EML 4230 Introduction to Composite Materials

Chapter 3 Micromechanical Analysis of a Lamina
Ultimate Strengths of a Unidirectional Lamina

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Courtesy of the Textbook
Mechanics of Composite Materials by Kaw

UNIVERSITY OF SOUTH FLORIDA

Longitudinal Tensile Strength

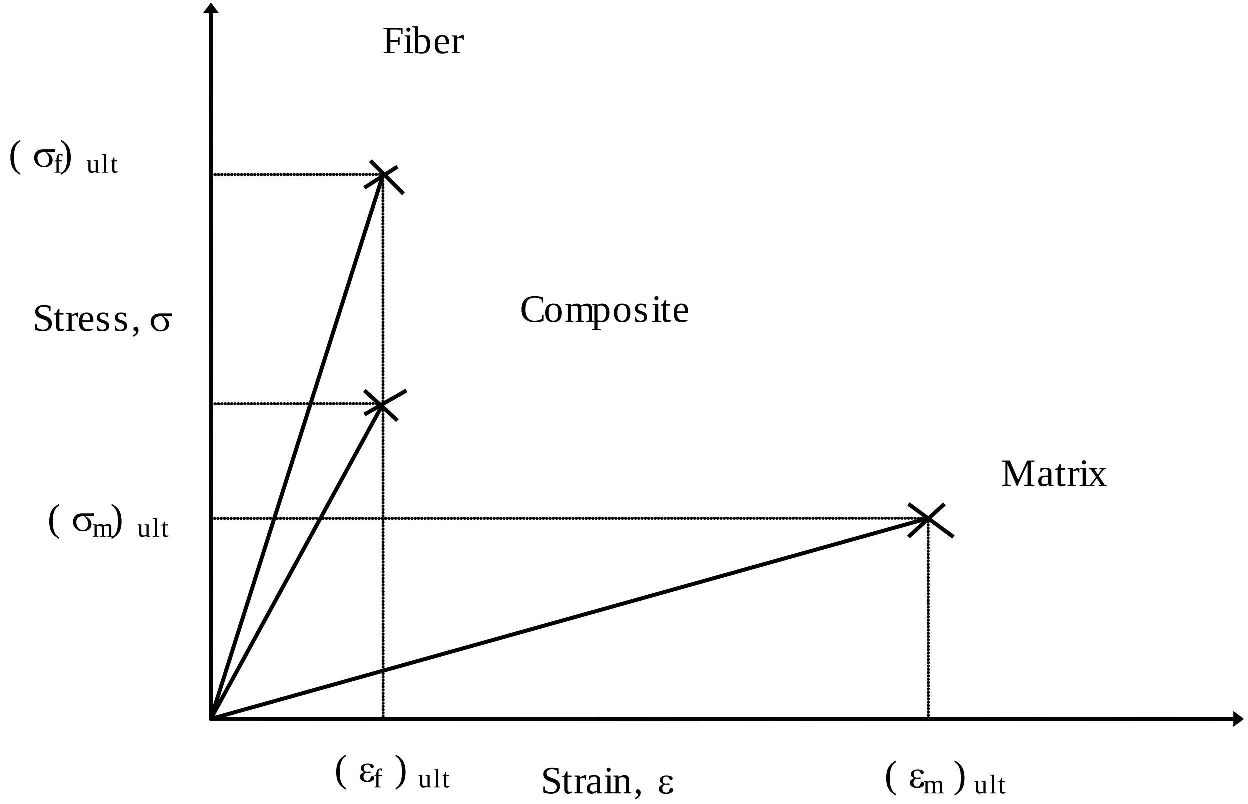


FIGURE 3.24
Stress–strain curve for a unidirectional composite under uniaxial tensile load along fibers.

Longitudinal Tensile Strength

$$(\varepsilon_f)_{ult} = \frac{(\sigma_f)_{ult}}{E_f},$$

and

$$(\varepsilon_m)_{ult} = \frac{(\sigma_m)_{ult}}{E_m}$$

$$(\sigma_1^T)_{ult} = (\sigma_f)_{ult} V_f + (\varepsilon_f)_{ult} E_m (1 - V_f)$$

Longitudinal Tensile Strength

$$(\sigma_m)_{ult}[1-(V_f)_{minimum}] > (\sigma_f)_{ult}(V_f)_{minimum} + (\varepsilon_f)_{ult} E_m[1-(V_f)_{minimum}]$$

$$(V_f)_{minimum} < \frac{(\sigma_m)_{ult} - E_m(\varepsilon_f)_{ult}}{(\sigma_f)_{ult} - E_m(\varepsilon_f)_{ult} + (\sigma_m)_{ult}}$$

$$(\sigma_m)_{ult} > (\sigma_f)_{ult}(V_f)_{critical} + (\varepsilon_f)_{ult} E_m[1-(V_f)_{critical}]$$

$$(V_f)_{critical} < \frac{(\sigma_m)_{ult} - E_m(\varepsilon_f)_{ult}}{(\sigma_f)_{ult} - E_m(\varepsilon_f)_{ult}}$$

Example

Example 3.13

Find the ultimate tensile strength for a Glass/Epoxy lamina with a 70% fiber volume fraction. Use the properties for glass and epoxy from Tables 3.1 and 3.2, respectively. Also, find the minimum and critical fiber volume fractions.

Example

Example 3.13

$$E_f = 85\,GPa, \text{ and } (\sigma_f)_{ult} = 1550\ MPa$$

$$(\varepsilon_f)_{ult} = \frac{1550 \times 10^6}{85 \times 10^9}$$

$$= 0.1823 \times 10^{-1}$$

Example

Example 3.13

$$E_m = 3.4\ GPa, \text{ and } (\sigma_m)_{ult} = 72\ MPa$$

$$(\varepsilon_m)_{ult} = \frac{72 \times 10^6}{3.4 \times 10^9}$$

$$= 0.2117 \times 10^{-1}$$

Example

Example 3.13

$$(\sigma_1^T)_{ult} = (1550\times10^6)(0.7)+(0.1823\times10^{-1})(3.4\times10^9)(1-0.7)$$
$$= 1104\ MP$$

Example

Example 3.13

$$(V_f)_{minimum} = \frac{72\times10^6 - (3.4\times10^9)(0.1823\times10^{-1})}{1550\times10^6 - (3.4\times10^9)(0.1823\times10^{-1}) + 72\times10^6}$$

$$= 0.6422\times10^{-2}$$

$$= 0.6422\%$$

Example

Example 3.13

$$(V_f)_{critical} = \frac{72\times10^6 - (3.4\times10^9)(0.1823\times10^{-1})}{1550\times10^6 - (3.4\times10^9)(0.1823\times10^{-1})}$$

