EML 4230 Introduction to Composite Materials

Chapter 4 Macromechanical Analysis of a Laminate

Hygrothermal Loads

Dr. Autar Kaw
Department of Mechanical Engineering
University of South Florida, Tampa, FL 33620

Courtesy of the Textbook

Mechanics of Composite Materials by Kaw

USF UNIVERSITY OF SOUTH FLORIDA

Mechanical Strains

$$\begin{bmatrix} \varepsilon_x^M \\ \varepsilon_y^M \\ \gamma_{xy}^M \end{bmatrix}_k = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_k - \begin{bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \end{bmatrix}_k = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_k - \begin{bmatrix} \alpha_x \Delta T \\ \alpha_y \Delta T \\ \alpha_{xy} \Delta T \end{bmatrix}_k$$

Hygrothermal Stresses

$$\begin{bmatrix} \sigma_x^T \\ \sigma_y^T \\ \gamma_{xy}^T \end{bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \varepsilon_x^M \\ \varepsilon_y^M \\ \gamma_{xy}^M \end{bmatrix}_k$$

Zero resultant stresses

$$\int_{-h/2}^{h/2} \begin{bmatrix} \sigma_x^T \\ \sigma_y^T \\ \tau_{xy}^T \end{bmatrix} dz = 0$$

$$\sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \sigma_x^T \\ \sigma_y^T \\ \tau_{xy}^T \end{bmatrix}_k dz = 0$$

Zero resultant stresses

$$\sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \varepsilon_x^M \\ \varepsilon_y^M \\ \gamma_{xy}^M \end{bmatrix}_k dz = 0$$

Deriving final formula

$$\sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left(\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_k - \begin{bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \end{bmatrix}_k \right) dz = 0$$

$$\sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left(\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} - \begin{bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \end{bmatrix}_k \right) dz = 0$$

Deriving final formula

$$\sum_{k=1}^{n}\int_{h_{k-1}}^{h_k}\begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}_k\left(\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{Bmatrix}\right)dz=$$

$$\sum_{k=1}^{n}\int_{h_{k-1}}^{h_k}\begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}_k\left(\begin{bmatrix}\varepsilon_x^T\\ \varepsilon_y^T\\ \gamma_{xy}^T\end{bmatrix}\right)dz$$

Deriving final formula

$$\sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left(\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \right) dz =$$

$$\sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left(\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k \Delta T \right) dz$$

Deriving final formula

$$\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix}$$

$$[\ N^T\ ] = \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \Delta T \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k (h_k - h_{k-1})$$

Other three equations – from zero resultant moments

$$\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix}$$

$$[\ M^T\ ] = \begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} = \frac{1}{2} \Delta T \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k (h_k^2 - h_{k-1}^2)$$

Final formula

$$\left[\frac{N^T}{M^T}\right] = \left[\frac{A\,|\,B}{B\,|\,D}\right]\left[\frac{\varepsilon^0}{\kappa}\right]$$

$$[\ N^T\ ] = \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \Delta T \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k (h_k - h_{k-1})$$

$$[\ M^T\ ] = \begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} = \frac{1}{2} \Delta T \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k (h_k^2 - h_{k-1}^2)$$

Final formula

$$\begin{bmatrix} \varepsilon_x^M \\ \varepsilon_y^M \\ \gamma_{xy}^M \end{bmatrix}_k = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_k - \begin{bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \end{bmatrix}_k$$

$$= \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_k - \begin{bmatrix} \alpha_x \Delta T \\ \alpha_y \Delta T \\ \alpha_{xy} \Delta T \end{bmatrix}_k$$

Example 4.5

Calculate the residual stresses at the bottom surface of the 90^{o} ply in a two ply [0/90] Graphite/Epoxy laminate subjected to a temperature change of -75^{o}C. Use the unidirectional properties of Graphite/Epoxy lamina from Table 2.1. Each lamina is 5 *mm* thick.

