EML 4230 Introduction to Composite Materials

Chapter 5 Design and Analysis of a Laminate
The Drive Shaft Problem

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Courtesy of the Textbook
Mechanics of Composite Materials by Kaw

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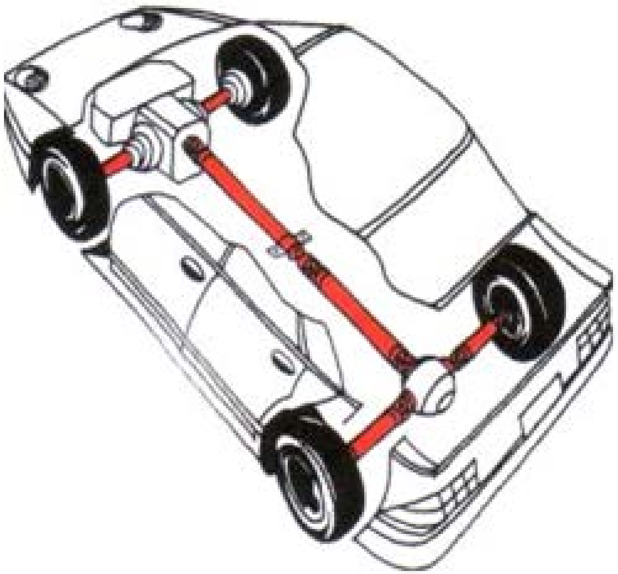
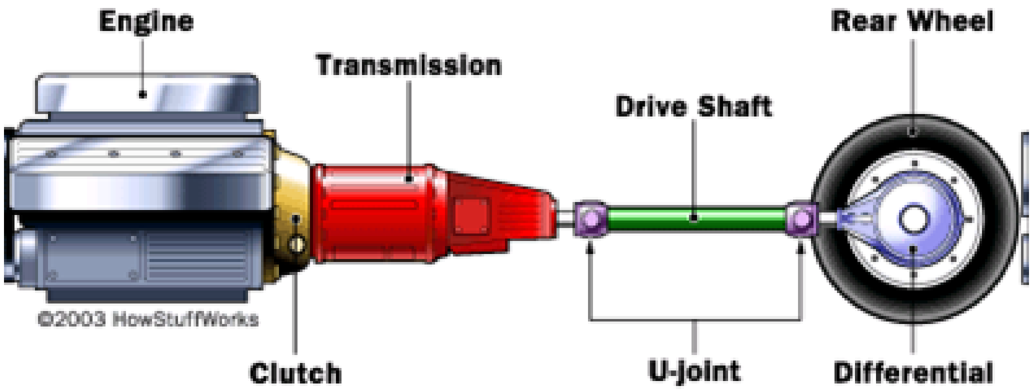
Problem Statement





A drive shaft for a Chevy Pickup truck is made of steel. Check whether replacing it with a drive shaft made of composite materials will save weight?

Design of a Composite Drive Shaft

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The Nissan 350Z uses a composite driveshaft for added power.

Why composite materials?

- <u>Light weight</u> — reduces energy consumption; increases amount of power transmitted to the wheels. About 17-22% of engine power is lost to rotating the mass of the drive train.

- <u>Fatigue resistant</u> — durable life.

- <u>Non-corrosive</u> — reduced maintenance cost and increased life.

- <u>Single piece</u> — reduces manufacturing cost.

Why composite materials?

- <u>Prevent injuries</u> – the composite drive shaft "broom" on failure

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Problem Description

Design Constraints

1. Maximum horsepower= 175 HP @4200 rpm
2. Maximum torque = 265 lb-ft @2800 rpm
3. Factor of safety = 3
3. Outside radius = 1.75 in
4. Length = 43.5 in

Torque in Drive Shaft

In first gear

- the speed is 2800 rpm (46.67 rev/s)
- assume ground speed of 23 mph (405 in/sec)