$$= 0.6732\times10^{-2}$$

$$= 0.6732\%$$

Longitudinal Tensile Strength

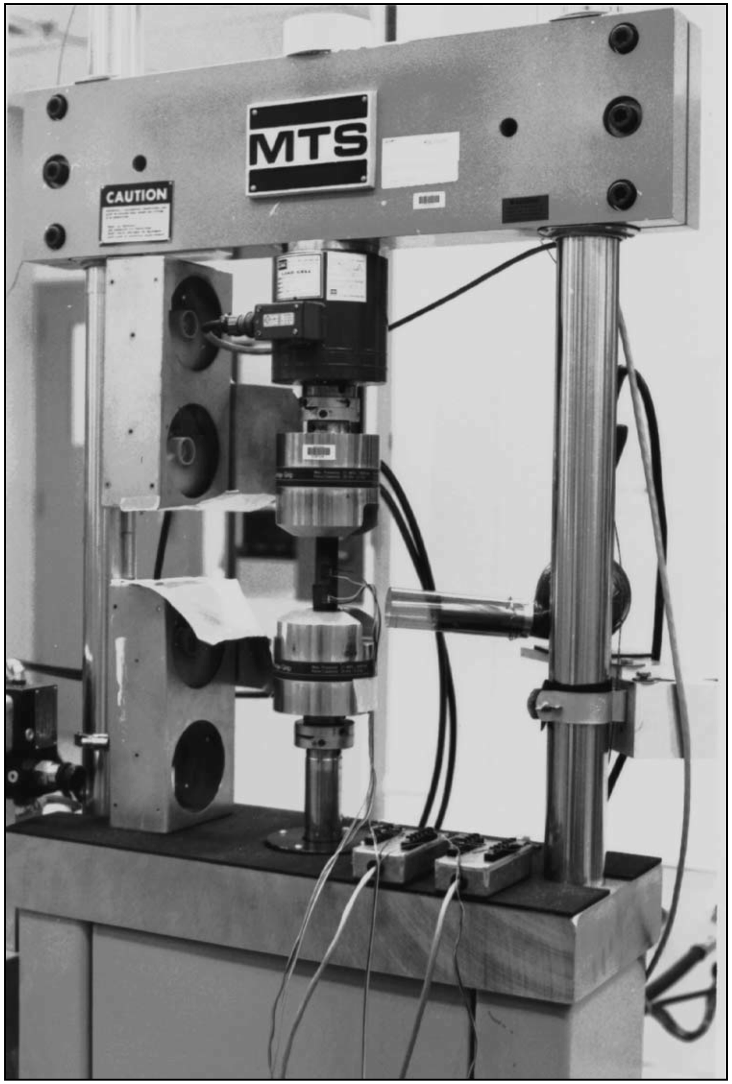


FIGURE 3.25
Tensile coupon mounted in the test frame for finding the tensile strengths of a unidirectional lamina. (Photo courtesy of Dr. R.Y. Kim, University of Dayton Research Institute, Dayton, OH.)

Longitudinal Tensile Strength

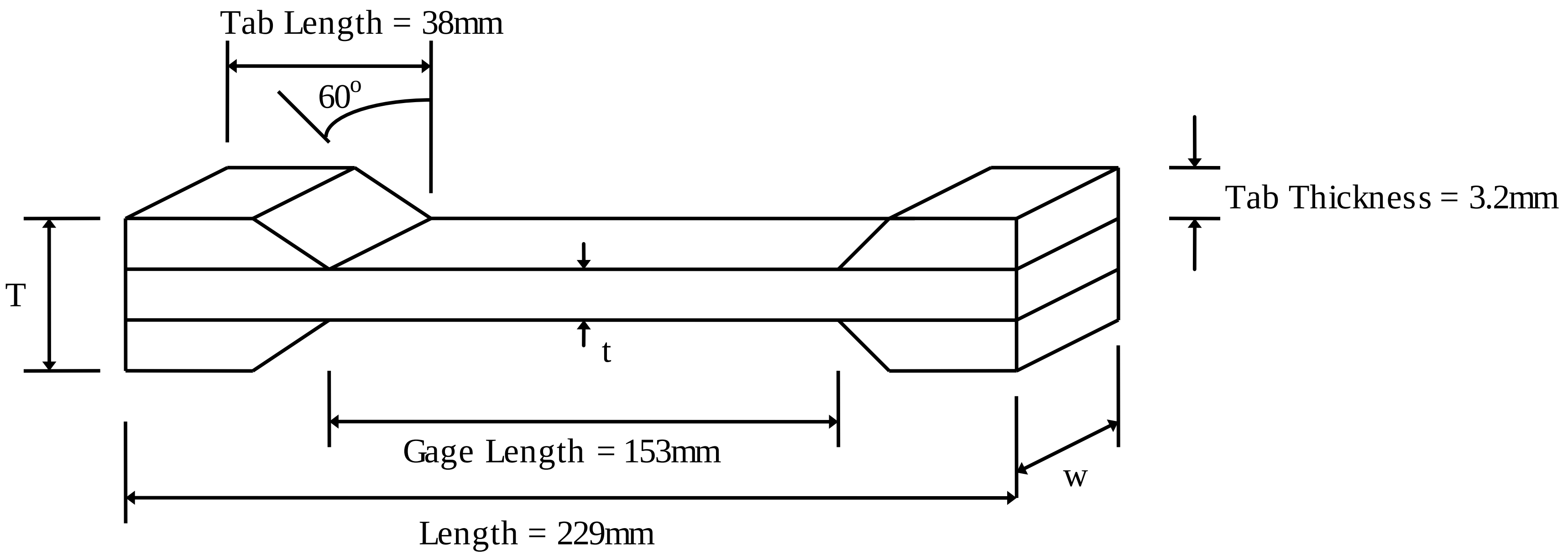


FIGURE 3.26
Geometry of a longitudinal tensile strength specimen.

