EML 4230 Introduction to Composite Materials

Chapter 4 Macromechanical Analysis of a Laminate
Classical Lamination Theory

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Courtesy of the Textbook
Mechanics of Composite Materials by Kaw

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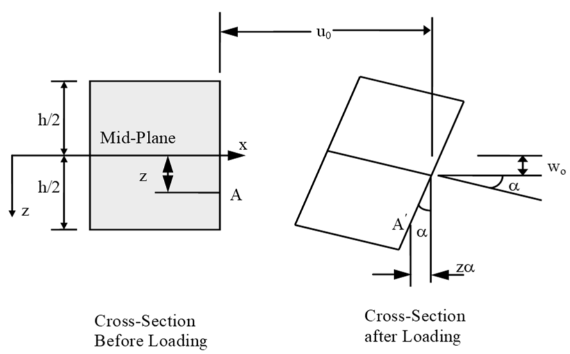
1

Classical Lamination Theory

- Each lamina is orthotropic.
- Each lamina is homogeneous.
- A line straight and perpendicular to the middle surface remains straight and perpendicular to the middle surface during deformation. $(\gamma_{xz} = \gamma_{yz} = 0)$.
- The laminate is thin and is loaded only in its plane (plane stress) $(\sigma_z = \tau_{xz} = \tau_{yz} = 0)$.
- Displacements are continuous and small throughout the laminate $(|u|,|v|,|w| << |h|)$, where *h* is the laminate thickness.
- Each lamina is elastic.
- No slip occurs between the lamina interfaces.

2

Midplane strains and curvatures



Relationship between displacements through the thickness of a plate to midplane displacements and curvatures.

3

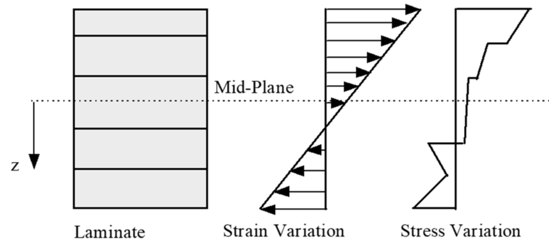
Global Strains in a Laminate

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}.$$

4

How do stresses and strains vary through the thickness

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}.$$



Strain and stress variation through the thickness of the laminate.

5

END

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