EML 4230 Introduction to Composite Materials

Chapter 4 Macromechanical Analysis of a Laminate
Laminate Modulus: Example

Dr. Autar Kaw
Department of Mechanical Engineering
University of South Florida, Tampa, FL 33620

Courtesy of the Textbook
Mechanics of Composite Materials by Kaw

USF UNIVERSITY OF SOUTH FLORIDA

Example 4.4

$$[\overline{Q}]_0 = \begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9)\ Pa$$

$$[\overline{Q}]_{90} = \begin{bmatrix} 10.35 & 2.897 & 0 \\ 2.897 & 181.8 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9)\ Pa$$

$$h_0 = -0.0075\ m$$

$$h_1 = -0.0025\ m$$

$$h_2 = 0.0025\ m$$

$$h_3 = 0.0075\ m$$

Example 4.4

$$A_{ij} = \sum_{k=1}^{3} [\overline{Q}_{ij}]_k (h_k - h_{k-1})$$

$$[A] = \begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9)[-0.0025 - (-0.0075)]$$

$$+ \begin{bmatrix} 10.35 & 2.897 & 0 \\ 2.897 & 181.8 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9)[0.0025 - (-0.0025)]$$

$$+ \begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9)[0.0075 - 0.0025]$$

Example 4.4

$$[A] = \begin{bmatrix} 1.870 \times 10^{9} & 4.345 \times 10^{7} & 0 \\ 4.345 \times 10^{7} & 1.013 \times 10^{9} & 0 \\ 0 & 0 & 1.076 \times 10^{8} \end{bmatrix} Pa - m$$

$$[A^{*}] = \begin{bmatrix} 5.353 \times 10^{-10} & -2.297 \times 10^{-11} & 0 \\ -2.297 \times 10^{-11} & 9.886 \times 10^{-10} & 0 \\ 0 & 0 & 9.298 \times 10^{-9} \end{bmatrix} \frac{1}{Pa - m}$$

Example 4.4

$$E_x = \frac{1}{hA_{11}^*} = \frac{1}{(0.015)(5.353 \times 10^{-10})} = 124.5\ GPa$$

$$E_y = \frac{1}{hA_{22}^*} = \frac{1}{(0.015)(9.886 \times 10^{-10})} = 67.43\ GPa$$

$$G_{xy} = \frac{1}{hA_{66}^*} = \frac{1}{(0.015)(9.289 \times 10^{-9})} = 7.17\ GPa$$

$$\nu_{xy} = -\frac{A_{12}^*}{A_{11}^*} = -\frac{-2.297 \times 10^{-11}}{5.353 \times 10^{-10}} = 0.04292$$

$$\nu_{yx} = -\frac{A_{12}^*}{A_{22}^*} = -\frac{-2.297 \times 10^{-11}}{9.886 \times 10^{-10}} = 0.02323$$

Example 4.4

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{3}[\overline{Q}_{ij}]_k (h_k^3 - h_{k-1}^3)$$

$$[D] = \frac{1}{3}\begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix}(10^9)[(-0.0025)^3 - (-0.0075)^3]$$

$$+\frac{1}{3}\begin{bmatrix} 10.35 & 2.897 & 0 \\ 2.897 & 181.8 & 0 \\ 0 & 0 & 7.17 \end{bmatrix}(10^9)[(0.0025)^3 - (-0.0025)^3]$$

$$+\frac{1}{3}\begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix}(10^9)[(0.0075)^3 - (0.0025)^3]$$

Example 4.4

$$[D] = \begin{bmatrix} 4.935\times 10^{4} & 8.148\times 10^{2} & 0 \\ 8.148\times 10^{2} & 4.696\times 10^{3} & 0 \\ 0 & 0 & 2.017\times 10^{3} \end{bmatrix} Pa - m^{3}$$

$$[D^{*}] = \begin{bmatrix} 2.032\times 10^{-5} & -3.526\times 10^{-6} & 0 \\ -3.526\times 10^{-6} & 2.136\times 10^{-4} & 0 \\ 0 & 0 & 4.959\times 10^{-4} \end{bmatrix} \frac{1}{Pa - m^{3}}$$

Example 4.4

$$E_x^f = \frac{12}{h^3 D_{11}^*} = \frac{12}{(0.015)^3 (2.032 \times 10^{-5})} = 175.0\ GPa$$

$$E_y^f = \frac{12}{h^3 D_{22}^*} = \frac{12}{(0.015)^3 (2.136 \times 10^{-4})} = 16.65\ GPa$$

$$G_{xy}^f = \frac{12}{h^3 D_{66}^*} = \frac{12}{(0.015)^3 (4.959 \times 10^{-4})} = 7.17\ GPa$$

$$\nu_{xy}^f = -\frac{D_{12}^*}{D_{11}^*} = -\frac{-3.526 \times 10^{-6}}{2.032 \times 10^{-5}} = 0.1735$$

$$\nu_{yx}^f = -\frac{D_{12}^*}{D_{22}^*} = -\frac{-3.526 \times 10^{-6}}{2.136 \times 10^{-4}} = 0.01651$$

END