Longitudinal Tensile Strength

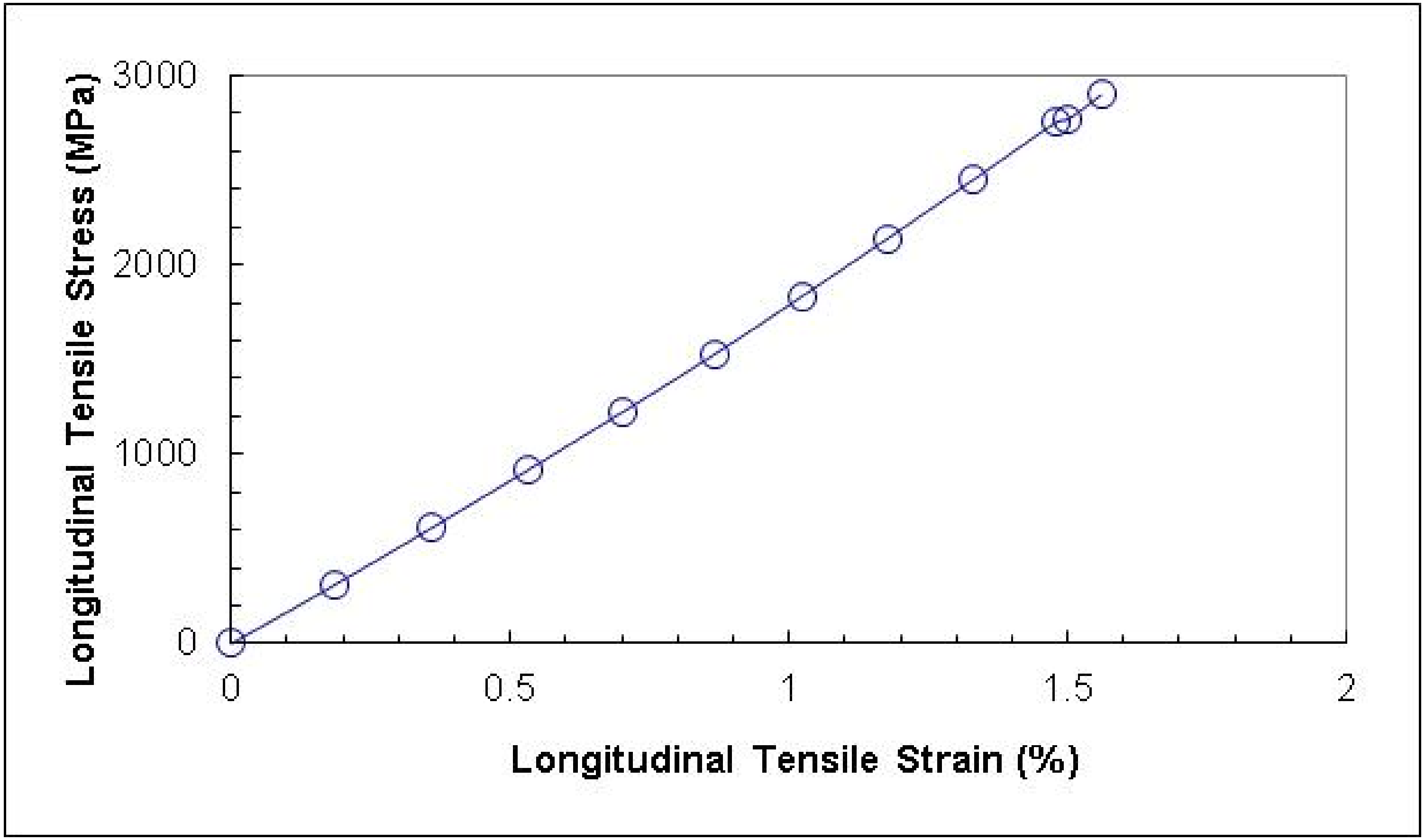


FIGURE 3.27
Stress–strain curve for a [0]8 laminate under a longitudinal tensile load. (Data courtesy of Dr. R.Y. Kim, University of Dayton Research Institute, Dayton, OH).

Example

Example 3.13

$$E_1 = 187.5\ GPa,$$

$$(\sigma_1^T)_{ult} = 2896\ MPa,$$

and

$$(\varepsilon_1^T)_{ult} = 1.560\%$$

Longitudinal Tensile Strength

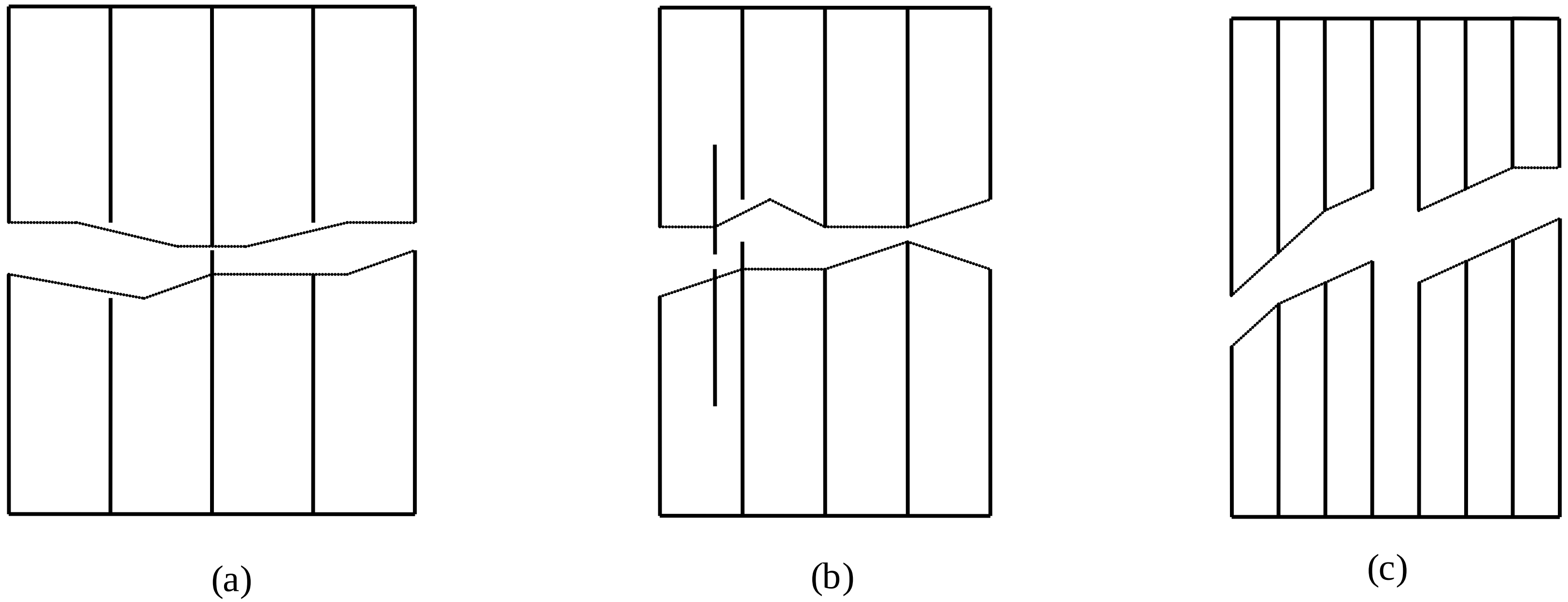


FIGURE 3.28
Modes of failure of unidirectional lamina under a longitudinal tensile load.

Longitudinal Compressive Strength

$$|\varepsilon_1| = \frac{|\sigma_1|}{E_1}$$

$$|\varepsilon_2| = \nu_{12}\frac{|\sigma_1|}{E_1}$$

Longitudinal Compressive Strength

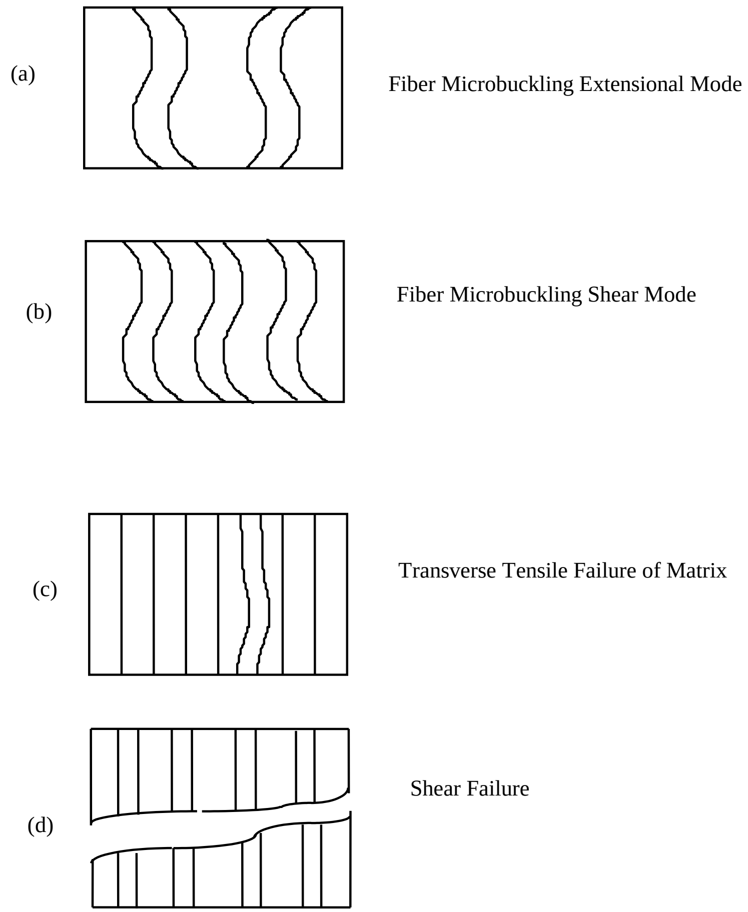


FIGURE 3.29
Modes of failure of a unidirectional lamina under a longitudinal compressive load.

