EML 4230 Introduction to Composite Materials

Chapter 3 Micromechanical Analysis of a Lamina

Coefficients of Thermal Expansion

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Courtesy of the Textbook

Mechanics of Composite Materials by Kaw

1

Coefficients of Thermal Expansion of Unidirectional Lamina

$$\alpha_1 = \frac{1}{E_1}(\alpha_f E_f V_f + \alpha_m E_m V_m)$$

$$\alpha_2 = (1+\nu_f)\alpha_f V_f + (1+\nu_m)\alpha_m V_m - \alpha_1 \nu_{12}$$

2

Derivation of Longitudinal Coefficient of Thermal Expansion, α_1

$$F_1 = \sigma_1 A_c = 0 = \sigma_f A_f + \sigma_m A_m$$

$$\sigma_f V_f + \sigma_m V_m = 0$$

$$\sigma_f = E_f(\varepsilon_f - \alpha_f \Delta T)$$

$$\sigma_m = E_m(\varepsilon_m - \alpha_m \Delta T)$$

$$(\varepsilon_f = \varepsilon_m = \varepsilon_1),$$

$$\varepsilon_f = \frac{\alpha_f E_f V_f + \alpha_m E_m V_m}{E_f V_f + E_m V_m}\Delta T$$

$$\varepsilon_1 = \alpha_1 \Delta T$$

$$\alpha_1 = \frac{\alpha_f E_f V_f + \alpha_m E_m V_m}{E_f V_f + E_m V_m}$$

3

Rewriting Longitudinal Thermal Expansion Coefficient Formula

$$\alpha_1 = \frac{\alpha_f E_f V_f + \alpha_m E_m V_m}{E_f V_f + E_m V_m}$$

$$= \frac{1}{E_1}(\alpha_f E_f V_f + \alpha_m E_m V_m)$$

$$= \left(\frac{\alpha_f E_f}{E_1}\right)V_f + \left(\frac{\alpha_m E_m}{E_1}\right)V_m$$

4

Derivation of Transverse Thermal Expansion Coefficient, α_2

$$(\epsilon_f)_1 = (\epsilon_m)_1 = \epsilon_1$$

$$(\sigma_f)_1 = E_f(\epsilon_f)_1 = E_f(\alpha_1 - \alpha_f)\Delta T$$

$$(\epsilon_f)_2 = \alpha_f \Delta T - \frac{\nu_f(\sigma_f)_1}{E_f}$$

$$(\sigma_m)_1 = E_m(\epsilon_m)_1 = E_m(\alpha_m - \alpha_1)\Delta T$$

$$(\epsilon_m)_2 = \alpha_m \Delta T - \frac{\nu_m(\sigma_m)_1}{E_m}$$

$$\epsilon_2 = (\epsilon_f)_2 V_f + (\epsilon_m)_2 V_m$$

$$\epsilon_2 = \left[\alpha_f \Delta T - \frac{\nu_f E_f(\alpha_1 - \alpha_f)\Delta T}{E_f}\right]V_f + \left[\alpha_m \Delta T - \frac{\nu_m E_m(\alpha_m - \alpha_1)\Delta T}{E_m}\right]V_m$$

5

Transverse Thermal Expansion Coefficient

$$\epsilon_2 = \alpha_2 \Delta T$$

$$\alpha_2 = [\alpha_f - \nu_f(\alpha_1 - \alpha_f)]V_f + [\alpha_m - \nu_m(\alpha_m - \alpha_1)]V_m$$

$$\nu_{12} = \nu_f V_f + \nu_m V_m$$

$$\alpha_2 = (1+\nu_f)\alpha_f V_f + (1+\nu_m)\alpha_m V_m - \alpha_1 \nu_{12}$$

6

Example, Coefficients of Thermal Expansion

Find the coefficients of thermal expansion for a Glass/Epoxy lamina with 70% fiber volume fraction. Use the properties of glass and epoxy from Tables 3.1 and 3.2, respectively.

$E_f = 85\ GPa$

$\nu_f = 0.2$

$\alpha_f = 5\times10^{-6}\ m/m/^0C$

$E_m = 3.4\ GPa$

$\nu_m = 0.3$

$\alpha_m = 63\times10^{-6}\ m/m/^\circ C$

$E_1 = 60.52\ GPa.$

$\nu_{12} = 0.2300$

7

Example, Coefficients of Thermal Expansion

$$\alpha_1 = \frac{1}{E_1}(\alpha_f E_f V_f + \alpha_m E_m V_m)$$

$$= \frac{1}{60.52\times10^9}[(5\times10^{-6})(85\times10^9)(0.7) + (63\times10^{-6})(3.4\times10^9)(0.3)]$$

$$= 5.978\times10^{-6}\ m/m/^0C$$

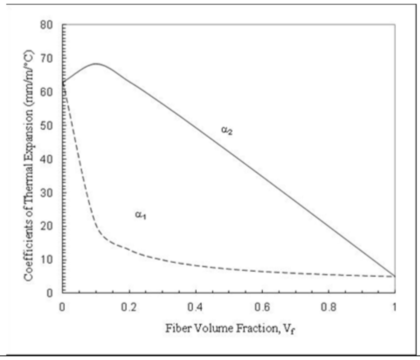
$$\alpha_2 = (1+\nu_f)\alpha_f V_f + (1+\nu_m)\alpha_m V_m - \alpha_1 \nu_{12}$$

$$= (1+0.2)(5.0\times10^{-6})(0.7) + (1+0.3)(63\times10^{-6})(0.3) - (5.978\times10^{-6})(0.23)$$

$$= 27.40\times10^{-6}\ m/m/^0C$$

8

How Coefficients of Thermal Expansion Vary as a Function of Fiber Volume Fraction

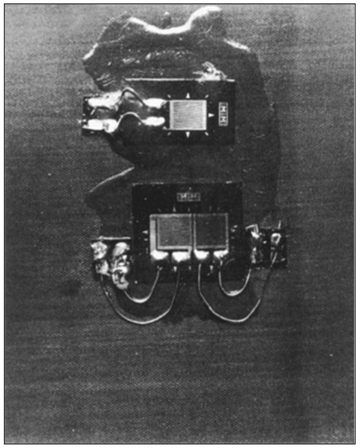


Longitudinal and transverse coefficients of thermal expansion as a function of fiber volume fraction for a glass/epoxy unidirectional lamina.

9

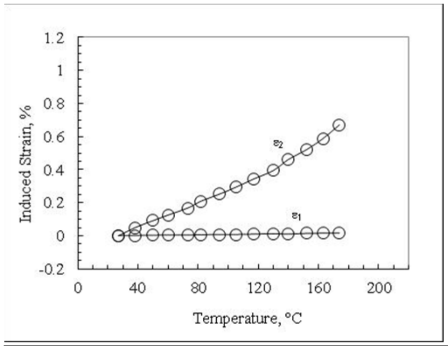
Experimental Setup to Find Thermal Expansion Coefficient

FIGURE 3.39
Unidirectional graphite/epoxy specimen with strain gages and temperature sensors for finding coefficients of thermal expansion.



10

Experimental Results for Coefficient of Thermal Expansion



$\alpha_1 = -1.3\times 10^{-6}\ m/m/^0C$, and

$\alpha_2 = 33.9\times 10^{-6}\ m/m/^0C$

Induced strain as a function of temperature to find the coefficients of thermal expansion of a unidirectional graphite/epoxy laminate.

11

END

12