Simultaneous Linear Equations and Matrix Algebra

Major: All Engineering Majors

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Transforming Numerical Methods Education for STEM Undergraduates

1

How do you find what you need to study and do for the course? (CHECK ALL THOSE APPLY) 0

What is handed out and said in class	0%
Weekly CANVAS announcements	0%
Go to CANVAS modules	0%
Go to Canvas syllabus link and see what is due	0%

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How many of the problems did you do from the end of chapter problem sets in the textbook to prepare for Test 1? 0

Almost 100%	0%
About 75%	0%
About 50%	0%
About 25%	0%
What is this problem set you talk about?	0%

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Which of the following prerequisite concepts of matrix algebra were you exposed to before you signed up for the Computational Methods course (CHOOSE ALL THAT APPLY) 0

What is a matrix	0%
Matrix Addition	0%
Matrix Multiplication	0%
Matrix Inverse	0%
Determinant	0%

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The ratio of net worth of Rihanna to net worth of A$AP Rocky is approximately

- 1 — 0%
- 7 — 0%
- 70 — 0%
- 170 — 0%

5

Given $[C] = \begin{bmatrix} 13 & 11 \\ 19 & 23 \end{bmatrix} \begin{bmatrix} 13 & 17 & 19 \\ 23 & 7 & 29 \\ 31 & 37 & 41 \end{bmatrix}$**, what is the value of** c_{21}**?**

- cannot be determined
- 298
- 534
- 776

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Velocity vs Time

The following data is given for the velocity of the rocket as a function of time. To find the velocity at $t = 21s$, you are asked to use a quadratic polynomial $v(t) = at^2 + bt + c$ to approximate the velocity profile.

t (s)	0	14	15	20	30	35
v (m/s)	0	227	362	517	602	901

- $\begin{bmatrix} 196 & 14 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 227 \\ 362 \\ 517 \end{bmatrix}$
- $\begin{bmatrix} 225 & 15 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 362 \\ 517 \\ 602 \end{bmatrix}$

SEE MORE

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Physical Problems

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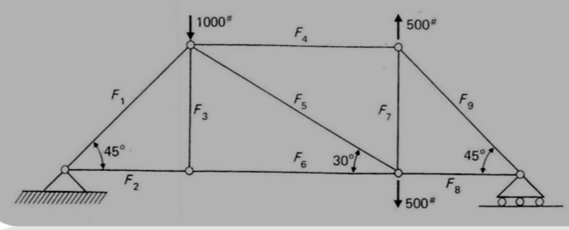
Truss Problem

$$\begin{bmatrix} 0.7071 & 0 & 0 & -1 & -0.8660 & 0 & 0 & 0 & 0 \\ 0.7071 & 0 & 1 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0.7071 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -0.7071 \\ 0 & 0 & 0 & 0 & 0.8660 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.7071 \end{bmatrix} F = \begin{bmatrix} 0 \\ -1000 \\ 0 \\ 0 \\ 500 \\ 0 \\ 0 \\ -500 \\ 0 \end{bmatrix}$$



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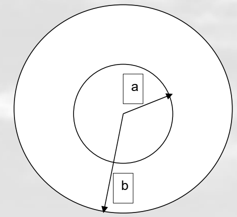
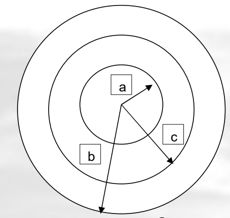
Pressure vessel problem



$$u_1 = c_1 r + \frac{c_2}{r}$$



$$u_1 = c_1 r + \frac{c_2}{r}$$
$$u_2 = c_3 r + \frac{c_4}{r}$$

$$\begin{bmatrix} 4.2857 \times 10^7 & -9.2307 \times 10^5 & 0 & 0 \\ 4.2857 \times 10^7 & -5.4619 \times 10^5 & -4.2857 \times 10^7 & 5.4619 \times 10^5 \\ -6.5 & -0.15384 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857 \times 10^7 & -3.6057 \times 10^5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887 \times 10^3 \\ 0 \\ 0.007 \\ 0 \end{bmatrix}$$

