 200.0 |
| Axial compressive strength | ksi | 289.8 | 224.8 | 40.02 |
| Transverse tensile strength | ksi | 11.16 | 224.8 | 1.015 |
| Transverse compressive strength | ksi | 6.09 | 224.8 | 1.015 |
| Shear strength | ksi | 5.22 | 5.08 | 3.045 |
| Specific gravity | - - - | 1.8 | 2.5 | 1.4 |

11

# Example

Table 3.4 Typical Properties of Matrices (USCS system of units)

| Property | Units | Epoxy | Aluminum | Polyamide |
|---|---|---|---|---|
| Axial modulus | Msi | 0.493 | 10.30 | 0.5075 |
| Transverse modulus | Msi | 0.493 | 10.30 | 0.5075 |
| Axial Poisson's ratio | - - - | 0.3 | 0.30 | 0.35 |
| Transverse Poisson's ratio | - - - | 0.3 | 0.30 | 0.35 |
| Axial shear modulus | Msi | 0.1897 | 3.915 | 0.1885 |
| Coefficient of thermal expansion | μin/in/°F | 35 | 12.78 | 50 |
| Coefficient of moisture expansion | in/in/lb/lb | 0.33 | 0.00 | 0.33 |
| Axial tensile strength | ksi | 10.44 | 40.02 | 7.83 |
| Axial compressive strength | ksi | 14.79 | 40.02 | 15.66 |
| Transverse tensile strength | ksi | 10.44 | 40.02 | 7.83 |
| Transverse compressive strength | ksi | 14.79 | 40.02 | 15.66 |
| Shear strength | ksi | 4.93 | 20.01 | 7.83 |
| Specific gravity | - - - | 1.2 | 2.7 | 1.2 |

12

# Example

***Example 3.1***

A glass/epoxy lamina consists of a 70% fiber volume fraction. Use properties of glass and epoxy from Table 3.1* and Table 3.2, respectively, to determine the

1. Density of lamina
2. Mass fractions of the glass and epoxy
3. Volume of composite lamina if the mass of the lamina is 4 kg
4. Volume and mass of glass and epoxy in part (3)

$$\rho_c = (2500)(0.7) + (1200)(0.3)$$
$$= 2110\ kg/m^3$$

13

$\rho_f = 2500\ kg/m^3.$

# Example

Example 3.1

b)

$$W_f = \frac{2500}{2110} \times 0.3 = 0.8294 \text{, and} \quad W_m = \frac{1200}{2110} \times 0.3 = 0.1706$$

$$W_f + W_m = 0.8294 + 0.1706 = 1.000$$

14

# Example

Example 3.1

c)

$$v_c = \frac{w_c}{\rho_c} = \frac{4}{2110} = 1.896 \times 10^{-3}\ m^3$$

15

# Example

Example 3.1

d)

$$v_f = V_f v_c = (0.7)(1.896 \times 10^{-3}) = 1.327 \times 10^{-3}\ m^3$$

16

## Example

Example 3.1

d)

$$v_m = V_m v_c$$
$$= (0.3)(0.1896 \times 10^{-3})$$
$$= 0.5688 \times 10^{-3}\ m^3$$

17

## Example

Example 3.1

d)

$$w_f = \rho_f v_f$$
$$= (2500)(1.327 \times 10^{-3})$$
$$= 3.318\ kg$$

18

## Example

Example 3.1

d)

$$w_m = \rho_m v_m$$
$$= (1200)(0.5688 \times 10^{-3})$$
$$= 0.6826\ \text{kg}$$

19

## Void Content

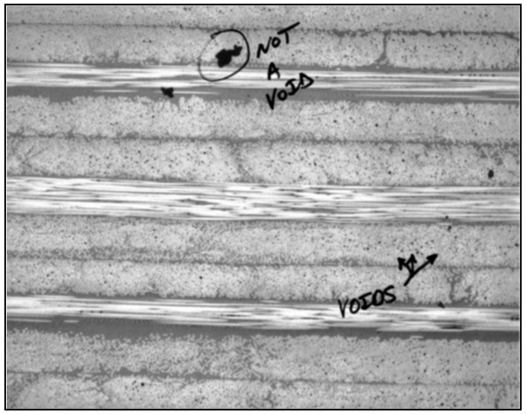


FIGURE 3.2 Photomicrographs of cross-section of a lamina with voids.

20

## Void Content

$$V_v = \frac{\nu_v}{\nu_c}$$

$$\nu_c = \nu_f + \nu_m + \nu_v$$

$$\nu_c = \frac{w_c}{\rho_{ce}}, \text{and} \qquad \nu_f + \nu_m = \frac{w_c}{\rho_{ct}}$$

21

## Void Content

$$\frac{w_c}{\rho_{ce}} = \frac{w_c}{\rho_{ct}} + \nu_v, \text{then}$$

$$\nu_v = \frac{w_c}{\rho_{ce}}\left(\frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}\right)$$

22

## Void Content

$$V_v = \frac{\nu_v}{\nu_c}$$

$$= \frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}$$

23

## Example

***Example 3.2***

A Graphite/Epoxy cuboid specimen with voids has dimensions of $a \times b \times c$, and its mass is $M_c$. After putting it in a mixture of sulfuric acid and hydrogen peroxide, the remaining graphite fibers have a mass $M_f$. From independent tests, the density of graphite and epoxy are $\rho_f$ and $\rho_m$, respectively. Find the volume fraction of the voids in terms of $a$, $b$, $c$, $M_f$, $M_c$, $\rho_f$, $\rho_m$.

24

## Example

$$\nu_c = \nu_f + \nu_m + \nu_v$$

25

## Example

$$\nu_f = \frac{M_f}{\rho_f}, \text{and} \quad \nu_m = \frac{M_c - M_f}{\rho_m}$$

26

## Example

$$\nu_c = abc$$

27

## Example

$$abc = \frac{M_f}{\rho_f} + \frac{M_c - M_f}{\rho_m} + \nu_v$$

$$V_v = \frac{\nu_v}{abc} = 1 - \frac{1}{abc}\left[\frac{M_f}{\rho_f} + \frac{M_c - M_f}{\rho_m}\right]$$

28

## Example

***Alternative Solution***

$$\rho_{ct} = \rho_f V'_f + \rho_m (1 - V'_f)$$

$$V_{f'} = \frac{volume\ of\ fibers}{volume\ of\ fibers + volume\ of\ matrix}$$

$$V'_f = \frac{\frac{M_f}{\rho_f}}{\frac{M_f}{\rho_f} + \frac{M_c - M_f}{\rho_m}}$$

29

## Example

***Alternative Solution***

$$V_{f'} = \frac{volume\ of\ fibers}{volume\ of\ fibers + volume\ of\ matrix}$$

$$V'_f = \frac{\frac{M_f}{\rho_f}}{\frac{M_f}{\rho_f} + \frac{M_c - M_f}{\rho_m}}$$

$$\rho_{ce} = \frac{M_c}{abc}$$

30

## Example

***Alternative Solution***

$$V_v = 1 - \frac{1}{abc}\left[\frac{M_f}{\rho_f} + \frac{M_c - M_f}{\rho_m}\right]$$

31

## Example

***Alternative Solution***

$$\rho_c = \frac{w_c}{w_c - w_i}\rho_w$$

32

# Example

***Alternative Solution***

$$\rho_c = \frac{w_c}{w_c + w_s - w_w}\rho_w$$

33

# END

34