EML 4230 Introduction to Composite Materials

Chapter 3 Micromechanical Analysis of a Lamina
Coefficients of Moisture Expansion

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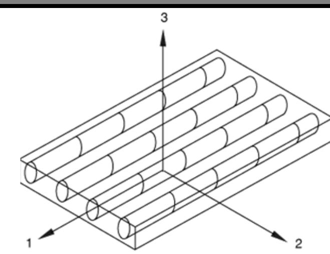
Courtesy of the Textbook
Mechanics of Composite Materials by Kaw

USF UNIVERSITY OF SOUTH FLORIDA

1

General Formulas for Coefficients of Moisture Expansion



$$\beta_1 = \frac{\beta_f\ \Delta C_f V_f E_f + \beta_m \Delta C_m V_m E_m}{E_1(\Delta C_f \rho_f V_f + \Delta C_m \rho_m V_m)} \rho_c$$

$$\beta_2 = \frac{V_f(1+\nu_f)\Delta C_f \beta_f + V_m(1+\nu_m)\Delta C_m \beta_m}{(V_m \rho_m \Delta C_m + V_f \rho_f \Delta C_f)} \rho_c - \beta_1 \nu_{12}$$

Source: https://docs.software.vt.edu/abaqusv2022/English/SIMACAEMATRefMap/simamat-c-damagefibercomposite.htm

2

Coefficients of Moisture Expansion with No Moisture Absorbed by Fiber

$$\beta_1 = \frac{\beta_f\ \Delta C_f V_f E_f + \beta_m \Delta C_m V_m E_m}{E_1(\Delta C_f \rho_f V_f + \Delta C_m \rho_m V_m)} \rho_c$$

$$\beta_2 = \frac{V_f(1+\nu_f)\Delta C_f \beta_f + V_m(1+\nu_m)\Delta C_m \beta_m}{(V_m \rho_m \Delta C_m + V_f \rho_f \Delta C_f)} \rho_c - \beta_1 \nu_{12}$$

If fiber does not absorb moisture

$$\beta_1 = \frac{E_m}{E_1}\frac{\rho_c}{\rho_m}\beta_m \qquad \beta_2 = (1+\nu_m)\frac{\rho_c}{\rho_m}\beta_m - \beta_1 \nu_{12}$$

If fiber does not absorb moisture AND we have a high E_f/E_m ratio (just shown for illustration)

$$\beta_1 = 0, \qquad \beta_2 = (1+\nu_m)\frac{\rho_c}{\rho_m}\beta_m$$

3

Derivation, Coefficient of Moisture Expansion, β_1

$$F_1 = \sigma_1 A_c = 0 = \sigma_f A_f + \sigma_m A_m, \text{ and } \quad \sigma_f V_f + \sigma_m V_m = 0$$

↓

$$\sigma_f = E_f(\varepsilon_f - \beta_f \Delta C_f), \text{and } \sigma_m = E_m(\varepsilon_m - \beta_m \Delta C_m) \quad (\varepsilon_f = \varepsilon_m = \varepsilon_1)$$

↓

$$\varepsilon_f = \frac{\beta_f \Delta C_f V_f E_f + \beta_m \Delta C_m V_m E_m}{E_f V_f + E_m V_m}$$

4

Derivation, Coefficient of Moisture Expansion, β_1

$$\varepsilon_f = \frac{\beta_f \Delta C_f V_f E_f + \beta_m \Delta C_m V_m E_m}{E_f V_f + E_m V_m}$$

$$\varepsilon_1 = \beta_1 \Delta C_c$$

$$\beta_1 = \frac{\beta_f \Delta C_f V_f E_f + \beta_m \Delta C_m V_m E_m}{(E_f V_f + E_m V_m)\Delta C_c}$$

$$\Delta C_c w_c = \Delta C_f w_f + \Delta C_m w_m$$

$$\Delta C_c = \Delta C_f W_f + \Delta C_m W_m$$

$$\beta_1 = \frac{\beta_f \Delta C_f V_f E_f + \beta_m \Delta C_m V_m E_m}{(E_f V_f + E_m V_m)(\Delta C_f W_f + \Delta C_m W_m)}$$

$$= \frac{\beta_f \Delta C_f V_f E_f + \beta_m \Delta C_m V_m E_m}{E_1 (\Delta C_f \rho_f V_f + \Delta C_m \rho_m V_m)} \rho_c$$

5

Example, Coefficients of Moisture Expansion

Find the two coefficients of moisture expansion for a Glass/Epoxy lamina with 70% fiber volume fraction. Use properties for glass and epoxy from Tables 3.1 and 3.2, respectively. Assume glass does not absorb moisture.

$\rho_f = 2500\ kg/m^3$

$\rho_m = 1200\ kg/m^3$

$\beta_m = 0.33\ m/m/kg/kg$

$E_m = 3.4\ GPa$

$\nu_m = 0.3$

$\rho_c = 2110\ kg/m^3$

$E_1 = 60.52\ GPa$

$\nu_{12} = 0.230$

6

Example, Coefficients of Moisture Expansion

$$\beta_1 = \frac{E_m}{E_1}\frac{\rho_c}{\rho_m}\beta_m$$

$$= \frac{3.4\times 10^9}{60.52\times 10^9}\frac{2110}{1200}(0.33)$$

$$= 0.3260\times 10^{-1}\ m/m/kg/kg$$

$$\beta_2 = (1+\nu_m)\frac{\rho_c}{\rho_m}\beta_m - \beta_1 \nu_{12}$$

$$= (1+0.3)\frac{2110}{1200}(0.33) - (0.3260\times 10^{-1})(0.230)$$

$$= 0.7468\ m/m/kg/kg$$

7

END

8