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### **Stacking Sequence**



Stresses in a Lamina in a Laminate  $\begin{bmatrix} \sigma_{x} \\ \sigma_{y} \\ \tau_{xy} \end{bmatrix}_{k} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{k} \begin{bmatrix} \varepsilon_{x} \\ \varepsilon_{y} \\ \gamma_{xy} \end{bmatrix}_{k}$   $= \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{k} \begin{bmatrix} \varepsilon_{x} \\ \varepsilon_{y} \\ \gamma_{xy} \end{bmatrix}_{k}$   $= \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{k} \begin{bmatrix} \varepsilon_{x} \\ \varepsilon_{y} \\ \varepsilon_{y} \\ \gamma_{xy} \end{bmatrix} + z \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{k} \begin{bmatrix} \kappa_{x} \\ \kappa_{y} \\ \kappa_{xy} \end{bmatrix}.$ 

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Forces & Midplane Strains/Curvatures  $\begin{bmatrix} N_{x} \\ N_{y} \\ N_{xy} \end{bmatrix} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_{k}} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{k} \begin{bmatrix} \varepsilon_{x}^{0} \\ \varepsilon_{y}^{0} \\ y_{xy}^{0} \end{bmatrix} dz$   $+ \sum_{k=1}^{n} \int_{h_{k-1}}^{h_{k}} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{k} \begin{bmatrix} \kappa_{x} \\ \kappa_{y} \\ \kappa_{xy} \end{bmatrix} z dz$  Forces & Midplane Strains/Curvatures

$$\begin{bmatrix} N_{x} \\ N_{y} \\ N_{xy} \end{bmatrix} = \begin{cases} \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{k}^{h_{k}} dz \\ + \begin{cases} \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{k}^{h_{k}} z dz \\ & & & & & \\ Kx \\ Ky \\ Kxy \end{bmatrix} \end{cases} \begin{bmatrix} \kappa_{x} \\ \kappa_{y} \\ \kappa_{xy} \end{bmatrix}$$

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Forces & Midplane Strains/Curvatures  $\begin{bmatrix} N_{x} \\ N_{y} \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{x}^{0} \\ \varepsilon_{y}^{0} \\ \gamma_{xy}^{0} \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_{x} \\ \kappa_{y} \\ \kappa_{xy} \end{bmatrix}$   $A_{ij} = \sum_{k=1}^{n} [(\overline{Q}_{ij})]_{k} (h_{k} - h_{k-1}), i = 1,2,6; j = 1,2,6,$   $B_{ij} = \frac{1}{2} \sum_{k=1}^{n} [(\overline{Q}_{ij})]_{k} (h_{k}^{2} - h_{k-1}^{2}), i = 1,2,6; j = 1,2,6$ 

## Integrating terms

$$\int_{h_{k-1}}^{h_k} dz = (h_k - h_{k-1}),$$
  
 $\int_{h_{k-1}}^{h_k} z dz = rac{1}{2} (h_k^2 - h_{k-1}^2),$ 

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#### **Stiffness Matrices**

[A] – Extensional stiffness matrix relating the resultant inplane forces to the in-plane strains.

[B] – Coupling stiffness matrix coupling the force and moment terms to the midplane strains and midplane curvatures.

Stresses in a Lamina in a Laminate

$$\begin{bmatrix} \sigma_{x} \\ \sigma_{y} \\ \tau_{xy} \end{bmatrix}_{k} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{k} \begin{bmatrix} \varepsilon_{x} \\ \varepsilon_{y} \\ \gamma_{xy} \end{bmatrix}_{k}$$
$$= \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{k} \begin{bmatrix} \varepsilon_{x}^{0} \\ \varepsilon_{y}^{0} \\ \gamma_{xy}^{0} \end{bmatrix} + z \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{k} \begin{bmatrix} \kappa_{x} \\ \kappa_{y} \\ \kappa_{y} \\ \kappa_{xy} \end{bmatrix}.$$

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Moments & Midplane Strains/Curvatures  $\begin{bmatrix} M_{x} \\ M_{y} \\ M_{xy} \end{bmatrix} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_{k}} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{k} \begin{bmatrix} \varepsilon_{x}^{0} \\ \varepsilon_{y}^{0} \\ y_{xy}^{0} \end{bmatrix} z dz$   $+ \sum_{k=1}^{n} \int_{h_{k-1}}^{h_{k}} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{k} \begin{bmatrix} \kappa_{x} \\ \kappa_{y} \\ \kappa_{xy} \end{bmatrix} z^{2} dz$ 



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#### Moments & Midplane Strains/Curvatures

$$\begin{bmatrix} M_{x} \\ M_{y} \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{x}^{0} \\ \varepsilon_{y}^{0} \\ \gamma_{xy}^{0} \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_{x} \\ \kappa_{y} \\ \kappa_{xy} \end{bmatrix}$$
$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} [(\overline{Q}_{ij})]_{k} (h_{k}^{3} - h_{k-1}^{3}), \qquad i = 1, 2, 6; j = 1, 2, 6.$$
$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} [(\overline{Q}_{ij})]_{k} (h_{k}^{2} - h_{k-1}^{2}), \qquad i = 1, 2, 6; j = 1, 2, 6$$

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Forces, Moments, Midplane Strains, Midplane Curvatures

$$\begin{bmatrix} N_x \\ N_y \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y^0 \\ \varepsilon_y^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

# Stiffness Matrices

[A] – Extensional stiffness matrix relating the resultant inplane forces to the in-plane strains.

[B] – Coupling stiffness matrix coupling the force and moment terms to the midplane strains and midplane curvatures.

[D] – Bending stiffness matrix relating the resultant bending moments to the plate curvatures.

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