Example 4.5

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_{12} \end{bmatrix} = \begin{bmatrix} 0.200 \times 10^{-7} \\ 0.225 \times 10^{-4} \\ 0 \end{bmatrix} m/m/^oC$$

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_{0^o} = \begin{bmatrix} 0.200 \times 10^{-7} \\ 0.225 \times 10^{-4} \\ 0 \end{bmatrix} m/m/^oC$$

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_{90^o} = \begin{bmatrix} 0.225 \times 10^{-4} \\ 0.200 \times 10^{-7} \\ 0 \end{bmatrix} m/m/^oC$$

Example 4.5

$$[\overline{Q}]_0 = \begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} GPa$$

$$[\overline{Q}]_{90} = \begin{bmatrix} 10.35 & 2.897 & 0 \\ 2.897 & 181.8 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} GPa$$

Example 4.5

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = +(-75)\begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix}(10^9)\begin{bmatrix} 0.200\ (10^{-7}) \\ 0.225\ (10^{-4}) \\ 0 \end{bmatrix}[0.000 - (-0.005)]$$

$$+(-75)\begin{bmatrix} 10.35 & 2.897 & 0 \\ 2.897 & 181.8 & 0 \\ 0 & 0 & 7.17 \end{bmatrix}(10^9)\begin{bmatrix} 0.225\ (10^{4}) \\ 0.200\ (10^{7}) \\ 0 \end{bmatrix}[0.005 - 0.000]$$

$$= \begin{bmatrix} -1.131 \times 10^5 \\ -1.131 \times 10^5 \\ 0 \end{bmatrix} Pa\text{ - }m.$$

Example 4.5

$$\begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} = \frac{1}{2}(-75)\begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix}(10^9)\begin{bmatrix} 0.200 \times 10^{-7} \\ 0.225 \times 10^{-4} \\ 0 \end{bmatrix}[(0.000)^2 - (-0.005)^2]$$

$$+ \frac{1}{2}(-75)\begin{bmatrix} 10.35 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix}(10^9)\begin{bmatrix} 0.225 \times 10^{-4} \\ 0.200 \times 10^{-7} \\ 0 \end{bmatrix}[(0.005)^2 - (0.000)^2]$$

$$= \begin{bmatrix} -1.538 \times 10^2 \\ 1.538 \times 10^2 \\ 0 \end{bmatrix} Pa \ m$$

Example 4.5

$$[A] = \begin{bmatrix} 9.608\times10^8 & 2.897\times10^7 & 0 \\ 2.897\times10^7 & 9.608\times10^8 & 0 \\ 0 & 0 & 7.170\times10^7 \end{bmatrix} Pa\text{-}m$$

$$[B] = \begin{bmatrix} -2.143\times10^6 & 0 & 0 \\ 0 & 2.143\times10^6 & 0 \\ 0 & 0 & 0 \end{bmatrix} Pa - m^2$$

$$[D] = \begin{bmatrix} 8.007\times10^3 & 2.414\times10^2 & 0 \\ 2.414\times10^2 & 8.007\times10^3 & 0 \\ 0 & 0 & 5.975\times10^2 \end{bmatrix} Pa - m^3$$

Example 4.5

$$\begin{bmatrix} -1.131\times10^{5} \\ -1.131\times10^{5} \\ 0 \\ -1.538\times10^{2} \\ 1.538\times10^{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 9.608\times10^{8} & 2.897\times10^{7} & 0 & -2.143\times10^{6} & 0 & 0 \\ 2.897\times10^{7} & 9.608\times10^{8} & 0 & 0 & 2.143\times10^{6} & 0 \\ 0 & 0 & 7.170\times10^{7} & 0 & 0 & 0 \\ -2.143\times10^{6} & 0 & 0 & 8.007\times10^{3} & 2.414\times10^{2} & 0 \\ 0 & 2.143\times10^{6} & 0 & 2.414\times10^{2} & 8.007\times10^{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 5.975\times10^{2} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \hline \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} -3.907\times10^{-4} \\ -3.907\times10^{-4} \\ 0 \\ \hline -1.276\times10^{-1} \\ 1.276\times10^{-1} \\ 0 \end{bmatrix} \begin{matrix} m/m \\ \\ 1/m \end{matrix}$$

$$\begin{bmatrix} N^T \\ \hline M^T \end{bmatrix} = \begin{bmatrix} A & B \\ \hline B & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \hline \kappa \end{bmatrix}$$

