EML 4230 Introduction to Composite Materials

Chapter 3 Micromechanical Analysis of a Lamina
Coefficients of Moisture Expansion

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Courtesy of the Textbook
Mechanics of Composite Materials by Kaw

USF UNIVERSITY OF SOUTH FLORIDA

Coefficients of Moisture Expansion

$$\beta_1 = \frac{\beta_f \ \Delta C_f V_f E_f + \beta_m \Delta C_m V_m E_m}{E_1 (\Delta C_f \rho_f V_f + \Delta C_m \rho_m V_m)} \rho_c$$

$$\beta_2 = \frac{V_f (1+\nu_f) \Delta C_f \beta_f + V_m (1+\nu_m) \Delta C_m \beta_m}{(V_m \rho_m \Delta C_m + V_f \rho_f \Delta C_f)} \rho_c - \beta_1 \nu_{12}$$

Coefficients of Moisture Expansion

$$\beta_1 = \frac{E_m}{E_1}\frac{\rho_c}{\rho_m}\beta_m$$

$$\beta_2 = (1+\nu_m)\frac{\rho_c}{\rho_m}\beta_m - \beta_1\nu_{12}$$

Coefficients of Moisture Expansion

$$\beta_1 = 0, \text{ and} \qquad \beta_2 = (1+\nu_m)\frac{\rho_c}{\rho_m}\beta_m$$

$$\downarrow$$

$$F_1 = \sigma_1 A_c = 0 = \sigma_f A_f + \sigma_m A_m, \text{ and} \qquad \sigma_f V_f + \sigma_m V_m = 0$$

$$\downarrow$$

$$\sigma_f = E_f(\varepsilon_f - \beta_f \Delta C_f), \text{and} \qquad \sigma_m = E_m(\varepsilon_m - \beta_m \Delta C_m)$$

$$\downarrow$$

$$\varepsilon_f = \frac{\beta_f \Delta C_f V_f E_f + \beta_m \Delta C_m V_m E_m}{E_f V_f + E_m V_m}$$

Coefficients of Moisture Expansion

$$\varepsilon_1 = \beta_1 \Delta C_c$$

$$\therefore \beta_1 = \frac{\beta_f \ \Delta C_f V_f E_f + \beta_m \Delta C_m V_m E_m}{(E_f V_f + E_m V_m) \Delta C_c}$$

$$\Delta C_c \, w_c = \Delta C_f \, w_f + \Delta C_m w_m$$

$$\therefore \Delta C_c = \Delta C_f W_f + \Delta C_m W_m$$

$$\beta_1 = \frac{\beta_f \ \Delta C_f V_f E_f + \beta_m \Delta C_m V_m E_m}{(E_f V_f + E_m V_m)(\Delta C_f W_f + \Delta C_m W_m)}$$

$$\therefore \beta_1 = \frac{\beta_f \Delta C_f V_f E_f \ + \beta_m \Delta C_m V_m E_m}{E_1 (\Delta C_f \rho_f V_f + \Delta C_m \rho_m \ V_m)} \rho_c$$

Example

Example 3.19

Find the two coefficients of moisture expansion for a Glass/Epoxy lamina with 70% fiber volume fraction. Use properties for glass and epoxy from Tables 3.1 and 3.2, respectively. Assume glass does not absorb moisture.

Example

Example 3.19

$\rho_f = 2500\ kg/m^3$

$\rho_m = 1200\ kg/m^3$

$\beta_m = 0.33\ m/m/kg/kg$

$E_m = 3.4\ GPa$

$\nu_m = 0.3$

$\rho_c = 2110\ kg/m^3$

$E_1 = 60.52\ GPa$

$\nu_{12} = 0.230$

Example

Example 3.19

$$\beta_1 = \frac{3.4\times10^9}{60.52\times10^9}\,\frac{2110}{1200}\,(0.33)$$

$$= 0.3260\times10^{-1}\ m/m/kg/kg$$

Example

Example 3.19

$$\beta_2 = (1+0.3)\frac{2110}{1200}(0.33) - (0.3260\times 10^{-1})(0.230)$$

$$= 0.7468\ m/m/kg/kg$$

END