EML 4230 Introduction to Composite Materials

Chapter 5 Design and Analysis of a Laminate
Ply by Ply Failure

Dr. Autar Kaw
Department of Mechanical Engineering
University of South Florida, Tampa, FL 33620

Courtesy of the Textbook
Mechanics of Composite Materials by Kaw

UNIVERSITY OF SOUTH FLORIDA

Ply by Ply Failure of a Laminate

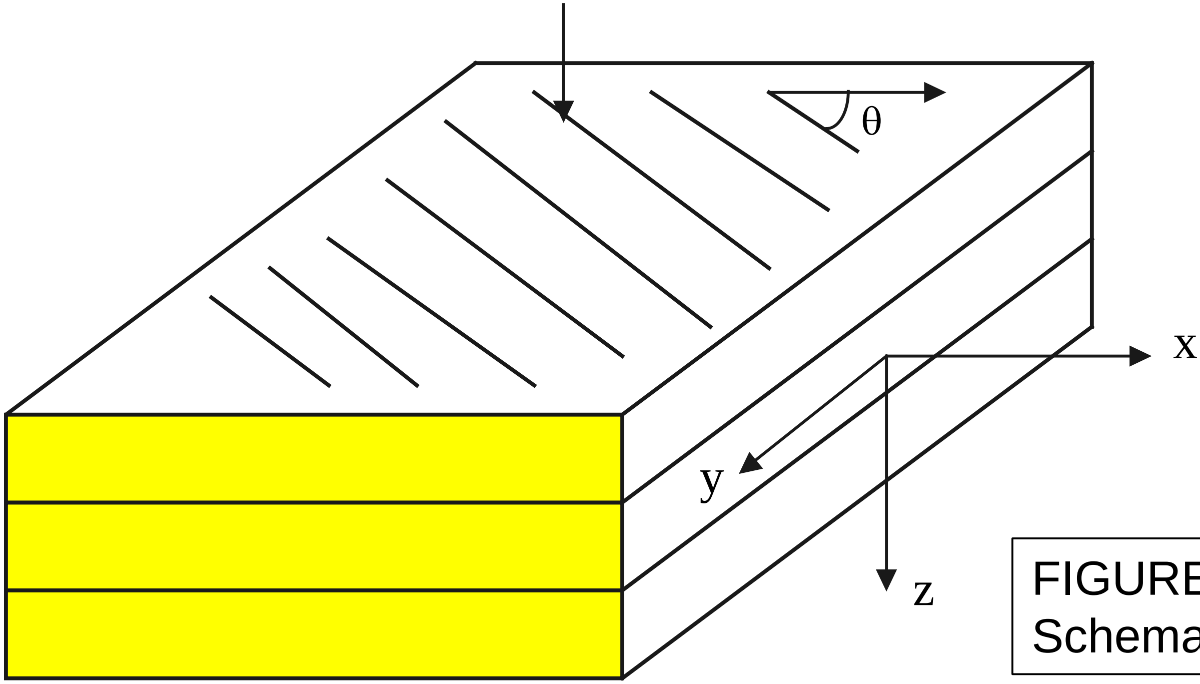
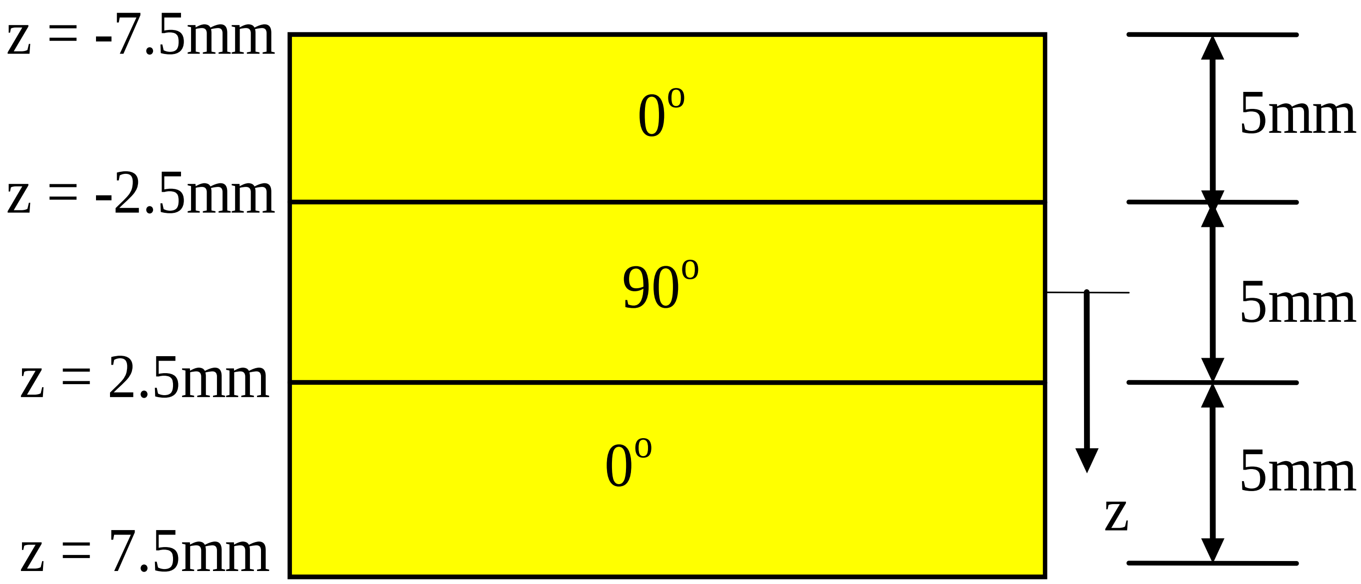


FIGURE 4.1
Schematic of a lamina

If a ply fails, has the whole laminate failed? Maybe. When the first ply fails, you may get catastrophic failure or you may be able to still apply for more load.