Example 4.5

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_{90^o} = \begin{bmatrix} -3.907\times10^{-4} \\ -3.907\times10^{-4} \\ 0 \end{bmatrix} + (0.005)\begin{bmatrix} -1.276\times10^{-1} \\ 1.276\times10^{-1} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1.029\times10^{-3} \\ 2.475\times10^{-4} \\ 0 \end{bmatrix} m/m$$

Example 4.5

$$\begin{bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \end{bmatrix} = \begin{bmatrix} 0.225 \times 10^{-4} \\ 0.200 \times 10^{-7} \\ 0 \end{bmatrix} (-75)$$

$$= \begin{bmatrix} -0.16875 \times 10^{-2} \\ -0.15000 \times 10^{-5} \\ 0 \end{bmatrix} m/m$$

Example 4.5

$$\begin{bmatrix} \varepsilon_x^M \\ \varepsilon_y^M \\ \gamma_{xy}^M \end{bmatrix} = \begin{bmatrix} -1.029\times10^{-3} \\ 2.475\times10^{-4} \\ 0 \end{bmatrix} - \begin{bmatrix} -0.16875\times10^{-2} \\ -0.1500\times10^{-5} \\ 0 \end{bmatrix} = \begin{bmatrix} 0.6585 \times10^{-3} \\ 0.2490 \times10^{-3} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{90^0} = \begin{bmatrix} 10.35 & 2.897 & 0 \\ 2.897 & 181.8 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9) \begin{bmatrix} 0.6585\times10^{-3} \\ 0.2490\times10^{-3} \\ 0 \end{bmatrix} = \begin{bmatrix} 7.535 \times10^{6} \\ 4.718\times10^{7} \\ 0 \end{bmatrix} Pa.$$

Example 4.5

Global Strains for Example 4.3

Ply #	Position	ε_x	ε_y	γ_{xy}
1 (0^0)	Top	2.475×10^{-4}	-1.029×10^{-3}	0.0
	Middle	-7.160×10^{-5}	-7.098×10^{-4}	0.0
	Bottom	-3.907×10^{-4}	-3.907×10^{-4}	0.0
2 (90^0)	Top	-3.907×10^{-4}	-3.907×10^{-4}	0.0
	Middle	-7.098×10^{-4}	-7.160×10^{-5}	0.0
	Bottom	-1.029×10^{-3}	2.475×10^{-4}	0.0

Example 4.5

Global Stresses for Example 4.3

Ply #	Position	σ_y	σ_y	τ_{xy}
$1(0^0)$	Top	4.718×10^7	7.535×10^6	0.0
	Middle	-9.912×10^6	9.912×10^6	0.0
	Bottom	-6.701×10^7	1.229×10^7	0.0
$2(90^0)$	Top	1.229×10^7	-6.701×10^7	0.0
	Middle	9.912×10^6	-9.912×10^6	0.0
	Bottom	7.535×10^6	4.718×10^7	0.0

Coefficients of Thermal and Moisture Expansion

To find coefficients of thermal and moisture expansion of laminates

- Symmetric laminates [B] = 0
- No bending occurs under thermal hygrothermal loads
- Assuming ΔT = 1 and ΔC = 0

$$\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} + \begin{bmatrix} N_x^C \\ N_y^C \\ N_{xy}^C \end{bmatrix}$$

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} \equiv \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}_{\substack{\Delta C = 0 \\ \Delta T = 1}} = \begin{bmatrix} A_{11}^* & A_{12}^* & A_{16}^* \\ A_{12}^* & A_{22}^* & A_{26}^* \\ A_{16}^* & A_{26}^* & A_{66}^* \end{bmatrix} \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix}$$

Coefficients of Thermal and Moisture Expansion

To find coefficients of thermal and moisture expansion of laminates

- Symmetric laminates [B] = 0
- No bending occurs under thermal hygrothermal loads
- Assuming ΔT = 0 and ΔC = 1

$$\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} + \begin{bmatrix} N_x^C \\ N_y^C \\ N_{xy}^C \end{bmatrix}$$

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} \equiv \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}_{\substack{\Delta T = 0 \\ \Delta C = 1}} = \begin{bmatrix} A_{11}^* & A_{12}^* & A_{16}^* \\ A_{12}^* & A_{22}^* & A_{26}^* \\ A_{16}^* & A_{26}^* & A_{66}^* \end{bmatrix} \begin{bmatrix} N_x^C \\ N_y^C \\ N_{xy}^C \end{bmatrix}$$

Example 4.6

Find the coefficients of thermal and moisture expansion of a $[0/\overline{90}]_S$ Graphite/Epoxy laminate. Use the properties of unidirectional Graphite/Epoxy lamina from table 2.1.