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Polynomial Regression

We are to fit the data

$$(T_1, \alpha_1), (T_2, \alpha_2), \ldots, (T_{n-1}, \alpha_{n-1}), (T_n, \alpha_n)$$

to the second order polynomial regression model

$$\alpha = a_0 + a_1 T + a_2 T^2$$

$$\begin{bmatrix} n & \left(\sum_{i=1}^{n} T_i\right) & \left(\sum_{i=1}^{n} T_i^2\right) \\ \left(\sum_{i=1}^{n} T_i\right) & \left(\sum_{i=1}^{n} T_i^2\right) & \left(\sum_{i=1}^{n} T_i^3\right) \\ \left(\sum_{i=1}^{n} T_i^2\right) & \left(\sum_{i=1}^{n} T_i^3\right) & \left(\sum_{i=1}^{n} T_i^4\right) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} \alpha_i \\ \sum_{i=1}^{n} T_i \alpha_i \\ \sum_{i=1}^{n} T_i^2 \alpha_i \end{bmatrix}$$

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END

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Inverse of a matrix

Definition of Inverse

A matrix [B] is inverse of [A] if [B][A]=[I].

Application of Inverse

$$[A][X] = [C]$$

$$[A]^{-1}[A][X] = [A]^{-1}[C]$$

$$[I][X] = [A]^{-1}[C]$$

$$[X] = [A]^{-1}[C]$$

Find Inverse of Matrix

Find inverse of

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \qquad [A]^{-1} = \begin{bmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \\ a'_{31} & a'_{32} & a'_{33} \end{bmatrix}$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \\ a'_{31} & a'_{32} & a'_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Setting up equations to find inverse

$$\begin{bmatrix}25 & 5 & 1\\64 & 8 & 1\\144 & 12 & 1\end{bmatrix}\begin{bmatrix}a'_{11} & a'_{12} & a'_{13}\\a'_{21} & a'_{22} & a'_{23}\\a'_{31} & a'_{32} & a'_{33}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}25 & 5 & 1\\64 & 8 & 1\\144 & 12 & 1\end{bmatrix}\begin{bmatrix}a'_{11}\\a'_{21}\\a'_{31}\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix} \longrightarrow \begin{bmatrix}a'_{11}\\a'_{21}\\a'_{31}\end{bmatrix}=\begin{bmatrix}0.04762\\-0.9524\\4.571\end{bmatrix}$$

$$\begin{bmatrix}25 & 5 & 1\\64 & 8 & 1\\144 & 12 & 1\end{bmatrix}\begin{bmatrix}a'_{12}\\a'_{22}\\a'_{32}\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix} \longrightarrow \begin{bmatrix}a'_{12}\\a'_{22}\\a'_{32}\end{bmatrix}=\begin{bmatrix}-0.08333\\1.417\\-5.000\end{bmatrix}$$

$$\begin{bmatrix}25 & 5 & 1\\64 & 8 & 1\\144 & 12 & 1\end{bmatrix}\begin{bmatrix}a'_{13}\\a'_{23}\\a'_{33}\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix} \longrightarrow \begin{bmatrix}a'_{13}\\a'_{23}\\a'_{33}\end{bmatrix}=\begin{bmatrix}0.03571\\-0.4643\\1.429\end{bmatrix}$$

