EML 4230 Introduction to Composite Materials

Chapter 4 Macromechanical Analysis of a Laminate
Classical Lamination Theory

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Courtesy of the Textbook
Mechanics of Composite Materials by Kaw

Laminate Stacking Sequence

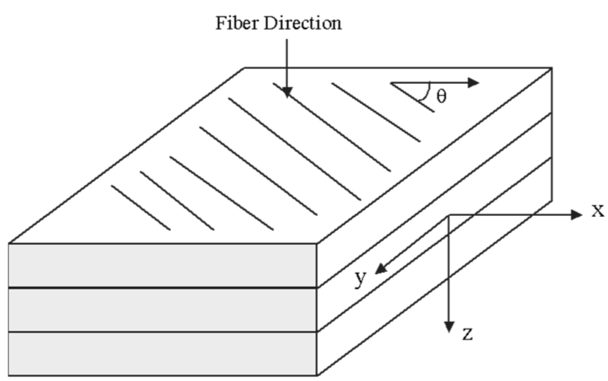


FIGURE 4.1
Schematic of a lamina

Laminate Behavior

- Elastic Moduli
- The Stacking Position
- Thickness
- Angles of Orientation
- Coefficients of Thermal Expansion
- Coefficients of Moisture Expansion

Strains in a

$$\sigma_{xx} = \frac{P}{A} \qquad (4.1)$$

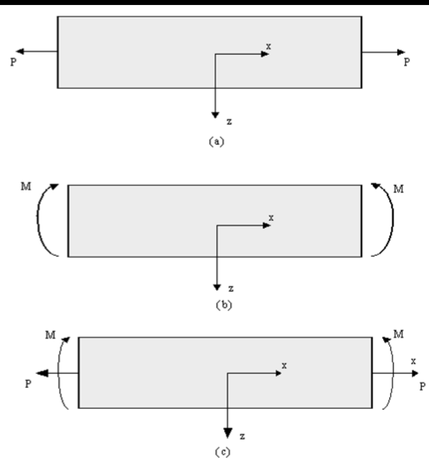


FIGURE 4.2
A beam under (a) axial load, (b) bending moment, and (c) combined axial and bending moment.

$$\varepsilon_{xx} = \frac{P}{AE}$$

$$\sigma_{xx} = \frac{Mz}{I}$$

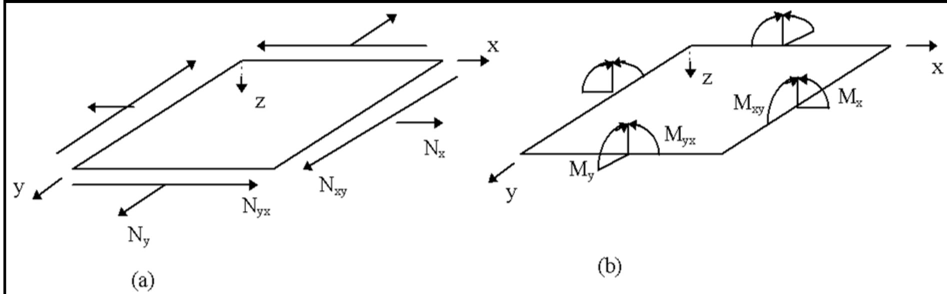
$$\varepsilon_{xx} = \frac{z}{\rho}$$

$$\varepsilon_{xx} = \left(\frac{1}{AE}\right)P + \left(\frac{z}{EI}\right)M$$

$$= \varepsilon_0 + z\left(\frac{1}{\rho}\right)$$

$$= \varepsilon_0 + z\kappa$$

Types of loads allowed in CLT analysis



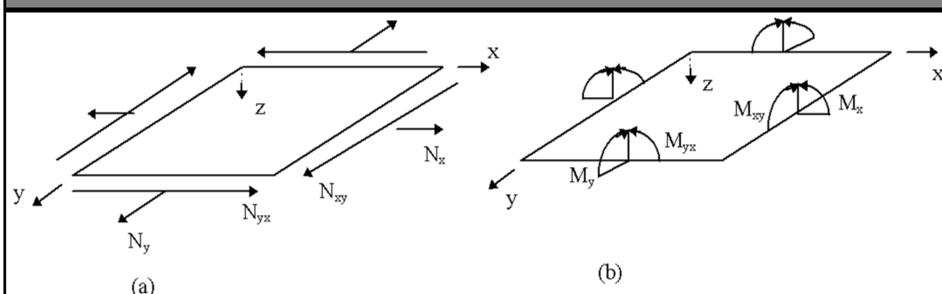
N_x = normal force resultant in the x direction (per unit length)

N_y = normal force resultant in the y direction (per unit length)

N_{xy} = shear force resultant (per unit length)

FIGURE 4.3
Resultant forces and moments on a laminate.

Figure 4.3



M_x = bending moment resultant in the yz plane (per unit length)

M_y = bending moment resultant in the xz plane (per unit length)

M_{xy} = twisting moment resultant (per unit length)

Classical Lamination Theory

- Each lamina is orthotropic.
- Each lamina is homogeneous.
- A line straight and perpendicular to the middle surface remains straight and perpendicular to the middle surface during deformation. $(\gamma_{xz} = \gamma_{yz} = 0)$.
- The laminate is thin and is loaded only in its plane (plane stress) $(\sigma_z = \tau_{xz} = \tau_{yz} = 0)$.
- Displacements are continuous and small throughout the laminate $(|u|, |v|, |w| << |h|)$, where *h* is the laminate thickness.
- Each lamina is elastic.
- No slip occurs between the lamina interfaces.

Figure 4.4

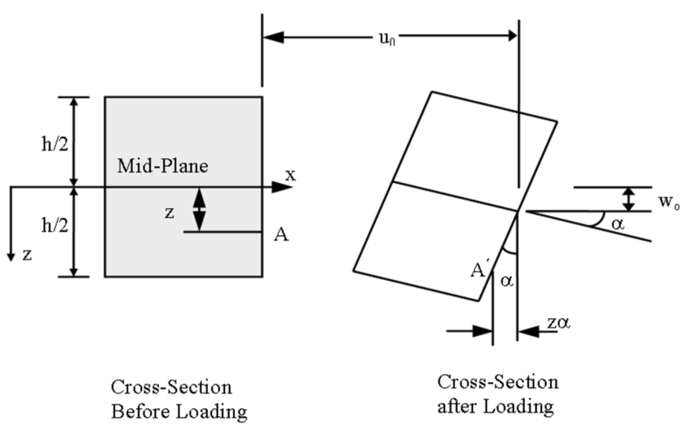


FIGURE 4.4
Figure showing the relationship between displacements through the thickness of a plate to midplane displacements and curvatures.

Global Strains in a Laminate

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}.$$

Figure 4.5

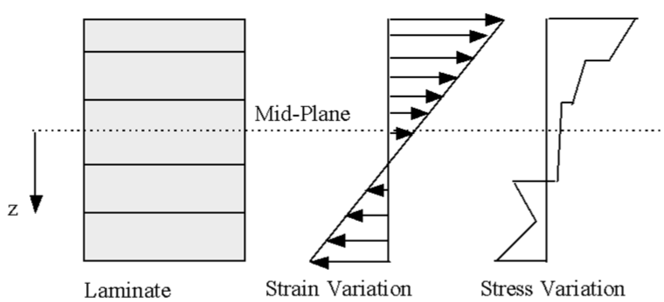


FIGURE 4.5
Strain and stress variation through the thickness of the laminate.

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