Longitudinal Compressive Strength

$$(\sigma_1^c)_{ult} = \frac{E_1 (\varepsilon_2^T)_{ult}}{\nu_{12}}$$

$$(\varepsilon_2^T)_{ult} = (\varepsilon_m^T)_{ult} (1 - V_f^{1/3})$$

$$(\varepsilon_2^T)_{ult} = (\varepsilon_m^T)_{ult} \left[\frac{d}{s} (\frac{E_m}{E_f} - 1) + 1 \right]$$

Longitudinal Compressive Strength

$$(\sigma_1^c)_{ult} = \min[S_1^c, S_2^c],$$

where

$$S_1^c = 2[V_f + (1 - V_f)\frac{E_m}{E_f}]\sqrt{\frac{V_f E_m E_f}{3(1 - V_f)}},$$

and

$$S_2^c = \frac{G_m}{1 - V_f}$$

Longitudinal Compressive Strength

$$(\tau_{12})_{ult} = (\tau_f)_{ult} V_f + (\tau_m)_{ult} V_m$$

$$(\sigma_1^c)_{ult} = 2[(\tau_f)_{ult} V_f + (\tau_m)_{ult} V_m]$$

Longitudinal Compressive Strength

Table 3.6.
Comparison of experimental and predicted values of longitudinal compressive strength of unidirectional laminae, V_f =0.50. (Source: Table 7.2 in Introduction to Composite Materials by D. Hull, 1981, 8 Cambridge University Press, 1981, Reprinted with the permission of Cambridge University Press)

Material	Experimental Strength	Equation 3.78(a)	Equation (3.78b)
	MPa	MPa	MPa
Glass/Polyester	600-1000	8700	2200
Type I Carbon/Epoxy	700-900	22800	2900
Kevlar 49/Epoxy	240-290	13200	2900

Example

Example 3.14

Find the longitudinal compressive strength of a Glass/Epoxy lamina with a 70% fiber volume fraction. Use the properties of glass and epoxy from Tables 3.1 and 3.2, respectively. Assume fibers are circular and are in a square array.

Example

Example 3.14.

From Table 3.1

$E_f = 85\ GPa,$

$\nu_f = 0.20,$

$(\sigma_f)_{ult} = 1550\ MPa,$

and

$(\tau_f)_{ult} = 35\ MPa$

From Table 3.2

$E_m = 3.4\ GPa,$

$\nu_m = 0.30,$

$(\sigma_m)_{ult} = 72\ MPa,$

and

$(\tau_m)_{ult} = 34\ MPa$

Example

Example 3.14.

$$E_1 = 60.52\ GPa \qquad \nu_{12} = 0.23$$

Example

Example 3.14.

$$\frac{d}{s} = \left[\frac{4(0.7)}{\pi}\right]^{\frac{1}{2}}$$

$$= 0.9441$$

Example

Example 3.14.

$$(\varepsilon_m)_{ult} = \frac{72\times 10^6}{3.40\times 10^9}$$

$$= 0.2117\times 10^{-1}$$

Example

Example 3.14.

$$(\varepsilon_2^T)_{ult} = 0.2117\times10^{-1}\left[0.9441\left(\frac{3.4\times10^9}{85\times10^9}-1\right)+1\right]$$

$$= 0.1983\times10^{-2}$$

Example

Example 3.14.

$$(\varepsilon_2^T)_{ult} = (0.2117 \times 10^{-1})\,(1 - 0.7^{1/3})$$

$$= 0.2373 \times 10^{-2}$$

Example

Example 3.14.

$$(\sigma_1^C)_{ult} = \frac{(60.52 \times 10^9)\,(0.1983 \times 10^{-2})}{0.23}$$

$$= 521.8\ MPa$$

Example

Example 3.14.

$$S_1^C = 2\left[0.7 + (1 - 0.7)\frac{3.4\times 10^9}{85\times 10^9}\right]\sqrt{\frac{(0.7)(3.4\times 10^9)(85\times 10^9)}{3(1-0.7)}}$$

$$= 21349\ MPa$$

Example

Example 3.14.

$$G_m = 1.308\ GPa$$

$$S_2^C = \frac{1.308 \times 10^9}{1 - 0.7}$$

$$= 4360\ MPa$$

Example

Example 3.14.

$$(\sigma_1^C)_{ult} = \min(21349, 4360) = 4360\ MPa$$

Example

Example 3.14.

$$(\sigma_1^C)_{ult} = 2\left[(35\times 10^6)(0.7) + (34\times 10^6)(0.3)\right]$$
$$= 69.4\ MPa$$

