EML 4230 Introduction to Composite Materials

Chapter 4 Macromechanical Analysis of a Laminate
Laminate Analysis Steps

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Courtesy of the Textbook
Mechanics of Composite Materials by Kaw

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Laminate Stacking Sequence

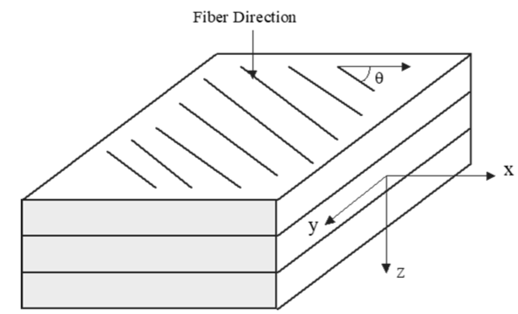

FIGURE 4.1
Schematic of a lamina

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Forces, Moments, Strains, Curvatures

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

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Steps

1. Find the value of the reduced stiffness matrix [Q] for each ply using its four elastic moduli, E_1, E_2, ν_{12}, G_{12} in Equation (2.93).
2. Find the value of the transformed reduced stiffness matrix $[\bar{Q}]$ for each ply using the [Q] matrix calculated in Step 1 and the angle of the ply in Equation (2.104) or Equations (2.137) and (2.138).
3. Knowing the thickness, t_k of each ply, find the coordinate of the top and bottom surface, h_i, $i = 1, \ldots\ldots, n$ of each ply using Equation (4.20).
4. Use the$[\bar{Q}]$ matrices from Step 2 and the location of each ply from Step 3 to find the three stiffness matrices [A], [B] and [D] from Equation (4.28).
5. Substitute the stiffness matrix values found in Step 4 and the applied forces and moments in Equation (4.29).

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Steps

6. Solve the six simultaneous Equations (4.29) to find the mid-plane strains and curvatures.
7. Knowing the location of each ply, find the global strains in each ply using Equation (4.16).
8. For finding the global stresses, use the stress-strain Equation (2.103).
9. For finding the local strains, use the transformation Equation (2.99).
10. For finding the local stresses, use the transformation Equation (2.94).

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Step 1: Find reduced stiffness matrix for each ply

Step 1: Find the reduced stiffness matrix [Q] for each ply

$$Q_{11} = \frac{E_1}{1 - \nu_{21}\ \nu_{12}} \qquad Q_{12} = \frac{\nu_{12}\ E_2}{1 - \nu_{21}\nu_{12}}$$

$$Q_{22} = \frac{E_2}{1 - \nu_{21}\ \nu_{12}} \qquad Q_{66} = G_{12}$$

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Step 2: Find transformed reduced stiffness for each ply

Step 2: Find the transformed stiffness matrix $[\bar{Q}]$ using the reduced stiffness matrix [Q] and the angle of the ply.

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \\
\bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(c^4 + s^4) \\
\bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})c^3s - (Q_{22} - Q_{12} - 2Q_{66})s^3c \\
\bar{Q}_{22} &= Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \\
\bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})cs^3 - (Q_{22} - Q_{12} - 2Q_{66})c^3s \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(s^4 + c^4)
\end{aligned}$$

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Step 3: Find co-ordinates of top and bottom surface of each ply

Step 3: Find the coordinate of the top and bottom surface of each ply.

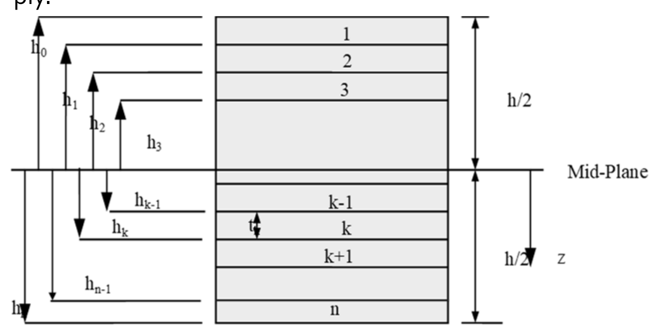

FIGURE 4.6
Coordinate locations of plies in the laminate.

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Step 4: Find the three stiffness matrices

Step 4: Find three stiffness matrices [A], [B], and [D]

$$A_{ij} = \sum_{k=1}^{n} [(\overline{Q}_{ij})]_k \; (h_k - h_{k-1}), \; i = 1,2,6; \; j = 1,2,6$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} [(\overline{Q}_{ij})]_k \; (h_k^2 - h_{k-1}^2), \; i = 1,2,6; \; j = 1,2,6$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} [(\overline{Q}_{ij})]_k \; (h_k^3 - h_{k-1}^3), \; i = 1, 2, 6; \; j = 1, 2, 6$$

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Step 5: Set up the six equations – six unknowns

Step 5: Substitute the three stiffness matrices [A], [B], and [D] and the applied forces and moments.

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

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Step 6: Solve the equations for midplane strains and curvatures

Step 6: Solve the six simultaneous equations to find the midplane strains and curvatures.

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

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Step 7: Find the global strains

Step 7: Find the global strains in each ply.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

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Step 8: Find the global stresses

Step 8: Find the global stresses using the stress-strain equation.

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

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Step 9: Find the local strains

Step 9: Find the local strains using the transformation equation.

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}/2 \end{bmatrix} \qquad \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{bmatrix} = [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}/2 \end{bmatrix}$$

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix} \qquad \begin{aligned} c &= \cos(\theta) \\ s &= \sin(\theta) \end{aligned}$$

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Step 10: Find the local stresses

Step 10: Find the local stresses using the transformation equation.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \qquad \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix} \qquad \begin{aligned} c &= \cos(\theta) \\ s &= \sin(\theta) \end{aligned}$$

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END

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