Diameter of tire = 27 in

Revolutions of tire =405/[π(27)] = 4.78 rev/s

Differential ratio = 3.42

Drive shaft speed = 4.78 x 3.42 = 16.35 rev/s

Torque in drive shaft = (265x46.7)/16.35 = 755 lb-ft

Maximum Frequency of Shaft

Maximum Speed = 100 mph (1760 in/sec)

Diameter of tire = 27 in

Revolutions of tire =1760/[π(27)] = 20.74 rev/s

Differential ratio = 3.42

Drive shaft speed = 20.74x3.42 = 71Hz

Design Parameters

- Torque Resistance.
  - Should carry load without failure
- Not rotate close to natural frequency.
  - Need high natural frequency otherwise whirling may take place
- Buckling Resistance.
  - May buckle before failing

Steel Shaft – Torque Resistance

$$\frac{\tau_{\max}}{FS} = \frac{Tc}{J}$$

Shear Strength, τ_{max} = 25 Ksi

Torque, T = 755 lb-ft

Factor of Safety, FS = 3

Outer Radius, c = 1.75 in

Polar moment of area, J = $\frac{\pi}{2} * \left(1.75^4 - c_{in}^4\right)$

c_{in} = 1.69 in

t = 1.75-1.69 = 0.06 in = 1/16 in

Steel Shaft - Natural Frequency

$$f_n = \frac{\pi}{2}\sqrt{\frac{gEI}{WL^4}}$$

Acceleration due to gravity, g= 32.2 ft/s²
Young's modulus, E = 30 Msi
Weight per unit length, W = 0.19011 lbf/in
Length, L = 43.5 in
Second Moment of Area, I = $\frac{\pi}{4}\left(1.75^4 - \left(1.75 - \frac{1}{16}\right)^4\right) = 0.9973\,\text{in}^4.$

$$f_n = 204\ \text{Hz}$$

Meets minimum of 71.1Hz

Steel Shaft - Torsional Buckling

$$T = 0.272 * 2\pi r_m^2 t E \left(\frac{t}{r_m}\right)^{3/2}$$

Mean radius, r_m = 1.6875 in
Thickness, t = 1/16 in
Young's modulus, E = 30 Msi
Critical Buckling Load, T = 5519 lb-ft

Meets minimum of 755 lb-ft

Designing with a composite

Load calculations for PROMAL

$$N_{xy} = \frac{T}{2\pi r_m^2} \quad \text{(Why)}$$

$$T = 755 \text{ lb-ft}$$

$$r = 1.75 \text{ in}$$

$$N_{xy} = 470.8 \text{ lb / in}$$

Neglecting centrifugal force contribution

Composite Shaft-Torque Resistance

Inputs to PROMAL:
- Glass/Epoxy from Table 2.1
- Lamina Thickness = 0.0049213 in
- Stacking Sequence: $(45/-45/45/-45/45)_s$
- Load N_{xy} = 470.8 lb / in

Outputs of PROMAL:
- Smallest Strength Ratio = 1.52 (not safe)

Thickness of Laminate:
- h = 0.0049213*10 = 0.04921 in

Composite Shaft - Natural Frequency

$$f_n = \frac{\pi}{2}\sqrt{\frac{gE_xI}{WL^4}}$$

$g = 32.2$ ft/s^2

$E_x = 1.814$ Msi

$I = 0.7942$ in^4

$W = 0.03438$ lbf/in

$L = 43.5$ in

Hence

$f_n = 105.6$ Hz (meets minimum 71.1 Hz)

Composite Shaft - Torsional Buckling

$$T = 0.272 * 2\pi r_m^2 t \left(E_x E_y^3\right)^{1/4} \left(\frac{t}{r_m}\right)^{3/2}$$

r_m = 1.75 -0.04921/2= 1.72539 in

t = 0.04921 in
E_x = 1.814 Msi
E_y = 1.814 Msi

T = 183 lb-ft (does not meet 755 lb-ft torque)

Comparison of Mass

	Steel	Glass/Epoxy (not acceptable design)
Specific Grav	7.850	1.785
Inside radius	1.6875"	1.7008"
Outside radius	1.75"	1.75"
Length	43.5"	43.5"
Mass	8.322 lbm	1.496 lbm

END