Longitudinal Compressive Strength

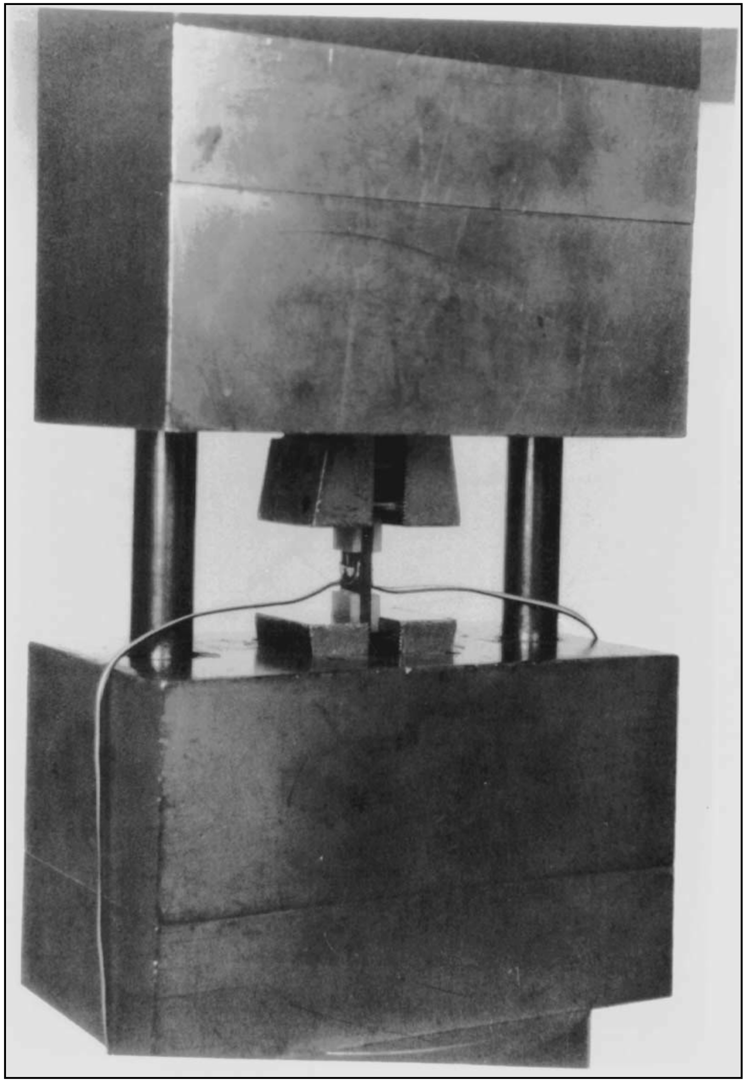


FIGURE 3.30
IITRI fixture mounted in a test frame for finding the compressive strengths of a lamina. (Data reprinted with permission from *Experimental Characterization of Advanced Composites*, Carlsson, L.A. and Pipes, R.B., Technomic Publishing Co., Inc., 1987, p. 76. Copyright CRC Press, Boca Raton, FL.)

Example

Example 3.14.

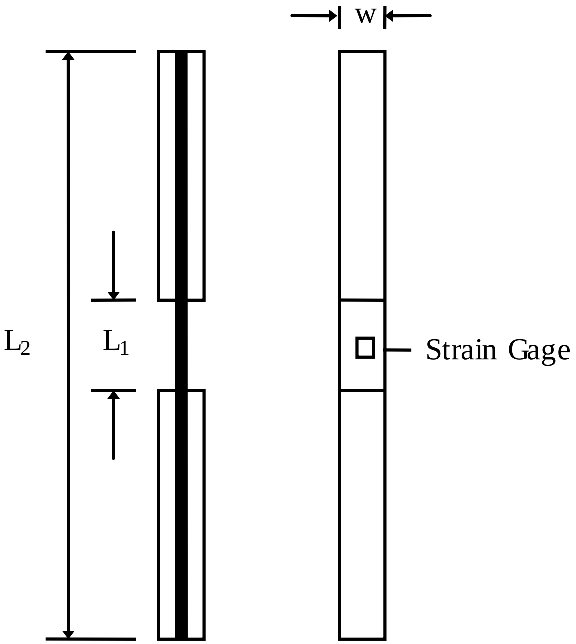
$$E_1^c = 199\ GPa,$$

$$(\sigma_1^c)_{ult} = 1908\ MPa,$$

and

$$(\varepsilon_1^c)_{ult} = 0.9550\%$$

Laminate Stacking Sequence



Specimen Dimensions		
L_1, mm	L_2, mm	w^*, mm
12.7 ± 1	127 ± 1.5	12.7 ± 0.1 or 6.4 ± 0.1

FIGURE 3.31
Geometry of a longitudinal compressive strength specimen. (Data reprinted with permission from *Experimental Characterization of Advanced Composites*, Carlsson, L.A. and Pipes, R.B., Technomic Publishing Co., Inc., 1987, p. 76. Copyright CRC Press, Boca Raton, FL.)

Transverse Tensile Strength

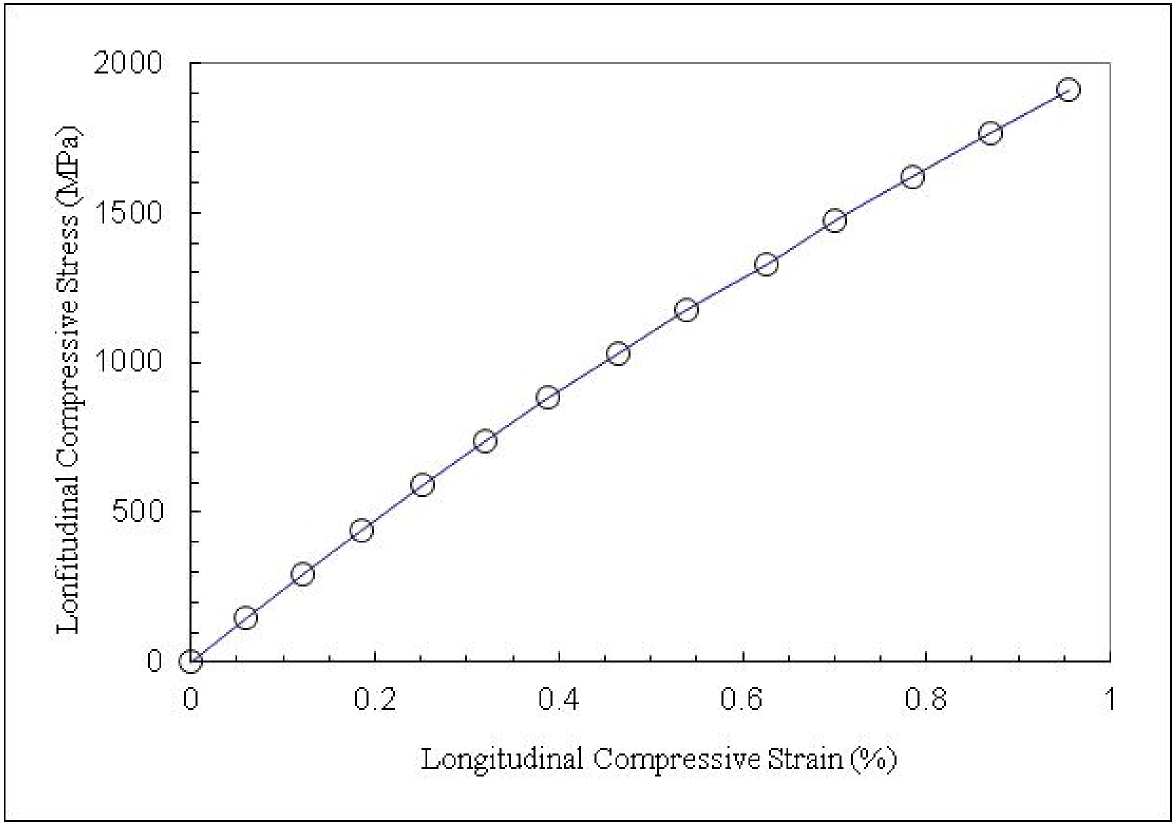


FIGURE 3.32
Stress–strain curve for a [0]24 graphite/epoxy laminate under a longitudinal compressive load. (Data courtesy of Dr. R.Y. Kim, University of Dayton Research Institute, Dayton, OH.)

