EML 4230 Introduction to Composite Materials

Chapter 5 Design and Analysis of Laminates
Special Cases of Laminates

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Courtesy of the Textbook
Mechanics of Composite Materials by Kaw

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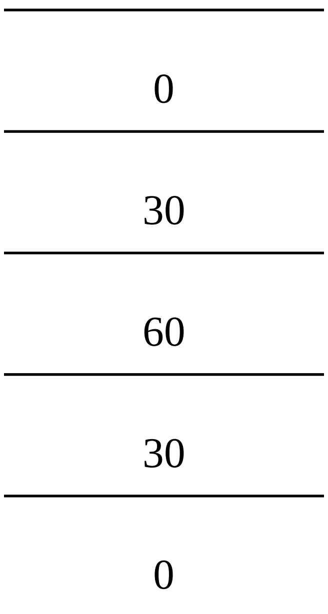
Symmetric Laminates



$$[0/30/\overline{60}]_S$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

Cross-Ply Laminates

0

90

90

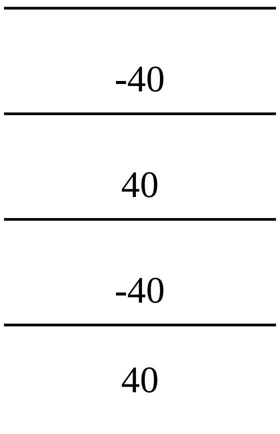
0

90

$$[0/90_2/0/90]$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

Angle Ply Laminates



$[-40/40/-40/40]$

If laminates consistes of
an even number of plies:

$$A_{16} = A_{26} = 0$$

If laminates consistes of
an odd number of plies:

Laminate is symmetric,
$[B] = 0$, and

$$A_{16}, A_{26}, D_{16}, D_{26} \approx 0$$

Antisymmetric Laminates

$$[45/60/-60/-45]$$

45

60

-60

-45

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & B_{26} \\ 0 & 0 & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & 0 \\ B_{16} & B_{26} & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

Balanced Laminates

30

40

-30

30

-30

-40

$$[30/40/-30/30/-30/-40]$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & B_{26} \\ 0 & 0 & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

Quasi-Isotropic Laminates

$$[A]=\begin{bmatrix} \frac{E}{1\text{-}v^2} & \frac{vE}{1\text{-}v^2} & 0 \\ \frac{vE}{1\text{-}v^2} & \frac{E}{1\text{-}v^2} & 0 \\ 0 & 0 & \frac{E}{2(1+v)} \end{bmatrix} h, \qquad [B]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$[D]=\begin{bmatrix} \frac{E}{12(1\text{-}v^2)} & \frac{vE}{12(1\text{-}v^2)} & 0 \\ \frac{vE}{12(1\text{-}v^2)} & \frac{E}{12(1\text{-}v^2)} & 0 \\ 0 & 0 & \frac{E}{24(1+v)} \end{bmatrix} h^3$$

Quasi-Isotropic Laminate

$$A_{11} = A_{22},$$

$$A_{16} = A_{26} = 0, \text{ and}$$

$$A_{66} = \frac{A_{11} - A_{12}}{2}$$

Examples:

$$[0/\pm 60],$$

$$[0/\pm 45/90]_S, \text{ and}$$

$$[0/36/72/-36/-72]$$

END