Problem Statement

The only load applied is a tensile normal load in the *x*-direction, that is, the direction parallel to the fibers in the 0° ply. Find the ply-by-ply failure load for a [0/90/0] Graphite/Epoxy laminate. Assume the thickness of each ply is *5 mm* and use properties of unidirectional Graphite/Epoxy lamina from Table 2.1.



Solution

Assume we apply N_x=1 N/m

Mid-plane strains (midplane curvature are zero)

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} 5.353 \times 10^{-10} \\ -2.297 \times 10^{-11} \\ 0 \end{bmatrix}$$

Local Stresses

Ply#	*Position*	σ_1	σ_2	τ_{12}
1 (0^0)	Top Middle Bottom	9.726×10^1 9.726×10^1 9.726×10^1	1.313×10^0 1.313×10^0 1.313×10^0	0.0 0.0 0.0
2 (90^0)	Top Middle Bottom	-2.626×10^0 -2.626×10^0 -2.626×10^0	5.472×10^0 5.472×10^0 5.472×10^0	0.0 0.0 0.0
3 (0^0)	Top Middle Bottom	9.726×10^1 9.726×10^1 9.726×10^1	1.313×10^0 1.313×10^0 1.313×10^0	0.0 0.0 0.0

Strength Ratios

Ply#	Position	Maximum Strain	Tsai-Wu
1 (0^0)	Top Middle Bottom	1.548×10^7 (1T) 1.548×10^7 (1T) 1.548×10^7 (1T)	1.339×10^7 1.339×10^7 1.339×10^7
2 (90^0)	Top Middle Bottom	7.254×10^6 (2T) 7.254×10^6 (2T) 7.254×10^6 (2T)	7.277×10^6 7.277×10^6 7.277×10^6
3 (0^0)	Top Middle Bottom	1.548×10^7 (1T) 1.548×10^7 (1T) 1.548×10^7 (1T)	1.339×10^7 1.339×10^7 1.339×10^7

What is the N_x that can be applied?

N_x=(1 N/m)x(7.277×10^6) =7.277×10^6 N/m

$N_x/h = 7.277 \times 10^6/0.015 = 485.1$ MPa

$\varepsilon^o{}_x = (5.353 \times 10^{-10}) \times (7.277 \times 10^6) = 0.003895$

Will the laminate take more load?

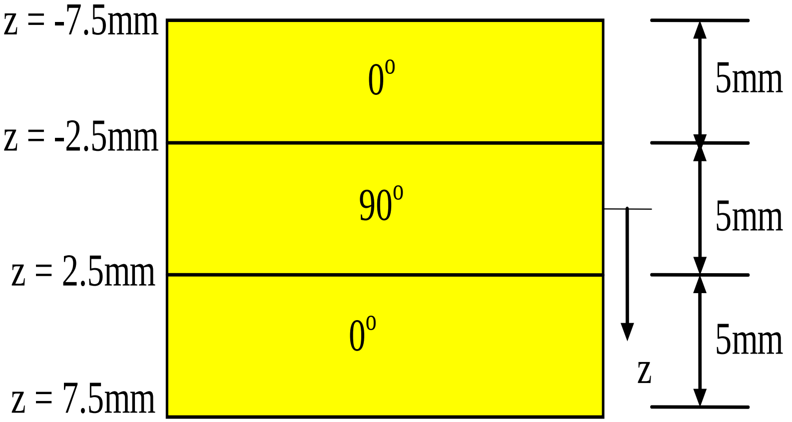
Degrade Ply#2

Q for the undamaged plies

$$[Q] = \begin{bmatrix} 181.8 & 2.897 & 0 \\ 2.897 & 10.35 & 0 \\ 0 & 0 & 7.17 \end{bmatrix} GPa$$

Q for the damaged plies

$$[Q] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{GPa}$$



Solution

Assume we apply N_x=1 N/m

Mid-plane strains (midplane curvatures are zero)

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} 5.525 \times 10^{-10} \\ -1.547 \times 10^{-10} \\ 0 \end{bmatrix}$$

Local Stresses

Ply#	Position	σ_1	σ_2	τ_{12}
1 (0^0)	Top Middle Bottom	1.0000×10^2 1.0000×10^2 1.0000×10^2	0.0 0.0 0.0	0.0 0.0 0.0
2 (90^0)	Top Middle Bottom	--- --- ---	--- --- ---	--- --- ---
3 (0^0)	Top Middle Bottom	1.0000×10^2 1.0000×10^2 1.0000×10^2	0.0 0.0 0.0	0.0 0.0 0.0