Transverse Tensile Strength

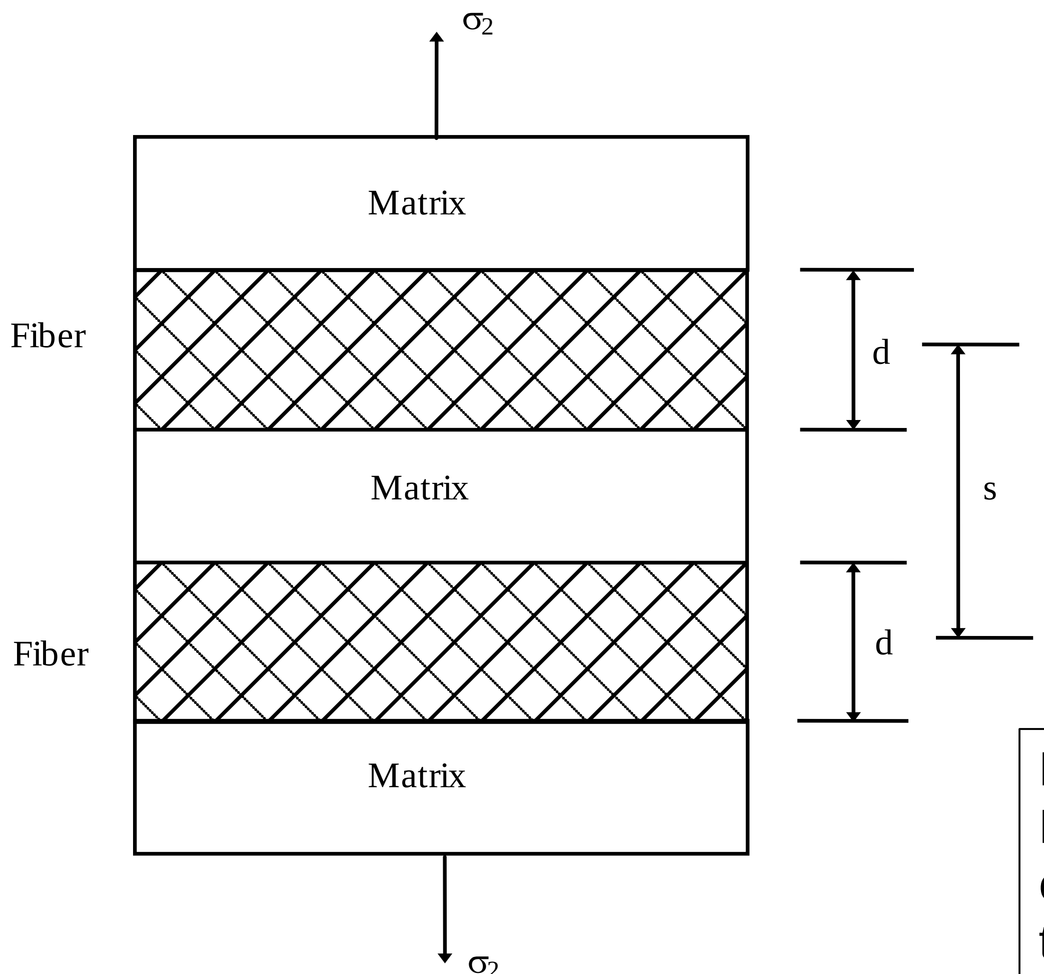


FIGURE 3.33 Representative volume element to calculate transverse tensile strength of a unidirectional lamina.

Transverse Tensile Strength

$$\delta_c = \delta_f + \delta_m$$

where:

$$\delta_c = s\,\varepsilon_c,$$

$$\delta_f = d\,\varepsilon_f,$$

and

$$\delta_m = (s - d)\,\varepsilon_m$$

Transverse Tensile Strength

$$\varepsilon_c = \frac{d}{s}\varepsilon_f + \left(1 - \frac{d}{s}\right)\varepsilon_m$$

$$E_f\,\varepsilon_f = E_m\,\varepsilon_m$$

$$\varepsilon_c = \left[\frac{d}{s}\frac{E_m}{E_f} + \left(1 - \frac{d}{s}\right)\right]\varepsilon_m$$

$$(\varepsilon_2^T)_{ult} = \left[\frac{d}{s}\frac{E_m}{E_f} + \left(1 - \frac{d}{s}\right)\right](\varepsilon_m^T)_{ult}$$

$$(\sigma_2^T)_{ult} = E_2\,(\varepsilon_2^T)_{ult}$$

Example

Example 3.15

Find the ultimate transverse tensile strength for a unidirectional Glass/Epoxy lamina with a 70% fiber volume fraction. Use properties of glass and epoxy from Table 3.1 and Table 3.2, respectively. Assume the fibers are circular and are arranged in a square array.

Example

Example 3.15

$$(\varepsilon_2^T)_{ult} = 0.1983 \times 10^{-2} \qquad E_2 = 10.37\ GPa$$

$$(\sigma_2^T)_{ult} = (10.37 \times 10^9)(0.1983 \times 10^{-2})$$

$$= 20.56\ MPa$$

Example

Example 3.15

$$E_2 = 9.963\ GPa,$$

$$(\sigma_2^T)_{ult} = 53.28\ MPa, \text{ and}$$

$$(\varepsilon_2^T)_{ult} = 0.5355\%$$

Transverse Tensile Strength

Example 3.15

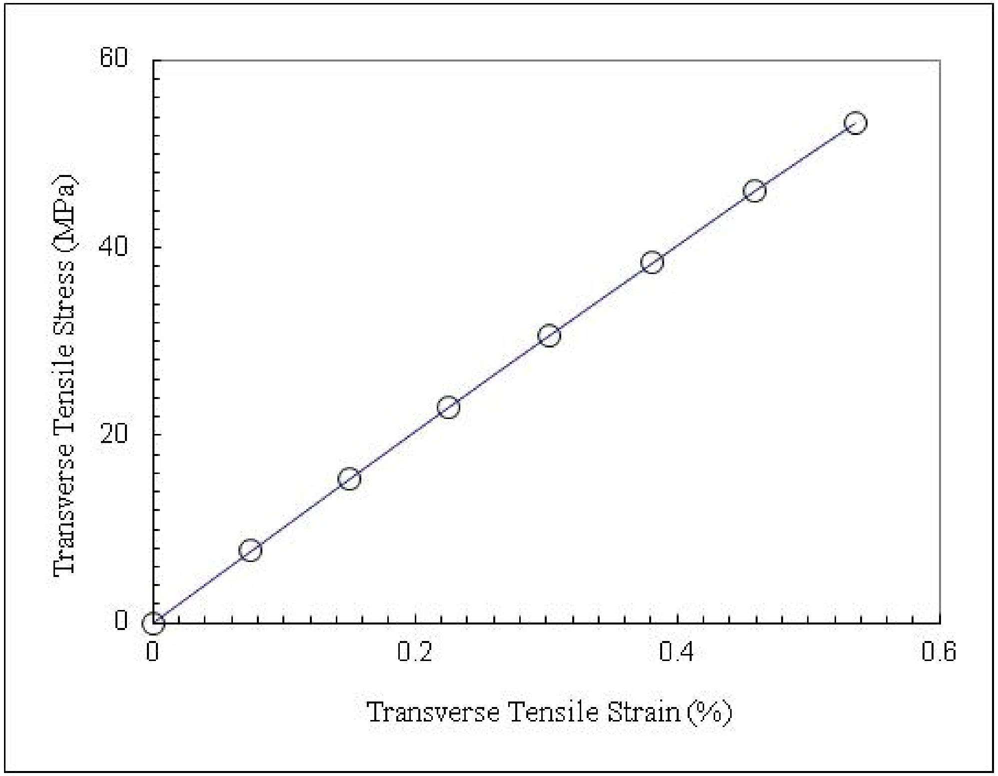


FIGURE 3.34
Stress–strain curve for a [90]16 graphite/epoxy laminate under a transverse tensile load. (Data courtesy of Dr. R.Y. Kim, University of Dayton Research Institute, Dayton, OH.)

