EML3041 Computational Methods Fall 2023 Week: Four from Sep 11 to Sep 15 Session: One

Answer the free-response Question#1 starting on a fresh sheet of paper. Solve the problem as if you were submitting them for a test. Put your last name, first name, and your first letter of last name in bold on top of the page.

1) The root of the equation f(x) = 0 is found by using the Newton-Raphson method. The initial estimate of the root is $x_0 = 3$ and f(3) = 5. The line tangent to the function f(x) at x = 3 makes an angle of 52.4° with the *x*-axis. What is the next estimate of the root x_1 ? Sketch the solution and find the answer.

Answer: -0.8505

2) For an equation like $x^2 = 0$, a root exists at x = 0. Why cannot the bisection method be adopted to solve this equation despite the root existing at x = 0? Write a sentence or so and sketch with labels to justify your answer.

3) The ideal gas law is given by

pv = RT

where p is the pressure, v is the specific volume, R is the universal gas constant, and T is the absolute temperature? This equation is only accurate for a limited range of pressure and temperature. Vander Waals came up with an equation that was accurate for larger range of pressure and temperature given by

$$(p + \frac{a}{v^2})(v - b) = RT$$

where *a* and *b* are empirical constants dependent on a particular gas.

Given the value of R = 0.08, a = 3.592, b = 0.04267, p = 10 and T = 300 (assume all units are consistent), if one is going to find the specific volume, v, for the above values.

- a) How would you write the nonlinear equation as a cubic polynomial equation for v such as to solve by bisection method? Make the coefficient to be 1 of the highest power of v. You only have to write the equation and not solve it.
- b) From the information given in the problem, and if you were using Newton-Raphson method, what would be a good initial estimate of the root?

Answer: a) $v^3 - 2.443v^2 + 0.3592v - 0.01533 = 0$ b) Answer intentionally not given