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Putting the solutions in matrix

$$\begin{bmatrix}a'_{11}\\a'_{21}\\a'_{31}\end{bmatrix}=\begin{bmatrix}0.04762\\-0.9524\\4.571\end{bmatrix} \quad \begin{bmatrix}a'_{12}\\a'_{22}\\a'_{32}\end{bmatrix}=\begin{bmatrix}-0.08333\\1.417\\-5.000\end{bmatrix} \quad \begin{bmatrix}a'_{13}\\a'_{23}\\a'_{33}\end{bmatrix}=\begin{bmatrix}0.03571\\-0.4643\\1.429\end{bmatrix}$$

$$[A]^{-1}=\begin{bmatrix}0.04762 & -0.08333 & 0.03571\\-0.9524 & 1.417 & -0.4643\\4.571 & -5.000 & 1.429\end{bmatrix}$$

$$\begin{bmatrix}25 & 5 & 1\\64 & 8 & 1\\144 & 12 & 1\end{bmatrix}\begin{bmatrix}0.04762 & -0.08333 & 0.03571\\-0.9524 & 1.417 & -0.4643\\4.571 & -5.000 & 1.429\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

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If $[A]_{n\times n}$ is the inverse of $[B]_{n\times n}$, then the following statements are true (Check all that apply)

0

- [B] is non-singular
- [B][A]=[I]
- [B] is inverse of [A}
- [A] is singular
- [A][B]=[I]

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Solve a Set of Equations Using Inverse

$$[A][X] = [C] \Longrightarrow [X] = [A]^{-1}[C]$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0.04762 & -0.08333 & 0.03571 \\ -0.9524 & 1.417 & -0.4643 \\ 4.571 & -5.000 & 1.429 \end{bmatrix} \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2917 \\ 19.67 \\ 1.15 \end{bmatrix}$$

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Naive Gauss Elimination Synopsis

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Naive Gaussian Elimination

A method to solve simultaneous linear equations of the form [A][X]=[C]

Two parts
1. Forward Elimination
2. Back Substitution

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Forward Elimination Part

The goal of forward elimination is to transform the coefficient matrix into an upper triangular matrix

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.21 \\ 0.735 \end{bmatrix}$$

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Back Substitution

Solve each equation starting from the last equation

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.21 \\ 0.735 \end{bmatrix}$$

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The goal of forward elimination steps in the Naive Gauss elimination method is to reduce the coefficient matrix to a (an) ________ matrix.

- Upper triangular
- Diagonal
- Lower triangular
- Identity

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Determinant of a Square Matrix Using Naïve Gauss Elimination

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Theorem of Determinants

If a multiple of one row of $[A]_{n\times n}$ is added or subtracted to another row of $[A]_{n\times n}$ to result in $[B]_{n\times n}$ then $\det(A) = \det(B)$

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Theorem of Determinants

The determinant of an upper triangular, lower triangular or diagonal matrix $[A]_{n\times n}$ is given by

$$\det(\mathrm{A}) = a_{11} \times a_{22} \times \ldots \times a_{ii} \times \ldots \times a_{nn}$$

$$= \prod_{i=1}^{n} a_{ii}$$

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Forward Elimination of a Square Matrix

Use forward elimination part to transform $[A]_{n\times n}$ to an upper triangular matrix, $[U]_{n\times n}$.

$$[A]_{n\times n} \rightarrow [U]_{n\times n}$$

$$\det(A) = \det(U)$$

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Using Naive Gaussian Elimination method, find the determinant of the following square matrix.

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

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Finding the Determinant

After forward elimination part

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

$$\begin{aligned} \det(\mathrm{A}) &= u_{11} \times u_{22} \times u_{33} \\ &= 25 \times (-4.8) \times 0.7 \\ &= -84.00 \end{aligned}$$

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What does det(A)=0 and det(A)≠0 mean for [A][X]=[C]

$\det(A) = 0$ implies [A][X]=[C] has no solution or infinite solutions

$\det(A) \neq 0$ implies [A][X]=[C] has a unique solution.

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The following system of equations

$x + y = 2$

$6x + 6y = 12$

has ___________ solution(s)

0

- no
- one
- more than one but a finite number of
- infinite

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If the determinant of a square matrix [A] is zero, then the following are (is) true (check all that apply)

0

- [A] does not have an inverse
- [A] has an inverse
- [A] is singular
- if [A][X]=[C] is a set of simultaneous linear equations, then [X] is unique
- if [A][X]=[C] is a set of simultaneous linear equations, then [X] is not unique

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