Transverse Compressive Strength

$$(\sigma_2^C)_{ult} = E_2 (\varepsilon_2^C)_{ult}, \text{where}$$

$$(\varepsilon_2^C)_{ult} = \left[\frac{d}{s} \frac{E_m}{E_f} + \left(1 - \frac{d}{s} \right) \right] (\varepsilon_m^C)_{ult}$$

Example

Example 3.16

Find the ultimate transverse compressive strength of a Glass/Epoxy lamina with 70% fiber volume fraction. Use the properties of glass and epoxy from Table 3.1 and Table 3.2, respectively. Assume the fibers are circular and are packed in a square array.

Example

Example 3.16

$$E_f = 85\ GPa,$$

$$E_m = 3.4\ GPa,$$

$$(\sigma_m^C)_{ult} = 102\ MPa,$$

$$E_2 = 10.37\ GPa,$$

and

$$\frac{d}{s} = 0.9441$$

Example

Example 3.16

$$(\varepsilon_m^C)_{ult} = \frac{102 \times 10^6}{3.4 \times 10^9}$$

$$= 0.0300$$

Example

Example 3.16

$$(\varepsilon_2^C)_{ult} = \left[0.9441 \frac{3.4 \times 10^9}{85 \times 10^9} + (1 - 0.9441) \right] (0.03)$$

$$= 0.2810 \times 10^{-2}$$

Example

Example 3.16

$$(\sigma_2^C)_{ult} = (10.37 \times 10^9)(0.2810 \times 10^{-2}) = 29.14\ MPa$$

Example

Example 3.16

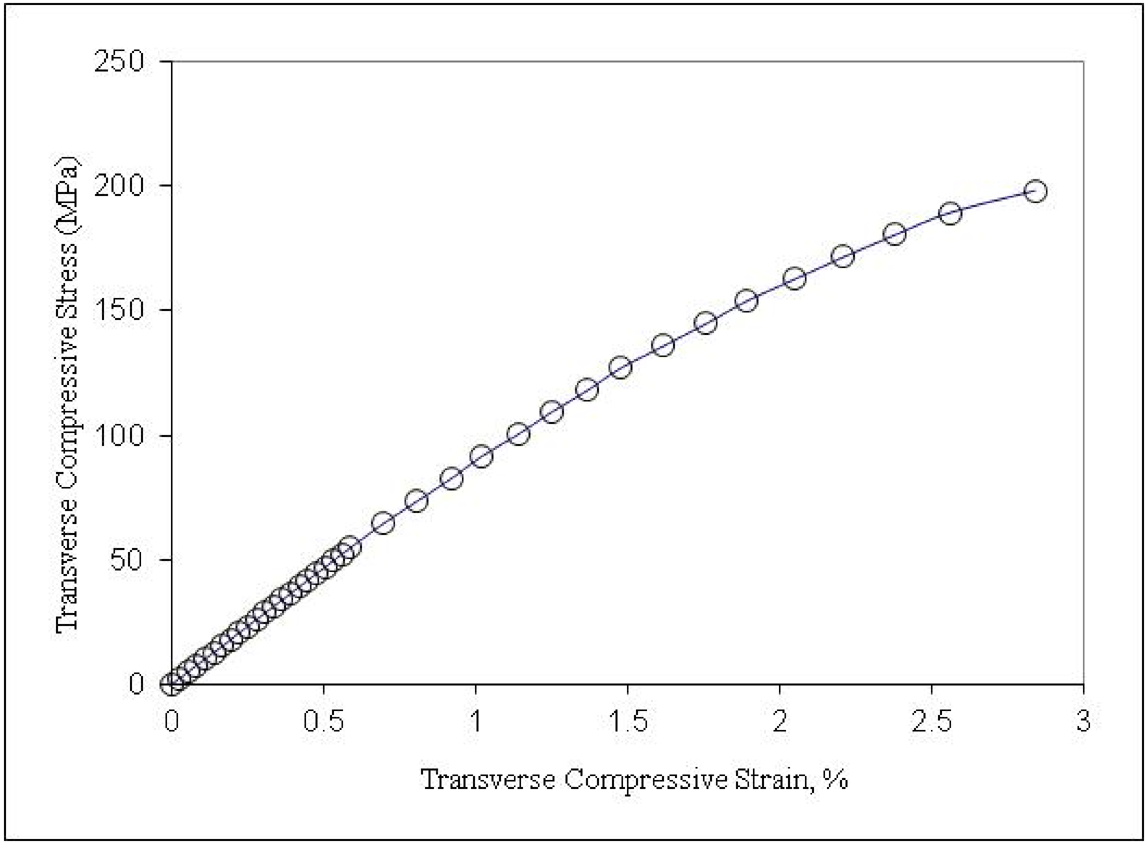


FIGURE 3.35
Stress–strain curve for a [90]40 graphite/epoxy laminate under a transverse compressive load perpendicular to the fibers.

Example

Example 3.16

$$E_2^c = 93\,GPa,$$

$$(\sigma_2^c)_{ult} = 198\;MPa, \text{ and}$$

$$(\varepsilon_2^c)_{ult} = 2.7\%$$

In-Plane Shear Strength

$$\Delta_c = \Delta_f + \Delta_m$$

where :

$$\Delta_c = s(\gamma_{12})_c \,,$$

$$\Delta_f = d(\gamma_{12})_f \,, \text{and}$$

$$\Delta_m = (s - d)(\gamma_{12})_m$$

In-Plane Shear Strength

$$(\gamma_{12})_c = \frac{d}{s}(\gamma_{12})_f + \left(1 - \frac{d}{s}\right)(\gamma_{12})_m$$

$$(\gamma_{12})_m G_m = (\gamma_{12})_f G_f$$

$$(\gamma_{12})_c = \left[\frac{d}{s}\frac{G_m}{G_f} + \left(1 - \frac{d}{s}\right)\right](\gamma_{12})_m$$

$$(\gamma_{12})_{ult} = \left[\frac{d}{s}\frac{G_m}{G_f} + \left(1 - \frac{d}{s}\right)\right](\gamma_{12})_{m\,ult}$$

$$(\tau_{12})_{ult} = G_{12}(\gamma_{12})_{ult}$$

$$= G_{12}\left[\frac{d}{s}\frac{G_m}{G_f} + \left(1 - \frac{d}{s}\right)\right](\gamma_{12})_{m\ ult}$$

Example

Example 3.17

Find the ultimate shear strength for a Glass/Epoxy lamina with 70% fiber volume fraction. Use properties for glass and epoxy from Tables 3.1 and 3.2, respectively. Assume the fibers are circular and are arranged in a square array.