Example 4.6

$$[A^*] = \begin{bmatrix} 5.353\times10^{-10} & -2.297\times10^{-11} & 0 \\ -2.297\times10^{-11} & 9.886\times10^{-10} & 0 \\ 0 & 0 & 9.298\times10^{-9} \end{bmatrix} \frac{1}{Pa\text{-}m}$$

Example 4.6

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \Delta T \sum_{k=1}^{3} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k (h_k - h_{k-1})$$

$$= (1) \begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9) \begin{bmatrix} 0.200 \times 10^{-7} \\ 0.225 \times 10^{-4} \\ 0 \end{bmatrix} [-0.0025 - (-0.0075)]$$

$$+ (1) \begin{bmatrix} 10.35 & 2.897 & 0 \\ 2.897 & 181.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9) \begin{bmatrix} 0.225 \times 10^{-4} \\ 0.200 \times 10^{-7} \\ 0 \end{bmatrix} [0.0025 - (-0.0025)]$$

$$+ (1) \begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9) \begin{bmatrix} 0.200 \times 10^{-7} \\ 0.225 \times 10^{-4} \\ 0 \end{bmatrix} [0.0075 - 0.0025] = \begin{bmatrix} 1.852 \times 10^3 \\ 2.673 \times 10^3 \\ 0 \end{bmatrix} Pa - m$$

Example 4.6

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} 5.353\times10^{-10} & -2.297\times10^{-11} & 0 \\ -2.297\times10^{-11} & 9.886\times10^{-10} & 0 \\ 0 & 0 & 9.298\times10^{-9} \end{bmatrix} \begin{bmatrix} 1.852\times10^{3} \\ 2.673\times10^{3} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9.303\times10^{-7} \\ 2.600\times10^{-6} \\ 0 \end{bmatrix} m/m.$$

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} \equiv \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}_{\substack{\Delta C = 0 \\ \Delta T = 1}} = = \begin{bmatrix} 9.303\times10^{-7} \\ 2.600\times10^{-6} \\ 0 \end{bmatrix} m/m/^\circ C$$

Example 4.6

$$\begin{bmatrix} N_x^C \\ N_y^C \\ N_{xy}^C \end{bmatrix} = \Delta C \sum_{k=1}^{3} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} (h_k - h_{k-1})$$

ΔC = 1 kg/kg

$$= (1) \begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9) \begin{bmatrix} 0 \\ 0.6 \\ 0 \end{bmatrix} [-0.0025 - (-0.0075)]$$

$$+ (1) \begin{bmatrix} 10.35 & 2.897 & 0 \\ 2.897 & 181.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9) \begin{bmatrix} 0.6 \\ 0 \\ 0 \end{bmatrix} [0.0025 - (-0.0025)]$$

$$+ (1) \begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} (10^9) \begin{bmatrix} 0 \\ 0.6 \\ 0 \end{bmatrix} [0.0075 - 0.0025] = \begin{bmatrix} 4.842 \times 10^7 \\ 7.077 \times 10^7 \\ 0 \end{bmatrix} Pa - m$$

Example 4.6

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} 5.353\times10^{-10} & -2.297\times10^{-11} & 0 \\ -2.297\times10^{-11} & 9.886\times10^{-10} & 0 \\ 0 & 0 & 9.298\times10^{-9} \end{bmatrix} \begin{bmatrix} 4.842\times10^{7} \\ 7.077\times10^{7} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2.430\times10^{-2} \\ 6.885\times10^{-2} \\ 0 \end{bmatrix} m/m$$

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} \equiv \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}_{\substack{\Delta C = 1 \\ \Delta T = 0}} = \begin{bmatrix} 2.430\times10^{-2} \\ 6.885\times10^{-2} \\ 0 \end{bmatrix} m/m\,/\,kg\,/\,kg$$

END