Strength Ratios

Ply#	Position	Max Strain	Tsai-Wu
1 (0^0)	Top Middle Bottom	1.5000×10^7 (1T) 1.5000×10^7(1T) 1.5000×10^7(1T)	1.5000×10^7 1.5000×10^7 1.5000×10^7
2 (90^0)	Top Middle Bottom	--- --- ---	--- --- ---
3 (0^0)	Top Middle Bottom	1.5000×10^7(1T) 1.5000×10^7(1T) 1.5000×10^7(1T)	1.5000×10^7 1.5000×10^7 1.5000×10^7

What is the N_x that can be applied?

N_x=(1 N/m)x(1.5x10^7) =1.5x10^7 N/m

N_x/h = 1.5x10^7/0.015 = 1000 MPa

$\varepsilon^o{}_x$ = (5.525x10^{-10})x(1.5x10^7) =0.008288

Stress-Strain Plot

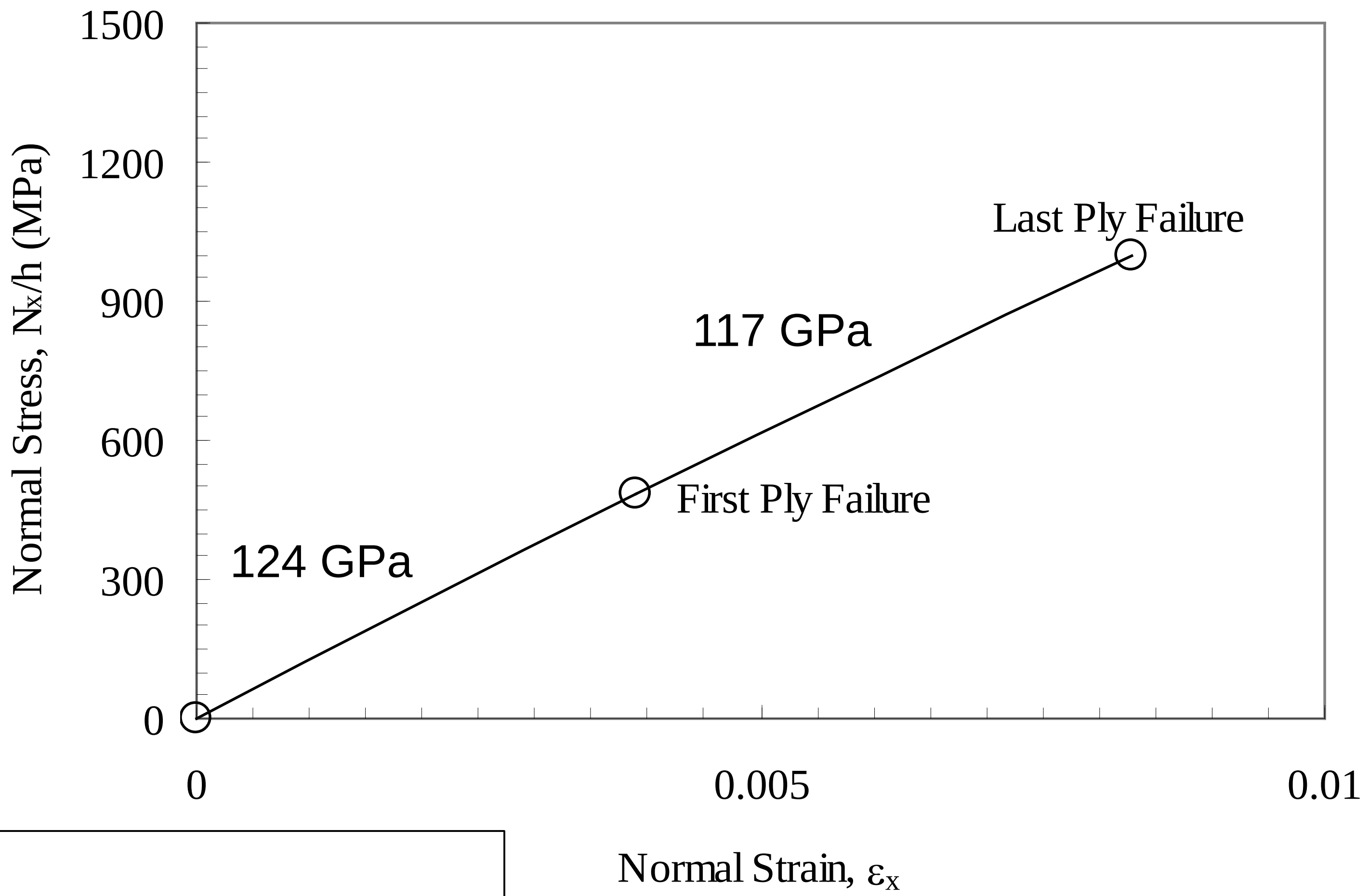


FIGURE 5.1
Stress-strain curve showing ply-by-ply failure of a laminated composite

END