Example

Example 3.17

$$G_f = 35.42\ GPa$$

$$G_m = 1.308\ GPa$$

$$G_{12} = 4.014\ GPa$$

$$\frac{d}{s} = 0.9441$$

$$(\tau_{12})_{mult} = 34\ MPa$$

Example

Example 3.17

$$(\gamma_{12})_{mult} = \frac{34 \times 10^6}{1.308 \times 10^9}$$

$$= 0.2599 \times 10^{-1}$$

Example

Example 3.17

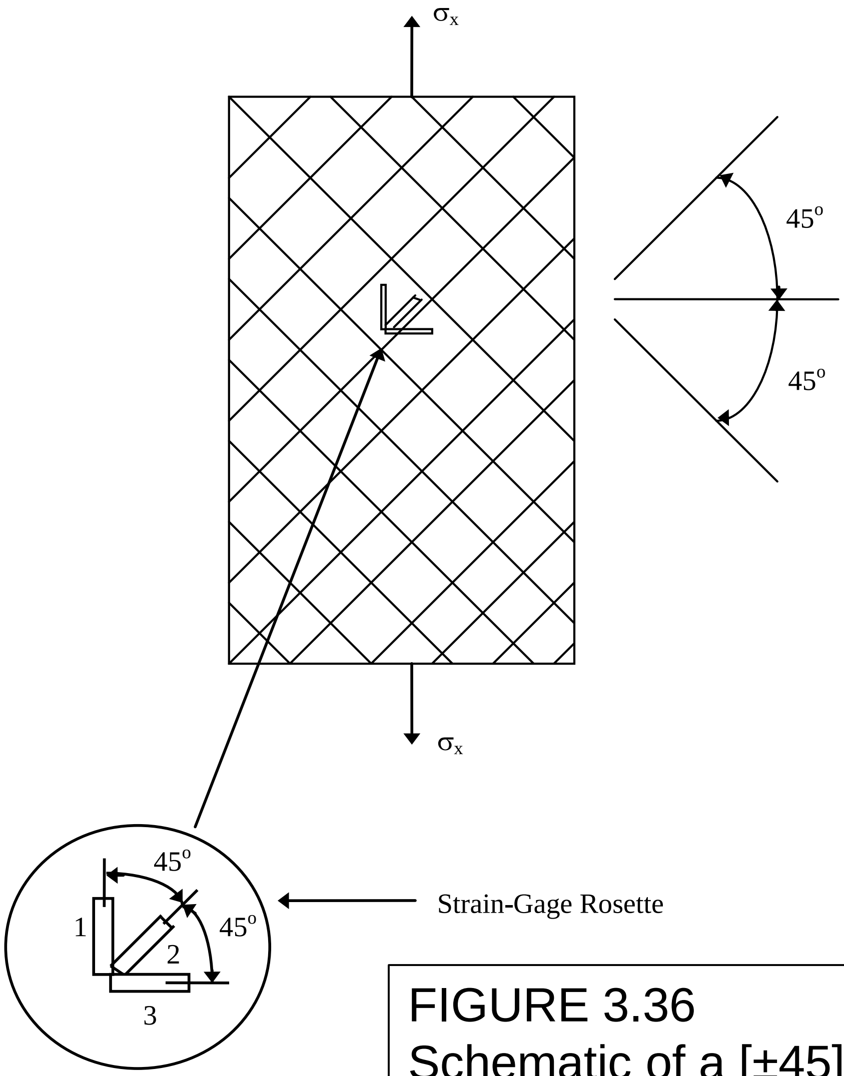


FIGURE 3.36
Schematic of a [±45]2S laminate shear test.

Example

Example 3.17

$$(\tau_{12})_{ult} = (4.014\times 10^9)\left[0.9441\frac{1.308\times 10^9}{35.42\times 10^9} + (1 - 0.9441)\right](0.2599\times 10^{-1})$$

$$= 9.469\ MPa$$

Example

Example 3.17

$$(\tau_{12})_{ult} = \frac{(\sigma_x)_{ult}}{2}, \text{and}$$

$$(\gamma_{12})_{ult} = (|\varepsilon_x|)_{ult} + (|\varepsilon_y|)_{ult}$$

Example

Example 3.17

$$G_{12} = 5.566\ GPa,$$

$$(\tau_{12})_{ult} = 87.57\ MPa, \text{and}$$

$$(\gamma_{12})_{ult} = 2.619\%$$

Example

Example 3.17

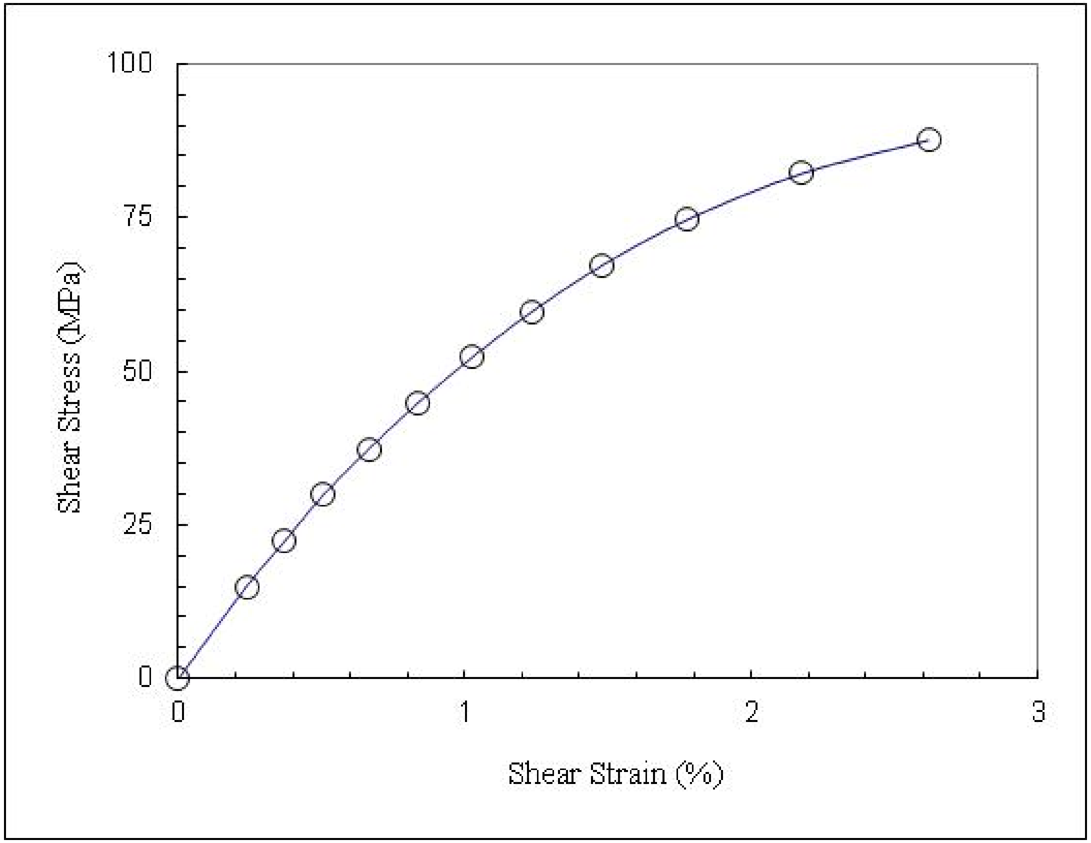


FIGURE 3.37
Shear stress–shear strain curve obtained from a [±45]2S graphite/epoxy laminate under a tensile load.

END