EML 4230 Introduction to Composite Materials

Chapter 4 Macromechanical Analysis of a Laminate
Laminate Modulus: Example

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Courtesy of the Textbook
Mechanics of Composite Materials by Kaw

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Q bars and co-ordinates

Find the in-plane and flexural stiffness constants for a three-ply [0 / $\overline{90}$ / 0] graphite/epoxy laminate. Use the unidirectional properties of graphite/epoxy from Table 2.1. Each lamina is 5 mm thick.

$$[\bar{Q}]_0 = \begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9)\ Pa$$

$$[\bar{Q}]_{90} = \begin{bmatrix} 10.35 & 2.897 & 0 \\ 2.897 & 181.8 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9)\ Pa$$

$$h_0 = -0.0075\ m$$
$$h_1 = -0.0025\ m$$
$$h_2 = 0.0025\ m$$
$$h_3 = 0.0075\ m$$

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The extensional stiffness matrix, A

$$A_{ij} = \sum_{k=1}^{3} [\bar{Q}_{ij}]_k (h_k - h_{k-1})$$

$$[A] = \begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9)[-0.0025 - (-0.0075)]$$

$$+ \begin{bmatrix} 10.35 & 2.897 & 0 \\ 2.897 & 181.8 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9)[0.0025 - (-0.0025)]$$

$$+ \begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9)[0.0075 - 0.0025]$$

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The extensional compliance matrix, A*

$$[A] = \begin{bmatrix} 1.870\times10^9 & 4.345\times10^7 & 0 \\ 4.345\times10^7 & 1.013\times10^9 & 0 \\ 0 & 0 & 1.076\times10^8 \end{bmatrix} Pa\text{-}m$$

$$[A^*] = \begin{bmatrix} 5.353\times10^{-10} & -2.297\times10^{-11} & 0 \\ -2.297\times10^{-11} & 9.886\times10^{-10} & 0 \\ 0 & 0 & 9.298\times10^{-9} \end{bmatrix} \frac{1}{Pa\text{-}m}$$

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In-plane Elastic moduli

$$[A^*] = \begin{bmatrix} 5.353\times10^{10} & -2.297\times10^{11} & 0 \\ -2.297\times10^{11} & 9.886\times10^{10} & 0 \\ 0 & 0 & 9.298\times10^{9} \end{bmatrix} \frac{1}{Pa\text{-}m}$$

$$E_x = \frac{1}{hA_{11}^*} = \frac{1}{(0.015)(5.353\times10^{-10})} = 124.5\ GPa$$

$$E_y = \frac{1}{hA_{22}^*} = \frac{1}{(0.015)(9.886\times10^{-10})} = 67.43\ GPa$$

$$G_{xy} = \frac{1}{hA_{66}^*} = \frac{1}{(0.015)(9.289\times10^{-9})} = 7.17\ GPa$$

$$\nu_{xy} = -\frac{A_{12}^*}{A_{11}^*} = -\frac{-2.297\times10^{-11}}{5.353\times10^{-10}} = 0.04292$$

$$\nu_{yx} = -\frac{A_{12}^*}{A_{22}^*} = -\frac{-2.297\times10^{-11}}{9.886\times10^{-10}} = 0.02323$$

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Bending stiffness matrix, D

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{3}[\bar{Q}_{ij}]_k (h_k^3 - h_{k-1}^3)$$

$$[D] = \frac{1}{3}\begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix}(10^9)[(-0.0025)^3 - (-0.0075)^3]$$

$$+\frac{1}{3}\begin{bmatrix} 10.35 & 2.897 & 0 \\ 2.897 & 181.8 & 0 \\ 0 & 0 & 7.17 \end{bmatrix}(10^9)[(0.0025)^3 - (-0.0025)^3]$$

$$+\frac{1}{3}\begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix}(10^9)[(0.0075)^3 - (0.0025)^3]$$

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Bending compliance Matrix, D*

$$[D] = \begin{bmatrix} 4.935\times10^{4} & 8.148\times10^{2} & 0 \\ 8.148\times10^{2} & 4.696\times10^{3} & 0 \\ 0 & 0 & 2.017\times10^{3} \end{bmatrix} Pa\text{-}m^3$$

$$[D^*] = \begin{bmatrix} 2.032\times10^{-5} & -3.526\times10^{-6} & 0 \\ -3.526\times10^{-6} & 2.136\times10^{-4} & 0 \\ 0 & 0 & 4.959\times10^{-4} \end{bmatrix} \frac{1}{Pa\text{-}m^3}$$

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Flexural Laminate Moduli

$$[D^*] = \begin{bmatrix} 2.032\times10^{-5} & -3.526\times10^{-6} & 0 \\ -3.526\times10^{-6} & 2.136\times10^{-4} & 0 \\ 0 & 0 & 4.959\times10^{-4} \end{bmatrix} \frac{1}{Pa\text{-}m^3}$$

$$E_x^f = \frac{12}{h^3 D_{11}^*} = \frac{12}{(0.015)^3(2.032\times10^{-5})} = 175.0\ GPa$$

$$E_y^f = \frac{12}{h^3 D_{22}^*} = \frac{12}{(0.015)^3(2.136\times10^{-4})} = 16.65\ GPa$$

$$G_{xy}^f = \frac{12}{h^3 D_{66}^*} = \frac{12}{(0.015)^3(4.959\times10^{-4})} = 7.17\ GPa$$

$$\nu_{xy}^f = -\frac{D_{12}^*}{D_{11}^*} = -\frac{-3.526\times10^{-6}}{2.032\times10^{-5}} = 0.1735$$

$$\nu_{yx}^f = -\frac{D_{12}^*}{D_{22}^*} = -\frac{-3.526\times10^{-6}}{2.136\times10^{-4}} = 0.01651$$

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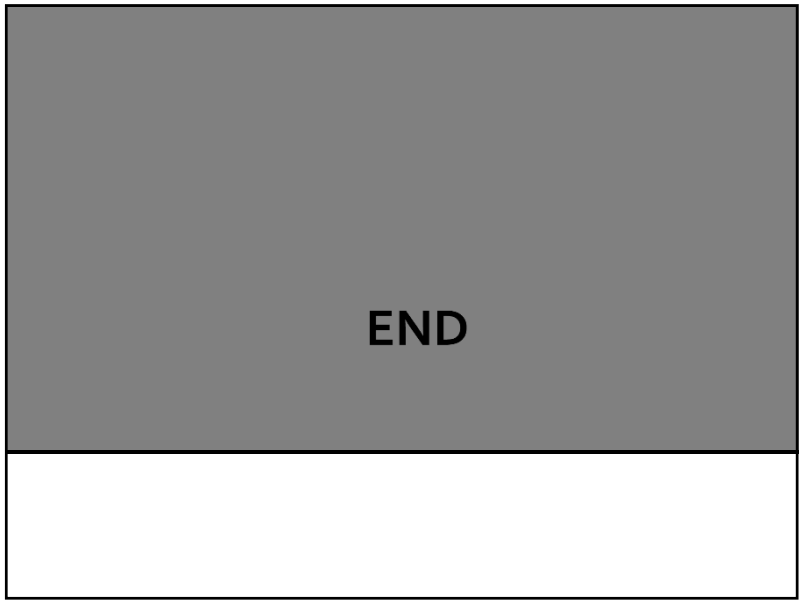

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